

DESIGN TOOL FOR PREDICTING THERMAL SYNCHRONOUS INSTABILITY

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INTRODUCTION

Thermal synchronous instability phenomena in rotor-bearing systems, also known as the Morton effect, can be generated by the temperature difference developing across the fluid film journal as a result of the viscous shearing within the lubricant of the bearing. This thermal instability phenomenon has attracted more attention recently both in the rotating machinery industry and research institutes [1-10].

The Morton effect can most easily occur when the journal is executing a synchronous orbit around the bearing center. This centered orbit causes one portion of the journal surface to always be at the minimum film thickness, while a diametrically opposite section of the journal surface is always at the maximum film thickness. Lower film thickness areas are generally associated with higher viscous shear stresses which produce higher temperatures. As a result, a hot spot will develop on the journal surface exposed to the minimum film thickness region and a cold spot will be formed on the surface at maximum film thickness. **This leads to a temperature gradient developing across the journal.** Such a gradient may create a thermal imbalance from the thermal bending that occurs across the shaft at the journal location. Under these conditions the bent shaft will decrease the bearing clearance and elevate the thermal gradient. The increased temperature gradient will then initiate more thermal bending. These actions describe a positive feedback mechanism which will drive the system unstable, producing very large synchronous response.

Most of the prior models of the Morton Effect have involved complex stability analyses [6, 8, 9]. In these models, an initial deformed rotor configuration is assumed and is then used as an input in a dynamic model of the rotor system. This dynamic model produces a synchronous orbit which causes a circumferential temperature distribution to develop on portions of the shaft within the bearings. A thermal bend, which depends on the temperature distribution and the rotor thermal characteristics, is subsequently initiated. This thermal bend affects the rotor dynamics and gives rise to the positive feedback behavior. The stability of the resulting feedback transfer function can then be analyzed by using Nyquist plots or another equivalent method.

In real machines satisfactory performance is established by using specified vibration, or unbalance threshold levels that are given in standards published by various organizations. One method of specifying such levels is to restrict the maximum allowable centrifugal force that acts on the machine. This restriction then limits the allowable imbalance for each speed value. Such a criterion determines whether the rotor will be “stable” or “unstable” during operation, i.e. if the total mechanical and thermal unbalance level exceeds the threshold level given by the allowable imbalance then the rotor will be “unstable.” This notion of practical stability will be utilized in the model presented in this document.

The current model for the Morton effect first requires an estimate for the initial mechanical imbalance in the rotor. This imbalance is then put into the dynamic rotor model to obtain the synchronous orbit. A fluid film bearing model is then used to establish the temperature

distribution which leads to the thermal imbalance. Finally, an unbalance threshold criterion is used to predict the system stability.

THEORETICAL MODEL

The Morton effect induced thermal instability in overhung rotors is focused in the current model. Figure 1 is the schematic of the rotor with overhung mass. H and C in the figure denote hot spot and cold spot, respectively. Y_d represents the thermal deflection at the overhung equivalent mass center of gravity.

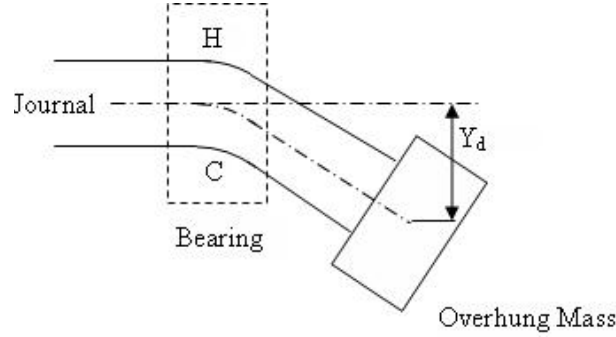


Figure 1. ROTOR WITH OVERHUNG MASS

The proposed solution method requires the calculation of the bearing journal response for an initial mechanical imbalance. A nominal initial mechanical imbalance U_m , has been defined as the imbalance created from a centrifugal force equal to 10% of the total static rotor weight W . This ratio selection is based on the experimental observation representing the most typical situation in reality. U_m will be calculated for the rotor running at its maximum continuous operating speed, ω_{MCOS} , which can be mathematically defined as:

$$U_m = \frac{0.1W}{\omega_{MCOS}^2} \quad (1)$$

This imbalance will be assumed to act at an angle of zero degrees with respect to a coordinate system on the rotor, and will be located at the center-of-gravity of the overhung mass.

The following procedure is necessary to obtain the thermal imbalance, U_t . First, the film thickness for the specific bearing type is determined. Then solve for the static equilibrium position of the journal and predict the steady state orbit at the journal for an assumed imbalance [14-16]. Next, the hot and cold spot regions for the journal must be determined and the temperature distribution on the journal can then be obtained. Finally, evaluate the rotor thermal bend and calculate the thermal imbalance resulting from the average hot spot temperature gradient using the following equation [1, 2, 7, 11, 12 , 13]

$$U_t = M_d Y_d \quad (2)$$

where M_d is the overhung mass.

In order to locate hot and cold spot regions, three methods have been used (Figure 2):
Method #1: hot spot is defined as the closest point to the bearing sleeve at time instant $t=0$. In this method, it is also assumed that O_{j0} , O_j and hot spot H all lie on the same straight line;
Method #2: hot spot is defined as the closest point to the bearing sleeve at time instant $t=t_m$. In this configuration, the hot spot H is located on the straight line O_b and O_j ; Method #3: hot spot is defined as the point which has the maximum temperature in the orbit. The corresponding equations can be derived to obtain the hot spot and cold spot based on the specific method [7].

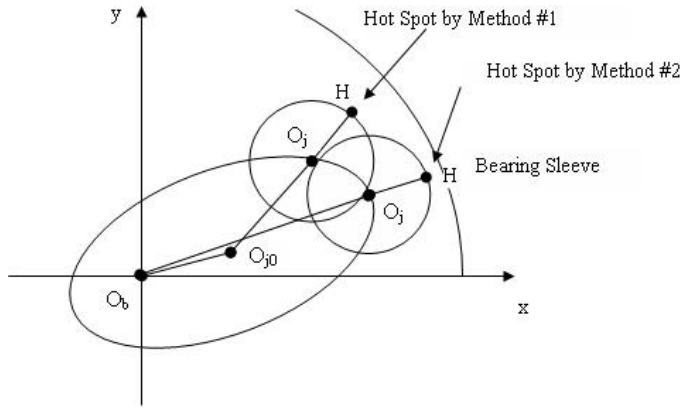


Figure 2. HOT AND COLD SPOT REGIONS

Obviously, for the plain journal bearing, the hot spot and cold spots using the Method #2 and #3 should be the same and results conducted show no big difference even using Method #1. However, this conclusion cannot be applied to the tilting pad journal bearing applications, due to the complexity of this kind of bearing configuration. The Method #2 is recommended for both plain journal bearing and tilting pad journal bearing applications based on experience from previous research results [7].

The resultant imbalance U from U_m and U_t can be represented as shown in Figure 3. The mechanical and thermal imbalances can be combined to produce the resultant imbalance, which can be represented as follows:

$$U = \sqrt{U_t^2 + U_m^2 - 2U_t U_m \cos(\omega t - \theta_{CH})} \quad (3)$$

The angle $(\omega t - \theta_{CH})$ is actually calculated by averaging similar angles at several different dynamic points in the synchronous orbit, where θ_{CH} is the phase angle of the vector connecting

hot spot to cold spot. Then φ , the phase angle difference between thermal and mechanical imbalances can also be obtained.

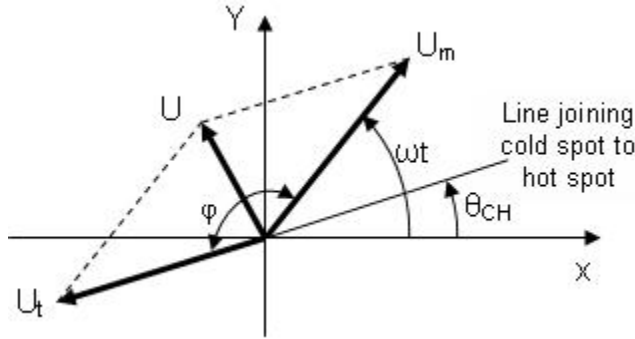


Figure 3. MECHANICAL, THERMAL, AND
RESULTANT IMBALANCES

To complete the system analysis, the final step requires that a threshold imbalance, U_{thr} , be defined. This total imbalance was assumed originally to be 50% larger than the applied mechanical imbalance. Hence the threshold imbalance force is equal to 15% of the rotor weight and can be mathematically expressed as:

$$U_{thr} = \frac{0.15 W}{\omega^2} \quad (4)$$

where W = rotor weight and ω = angular journal design speed.

The rotor is considered to be unstable whenever U exceeds U_{thr} . As a result, the threshold speed for instability, ω_{thr} occurs when $U = U_{thr}$. This instability criterion can be obtained graphically from the intersection of the U vs. ω and the U_{thr} vs. ω curves and it determines the onset of the Morton effect in the rotor system.

The original version of the thermal instability design program was based on the speed dependent threshold level which gives a quadratic curve, decreasing as speed increases and increasing as speed decreases. This threshold works fine as long as the speed region of interest is very close to MCOS. A problem can occur if the top speed is well beyond the MCOS, since at some speed the threshold would be crossed, even if no thermal increase in imbalance occurs. This situation did not occur in cases evaluated previously, but false predictions of a potential problem above MCOS could be made by designers using the program without proper understanding of the theory.

To prevent this false positive from occurring, it has been recently decided to redefine the imbalance threshold, U_{thr} . The total imbalance was assumed to be 50% larger than the original mechanical imbalance. Hence the threshold imbalance force is equal to 15% of the rotor weight and can be mathematically expressed as:

$$U_{thr} = \frac{0.15W}{\omega_{MCOS}^2} \quad (5)$$

where W = rotor weight and ω = angular journal design speed.

The rotor is considered to be unstable whenever U exceeds U_{thr} . As a result, the threshold speed for instability, ω_{thr} occurs when $U = U_{thr}$. This instability criterion can be obtained graphically from the intersection of the U vs. ω and the U_{thr} vs. ω curves. The difference now, is that the threshold imbalance is a constant amount and is not speed dependent with this new equation for the threshold imbalance level. The problem with the false instability at higher speed is eliminated with this new definition of the threshold level.

COMPUTER PROGRAM

Based on the model presented above a user friendly program was developed for overhung rotors [14]. The current document will only address the overhung condition and the current program only considers rotors with overhung sections and **bearing type can only be tilting pad**. The stability plot gives both the resultant imbalance U vs. ω curve and the threshold imbalance U_{thr} vs. ω curve within the entire range of running speed. The intersection of two curves determines the onset of Morton effect induced thermal instability.

The prior analysis results have required an extensive amount of information from both forced response analysis and also the bearing analysis programs, in addition to other parameters unique to the thermal instability evaluation process. This was a great issue for most designers as the typical time to complete the data collection was a day or more. If the standard rotor analysis for forced response is available, the new program will be able to produce the stability plot for one rotor overhang in less than 5 minutes, since almost all of the labor intensive portions have been automated.

The overhang mass and length of the overhang are key parameters that can influence the results. Many times the end results are unknown and the work has been almost totally proprietary.

The following example rotors will give a more detailed explanation of the new design program procedure. It is assumed that the Dyrobes computer analysis program [15,16] is available and the user is familiar with the basic data input and method for a standard rotor bearing analysis including the use of the tilting pad analysis, BePerf. A new bearing option type 15 is used where the user supplies only the **TDI file name and path** inside the rotor model instead of the brg bearing file, hence reducing the work and time of acting on a bearing change. In the analysis option a new type 15 has been added to allow a Morton thermal analysis of a single rotor or a train of rotors by providing a brief description of which overhang bearing is selected and a few other details. The additional parameters required for Morton Analysis will be discussed in detail in the 3rd example Case 4 train analysis.

Table 1 gives a brief summary of some of the machines examined over the past 21 years. It is easy to consider any problem that is not easily explained to be a Morton issue. The current analysis was used to prove a serious vibration was not due to a Morton stability issue but rather a long coupling thermal problem [17, 18, 19].

Table 1 SUMMARY RESULTS FOR PRIOR STUDIES

No	TYPE	Analysis	Field
1	turbocharger	fix found	stable
2	compressor	stable	stable
3	compressor	Weak instability	-----
4	compressor	instability	Re-design
5	compressor	Strong instability	confirmed
6	compressor	instability	confirmed
7	compressor	weak instability	confirmed
8	compressor	stable	stable
9	expander	stable	stable
10	gear pinion	unstable	unstable
11	expander rigid ped	unstable	-----
	expander flex ped	stable	stable
12	4 stg compressor	unstable	-----
13	turbine drive	unstable blind Solution	Unstable stable
14	compressor	Unstable above MCOS	-----
15	pinion	Stable balanced Unstable unbalanced	Prediction Prediction
16	turborotor	Soft instability	Prediction
17	Turbine rotor	Unstable above 6500 rpm	Prediction
18	Train analysis	2 of 4 bearings unstable	Prediction

EXAMPLE ROTOR ANALYSIS

The first example is referred to as Case 2 which is a made up rotor analysis typical of a double overhung turbocompressor rotor supported by two tilting pad bearings. Figure 4 gives the Dyrobes forced response model. The Morton analysis requires the response analysis of the overhang section of interest, bearing 1 in this first example. The approximate cg location of the overhang is computed in the solution and the required imbalance is applied to allow the response at bearing 1 to be provided to the Morton analysis program. Figure 5 shows the Morton analysis model for Bearing 2 overhang.

Case 2 Model for a double overhung turbocompressor

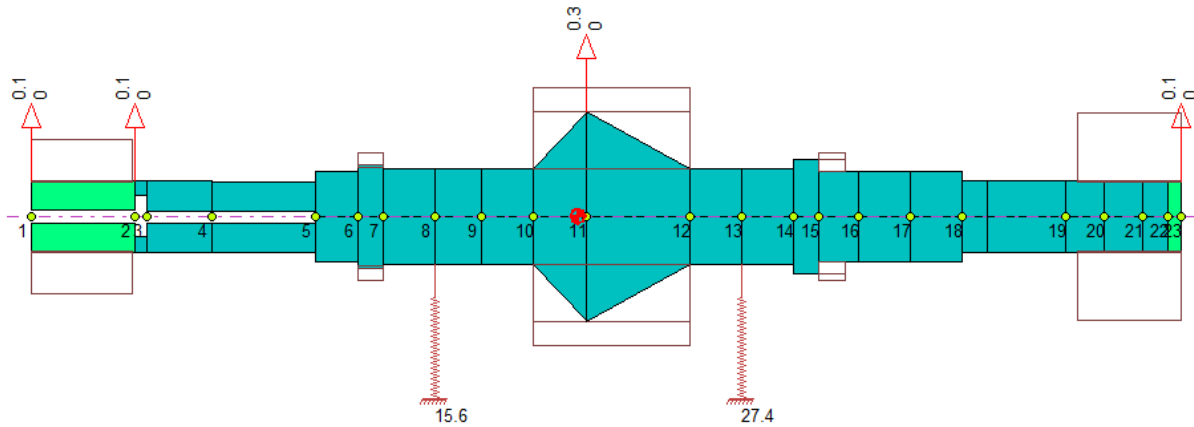


Figure 4 Basic rotor analysis model for Dyrobes model Case 2

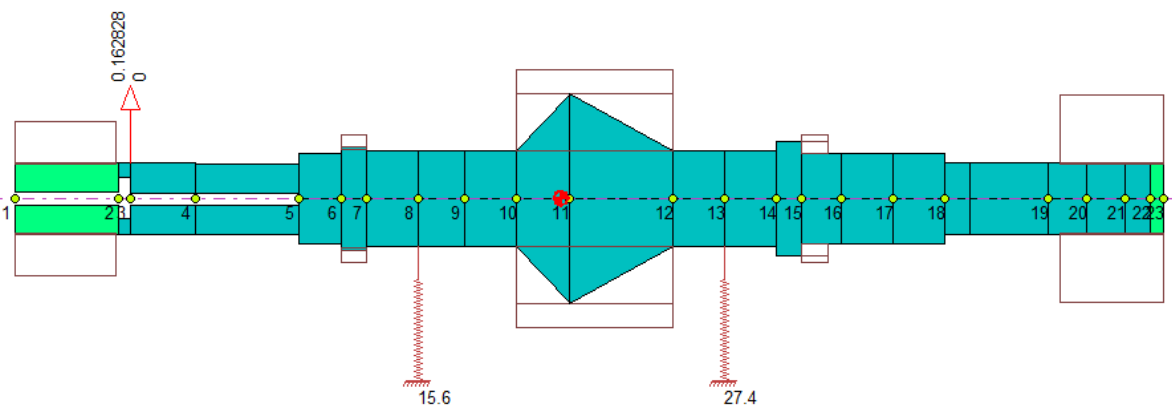


Figure 5 Morton rotor model for analysis of Case 2 bearing 1

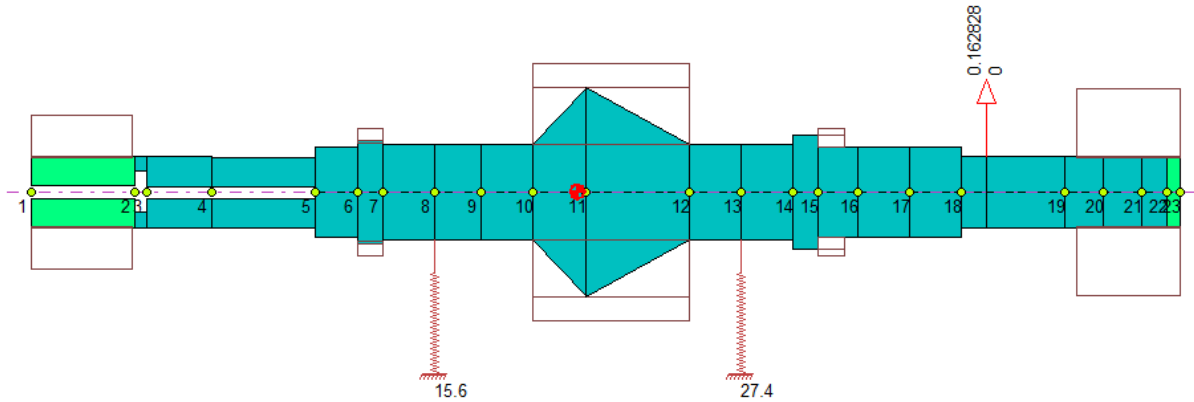


Figure 6 Morton rotor model for analysis of Case 2 bearing 2

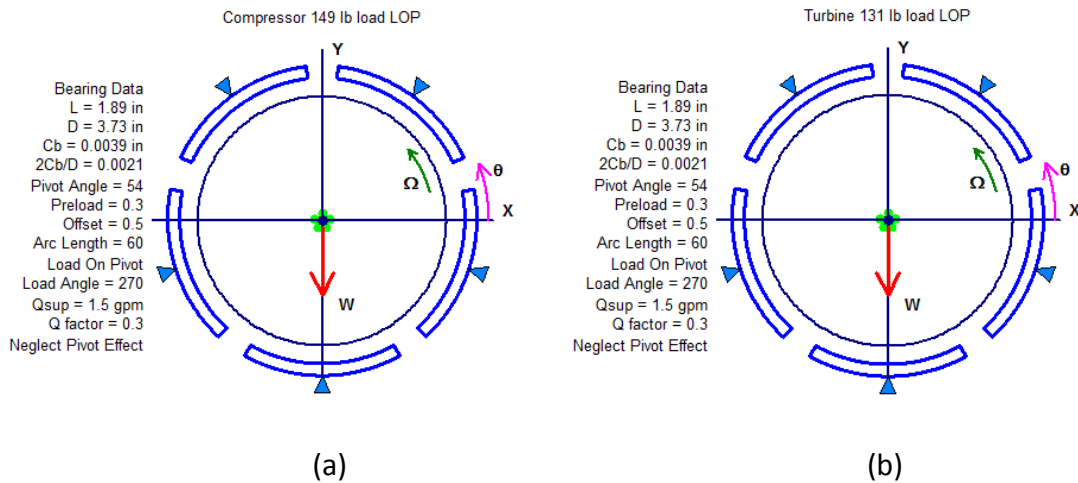


Figure 7 Bearings for rotor analysis for Case 2 (a) bearing 1 (b) bearing 2

The final result of the Morton analysis is a stability plot that plots the total mechanical plus thermal imbalance versus rotor speed. If the imbalance goes beyond a 50% increase the system is suspect to have a problem with thermal synchronous instability. The shape of the total imbalance plot is also important. The higher the slope and hence more rapid increase of imbalance indicates the more likely chance of having a rotor with a violent synchronous instability. The Case 2 results indicates only a slight chance of having any problem with this machine. The bearing 1 left overhang has a greater chance of showing a problem and bearing 2 right overhang has even less of a chance to have a synchronous instability.

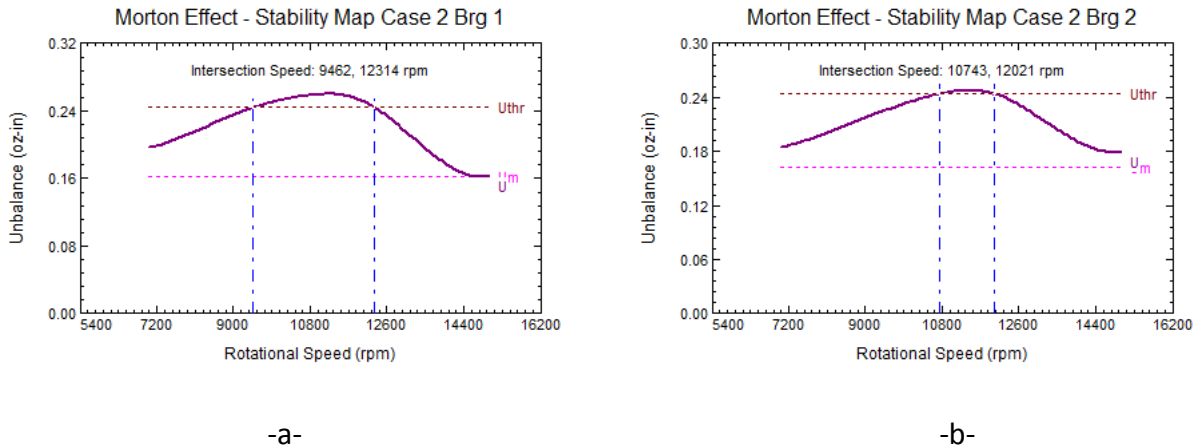


Figure 8 Stability Results for Case 2 (a) bearing 1 (b) bearing 2

The second example rotor, Figure 8 is a large turbine model taken from the example folder of Dyrobes. A four pad tilting design was selected for this very heavy machine, 38,000 lb total weight. The design speed is not given but a likely speed would be around 3600 rpm or maybe as high as 5000 rpm. A design speed of 5000 rpm was selected for this example stability check. The bearings shown in Figure 10 for loads of 20,000 and 17,000 lb for bearing 1 and 2 .

Case 3 Turbine Model from Examples of DyRoBeS

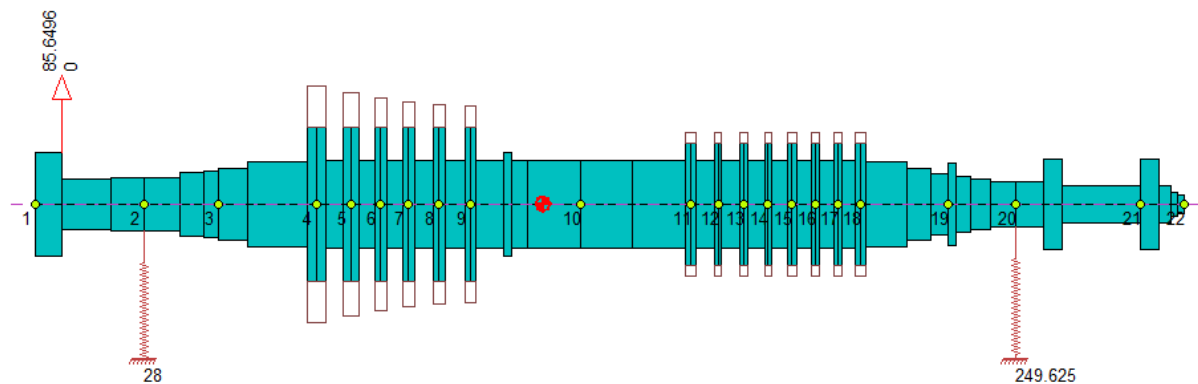


Figure 9 Morton rotor model for analysis of Case 3 bearing 1

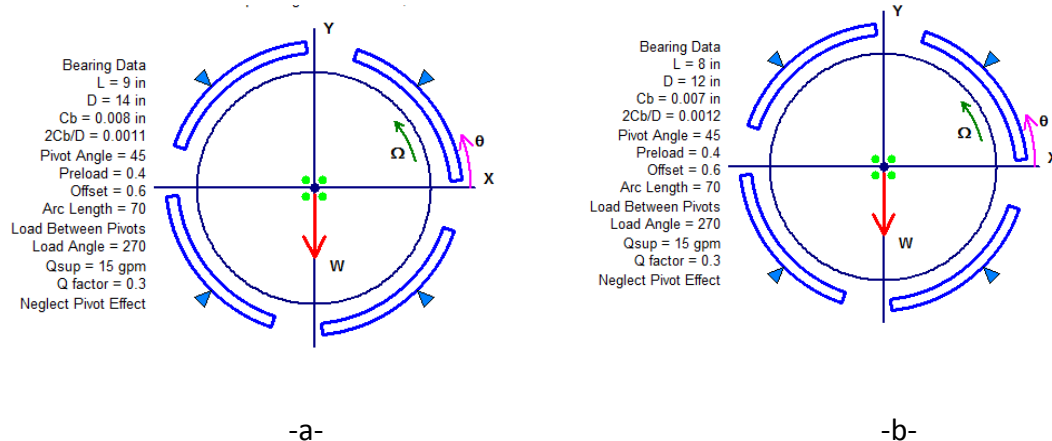


Figure 10 Bearings for rotor analysis for Case 3 (a) bearing 1 (b) bearing 2

The results of the Morton thermal stability are given in Figure 11 and now a condition that would most likely have a violent instability if the design speed is at or above 6600 rpm.

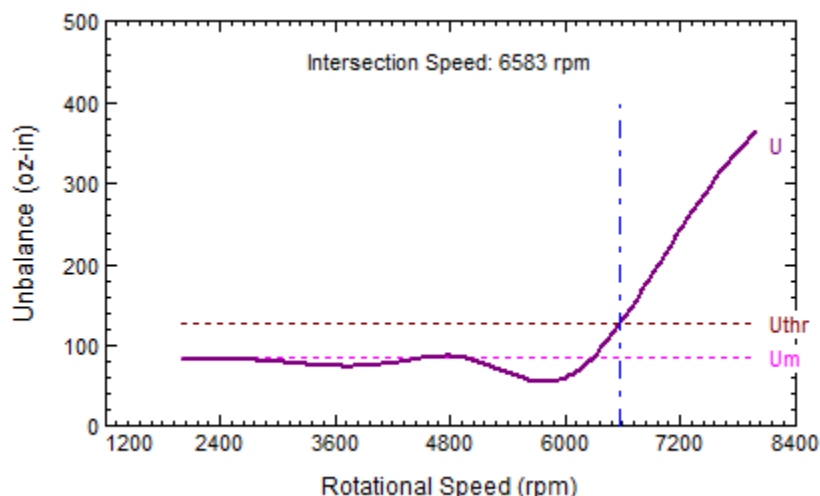


Figure 11 Stability for Bearing 1 overhang showing a rapid increase above 6000 rpm

The rotor model for station 20 has sub-elements for the overhang and a long section has no subelements. This makes it hard to get a good placement of the overhang cg. The overhang should have a generous number of sub-elements to allow a best placement of the overhang imbalance location. Figure 12 shows the imbalance next to the thrust disk and just by looking it seems wrong. By adding elements, the cg is shown to be more midway between the 2 disks. The Morton stability was run for both models and the results are shown in Figure 14.

Modified station 20 sub elements to improve accuracy of overhang cg

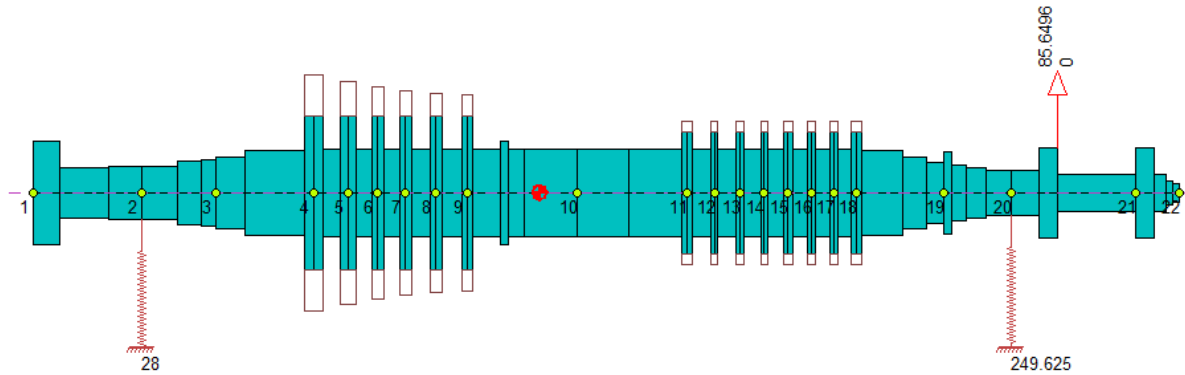


Figure 12 Morton original rotor model for analysis of Case 3 bearing 2

Above is original model and below added sub elements but same length

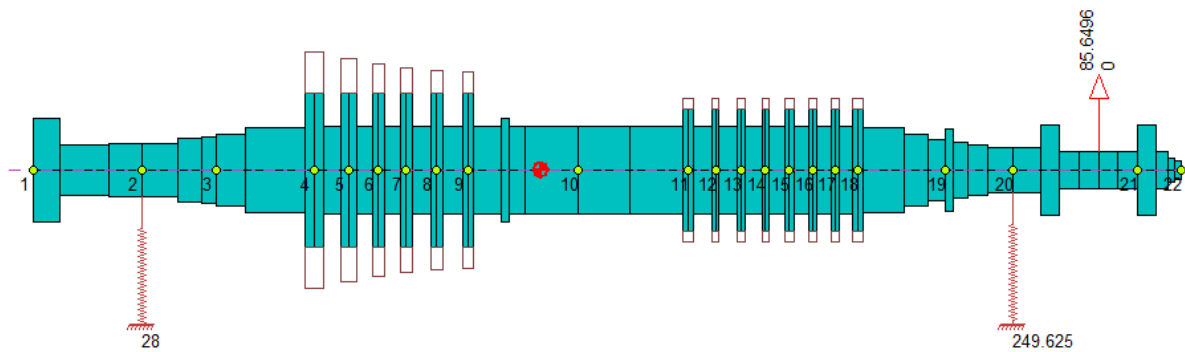
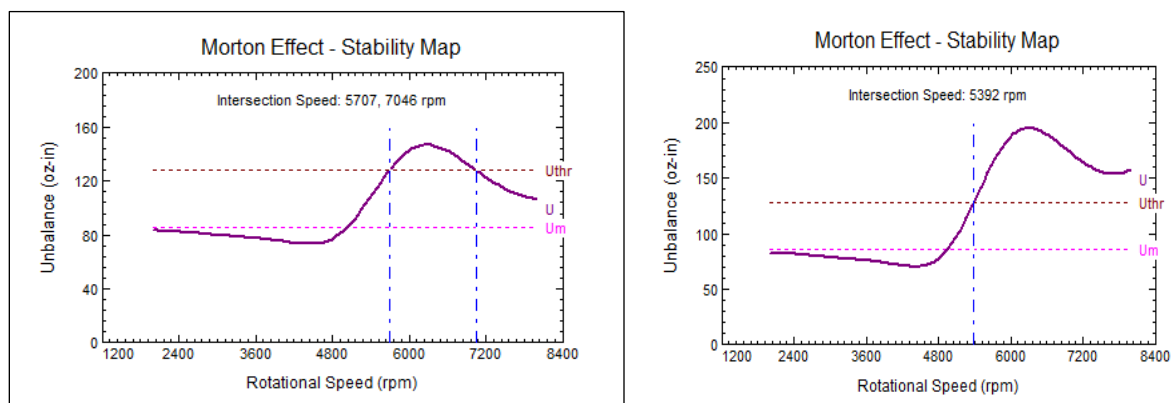


Figure 13 Morton revised rotor model for analysis of Case 3 bearing 2



-a- Original Model Brg 2 Case 3

-b- Revised Model Brg 2 Case 3

Figure 14 Stability Results for Case 3 (a) original (b) revised

The 3rd example, Case 4, is a more complex situation since the model consists of 3 shafts and now there are 4 rotor overhangs to examine. It is best to have four or more folders to allow the results to be less confusing when it is time to write a report.

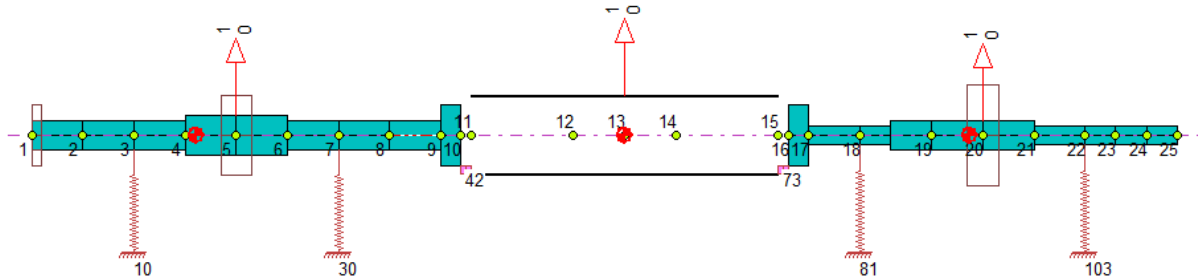
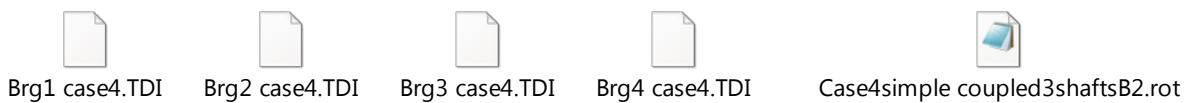
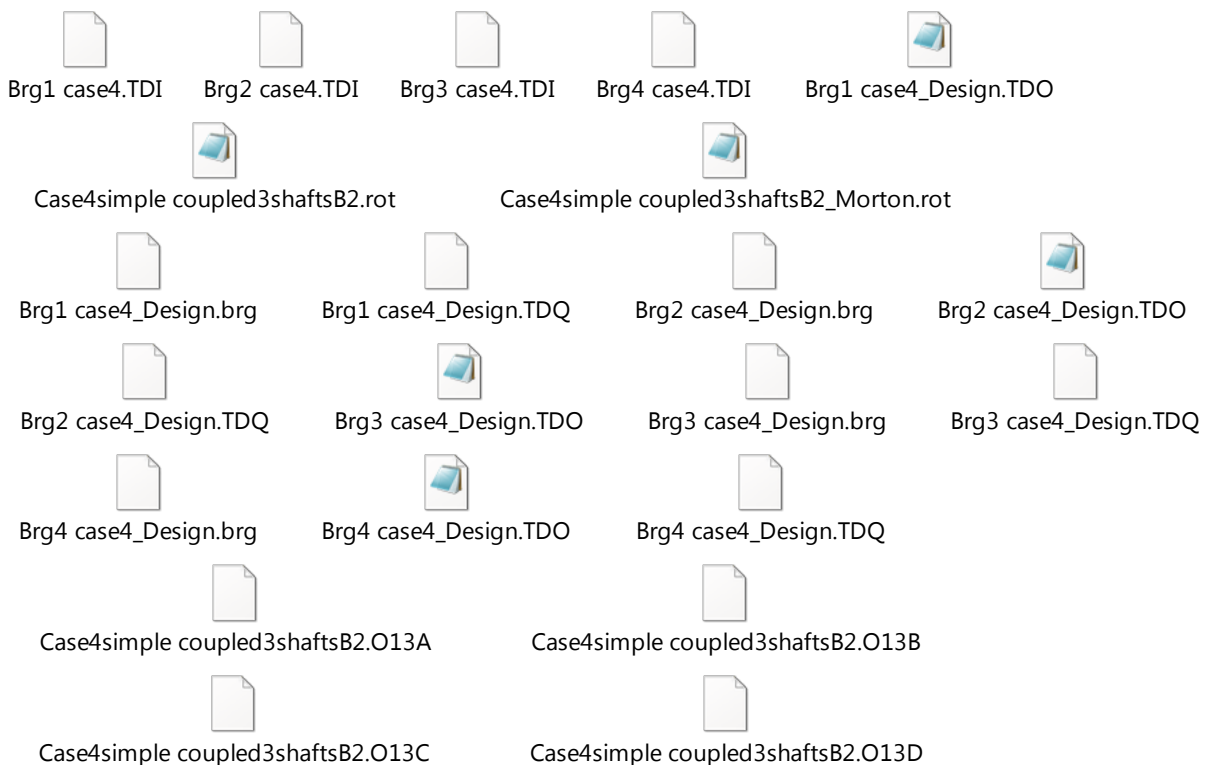


Figure 15 Morton rotor train model for analysis of Case 4 bearings 1, 2, 3, and 4

Files needed in a new folder to run this Train Analysis for bearing 2:



Files in the new folder after the Morton Analysis runs for bearing 2:



It is suggested to have a folder system to store all the output for each run. The folders used for this example Case 4 are given in Figure 16.

Figure 16 Morton rotor train folders for analysis of Case 4 bearings 1, 2, 3, and 4

The system must be put into Dyrobes using the TDI option 15 for bearings.

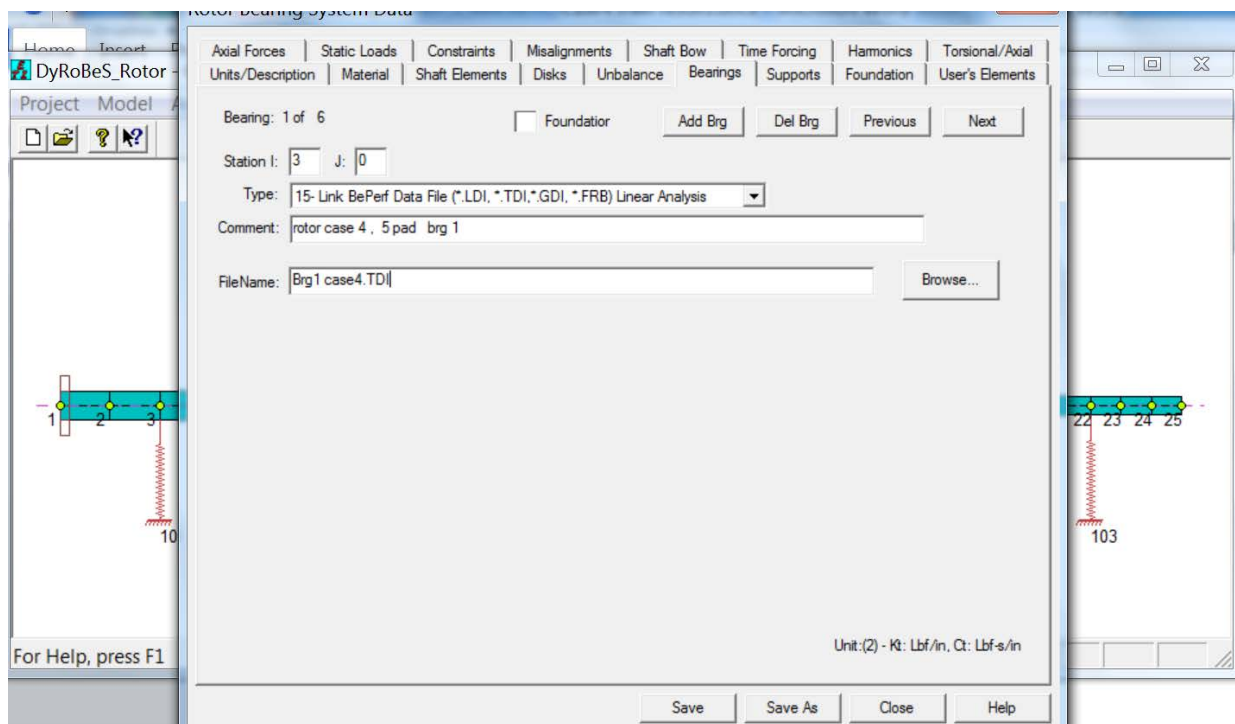


Figure 17 Morton analysis requires bearing option 15

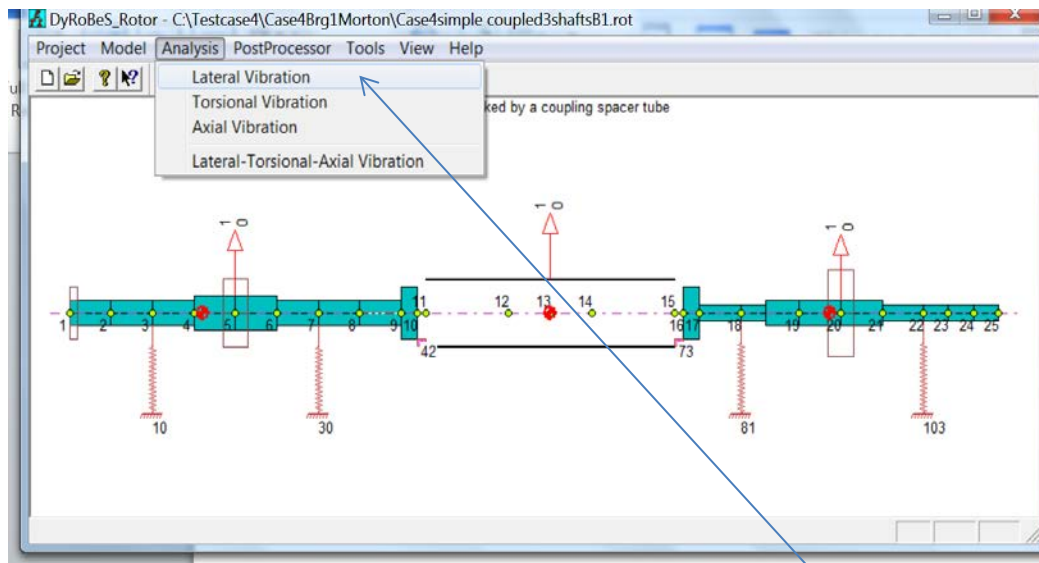


Figure 18 Morton rotor train Analysis select option - Lateral Vibration

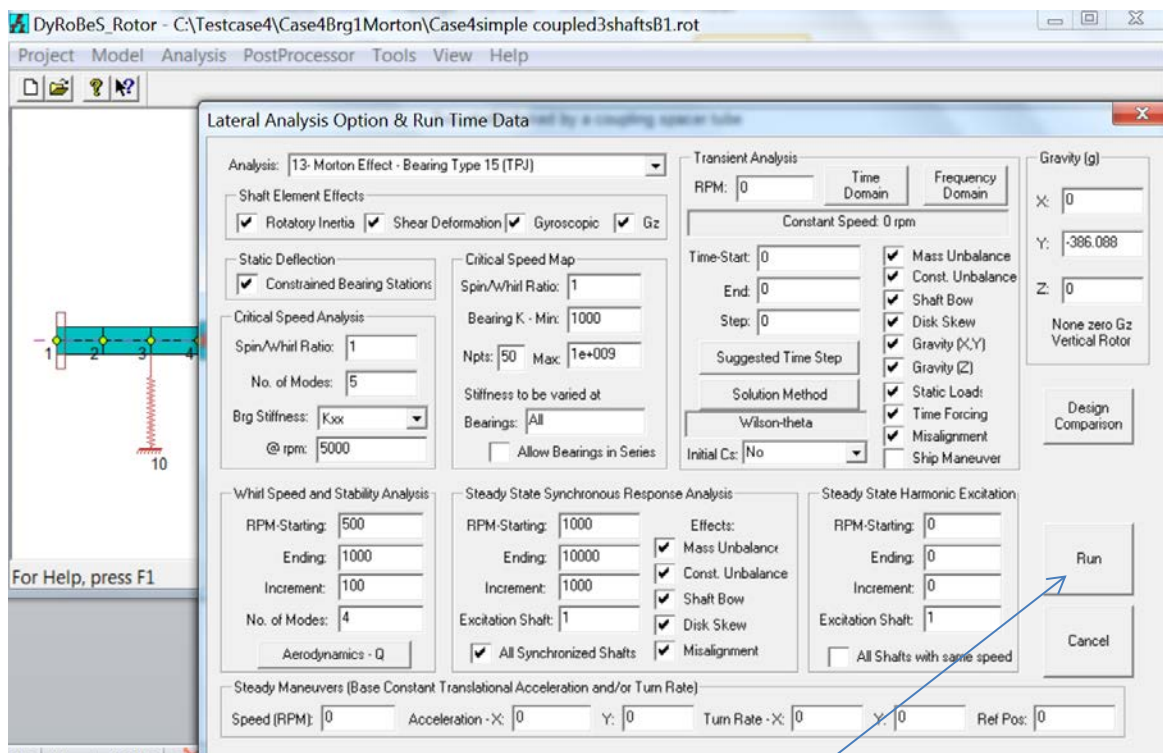


Figure 17 Morton rotor train Analysis option - select RUN

Process view help

C:\Testcase4\Case4Brg1Morton\Case4simple coupled3shaftsB1.rot

Morton Effect Additional Input Parameters

Bearing and Overhang Parameters

Bearing No.: 1 @ Station: 3 Overhang: Left

Bearing Pad Thickness: 0.5 (in) Additional Overhang Mass: 0 (Lb)

File: Brg1 case4.TDI

OK

Cancel

Lubricant Parameters

Supply Pressure: 20 (psi)

Ambient Temperature: 70 (degF)

Inlet Temperature = 120 degF - Lubricant: Amokon ISO-VG 32

Rotor Parameters

Design Speed (N): 10000 (RPM)

Journal Coeff. of Thermal Expansion: 6.67e-006 (/degF)

Mechanical Overhang Unbalance (UNBM): 1 - 56347xW/N^2 0.161478 (oz-in)

Shaft: 1, Weight: 286.578 Lb, Overhang station: from 1 to 3

Speed (RPM) Range of Interest

Start: 1000 End: 15000 Increment: 1000

Pivot thickness(height) a good guess if no print

0 as specified
56347 x W / N^2 for speeds below 11000 rpm
4W/N speeds above 11000
8W/N speeds above 11000
ISO G2.5 larger level

Figure 18 Morton rotor train Additional Input Parameters for Brg 1
- appears after RUN selected

Case4 train results.docx - Microsoft Word

Morton Effect Additional Input Parameters

Bearing and Overhang Parameters
 Bearing No.: 2 @ Station: 7 Overhang: Right
 Bearing Pad Thickness: 0.5 (in) Additional Overhang Mass: 25 (Lb)
 File: Brg2 case4.TDI

Lubricant Parameters
 Supply Pressure: 20 (psi)
 Ambient Temperature: 70 (degF)
 Inlet Temperature = 120 degF - Lubricant: Amokon ISO-VG 32

Rotor Parameters
 Design Speed (N): 10000 (RPM)
 Journal Coeff. of Thermal Expansion: 6.67E-006 (/degF)
 Mechanical Overhang Unbalance (UNBM): 1 - 56347xw/N^2 0.161478 (oz-in)
 Shaft: 1, Weight: 286.578 Lb, Overhang station: from 7 to 10

Speed (RPM) Range of Interest
 Start: 1000 End: 15000 Increment: 1000

OK Cancel

½ spacer weight added to Morton analysis calculation

Figure 19 Morton rotor train Additional Input Parameters for Brg 2

r - C:\Testcase4\Case4Brg3Morton\Case4simple coupled3shaftsB3.rot

Morton Effect Additional Input Parameters

Bearing and Overhang Parameters

Bearing No.: 3 @ Station: 18 Overhang: Left

Bearing Pad Thickness: 0.5 (in) Additional Overhang Mass: 25 (Lb)

File: Brg3 case4.TDI

Lubricant Parameters

Supply Pressure: 20 (psi)

Ambient Temperature: 70 (degF)

Inlet Temperature = 120 degF - Lubricant: Amokon ISO-VG 32

Rotor Parameters

Design Speed (N): 10000 (RPM)

Journal Coeff. of Thermal Expansion: 6.67e-006 (/degF)

Mechanical Overhang Unbalance (UNBM): 1 - 56347xw/N² 0.120339 (oz-in)

Shaft: 3, Weight: 213.569 Lb, Overhang station: from 16 to 18

Speed (RPM) Range of Interest

Start: 1000 End: 15000 Increment: 1000

OK Cancel

Figure 20 Morton rotor train Additional Input Parameters for Brg 3

A:\Testcase4\Case4Brg4Morton\Case4simple coupled3shaftsB4.rot

Analysis PostProcessor Tools View Help

Morton Effect Additional Input Parameters

Bearing and Overhang Parameters

Bearing No.: 4 @ Station: 22 Overhang: Right

Bearing Pad Thickness: 0.5 (in) Additional Overhang Mass: 0 (Lb)

File: Brg4 case4.TDI

Lubricant Parameters

Supply Pressure: 20 (psi)

Ambient Temperature: 70 (degF)

Inlet Temperature = 120 degF - Lubricant: Amokon ISO-VG 32

Rotor Parameters

Design Speed (N): 10000 (RPM)

Journal Coeff. of Thermal Expansion: 6e-006 (/degF)

Mechanical Overhang Unbalance (UNBM): 1 - 56347xw/N^2 0.120339 (oz-in)

Shaft: 3, Weight: 213.569 Lb, Overhang station: from 22 to 28

Speed (RPM) Range of Interest

Start: 1000 End: 15000 Increment: 1000

OK Cancel

Figure 21 Morton rotor train Additional Input Parameters for Brg 4

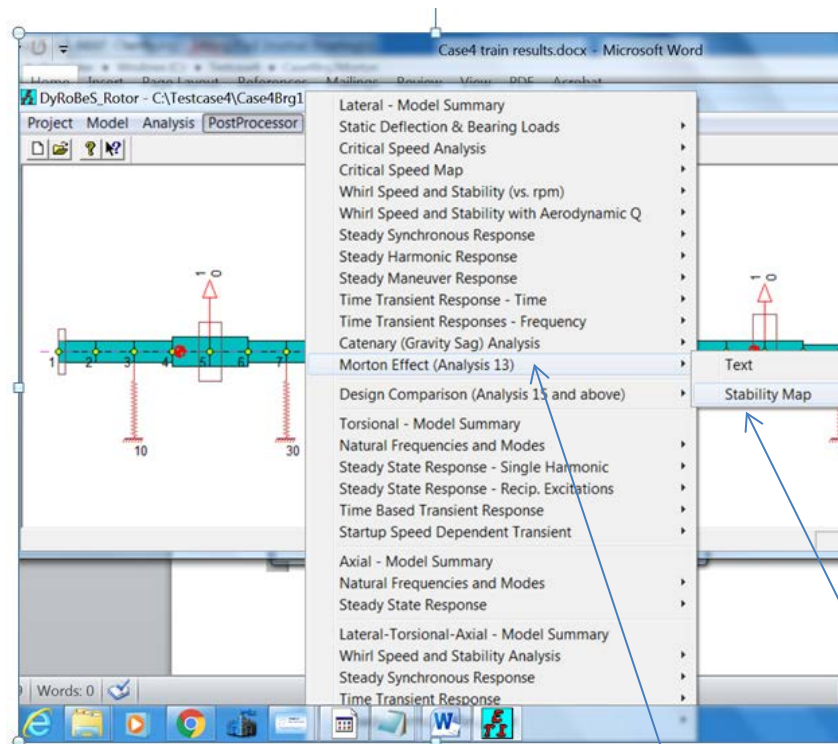


Figure 22 Morton rotor train Post processor - select Morton Effect then Stability map

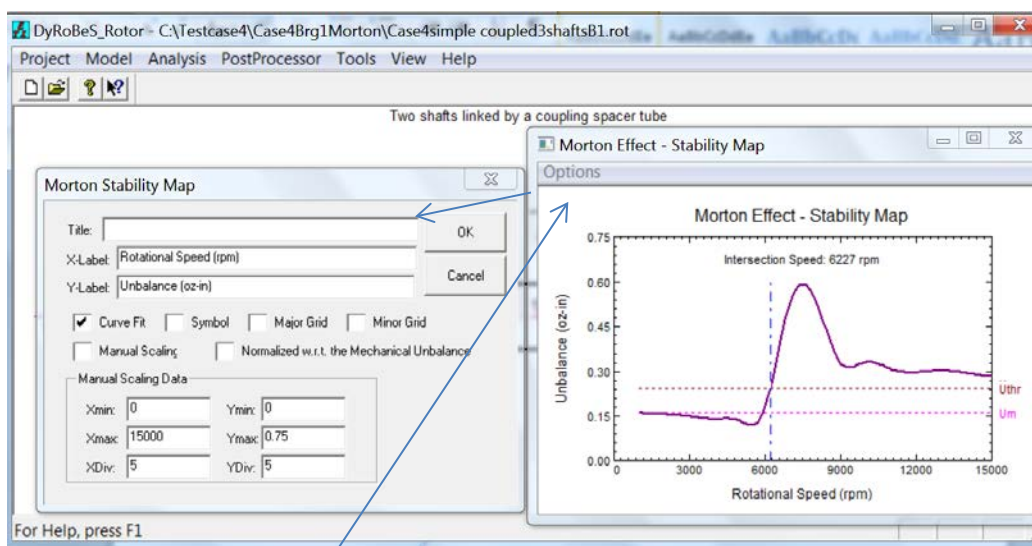


Figure 23 Plot options to change scale, labels, title, etc.

Now, the process is to run analysis, plot the stability map, then collect and evaluate the results.

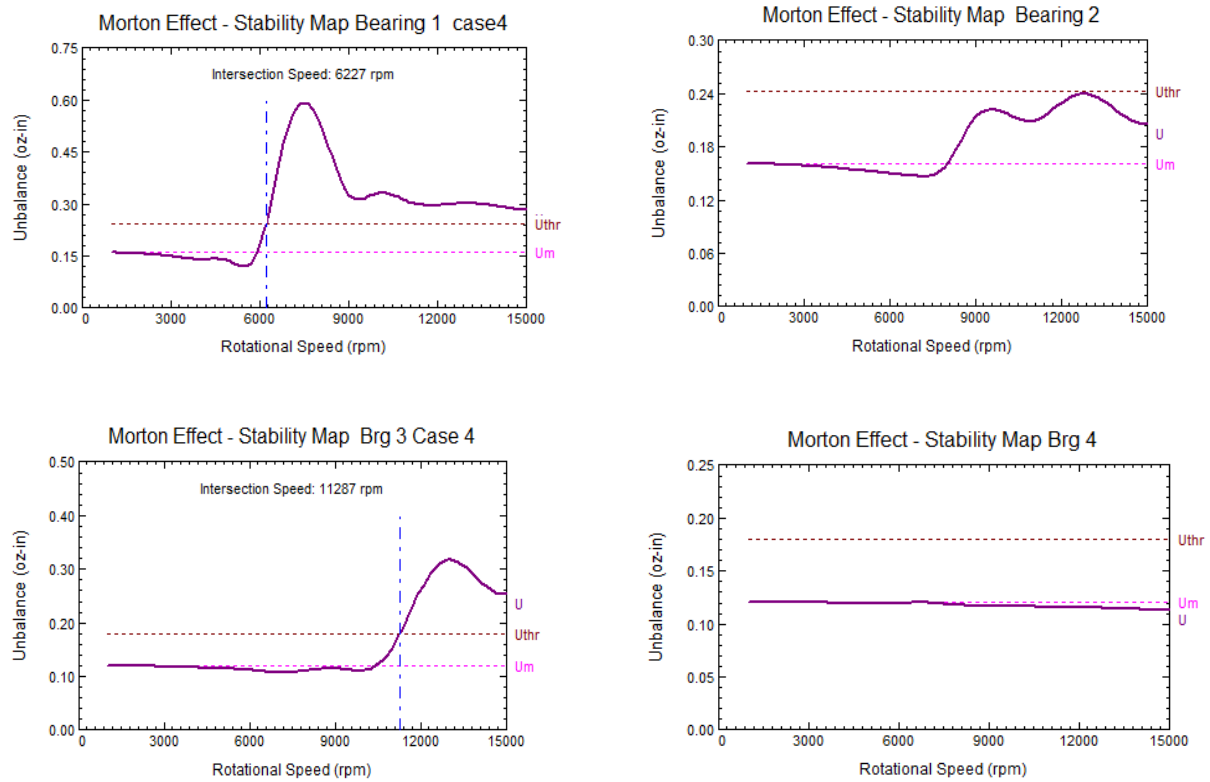


Figure 24 Morton Analysis results for Case 4, Brg 1, 2, 3, 4 plotted to different scales

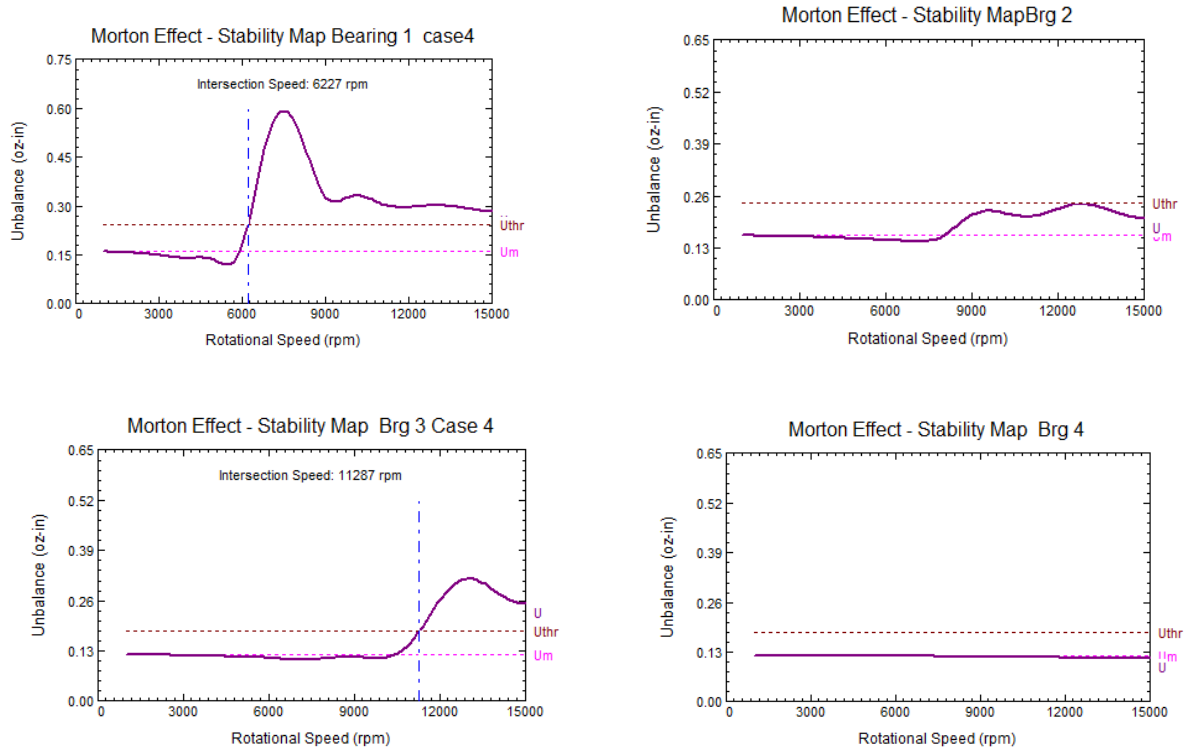


Figure 25 Morton Analysis results for Case 4, Brg 1, 2, 3, 4 plotted to almost same scales

It is clear that the bearing 1 overhang is a potential problem for operation between 6000 and 9000 rpm while bearing 3 is an issue for speeds above 11000 rpm.

Bearing redesign, shaft modifications, and reduced weight added parts can lower the odds of having a problem on test stand and in the field.

This is what we do for a living. Have at it!!

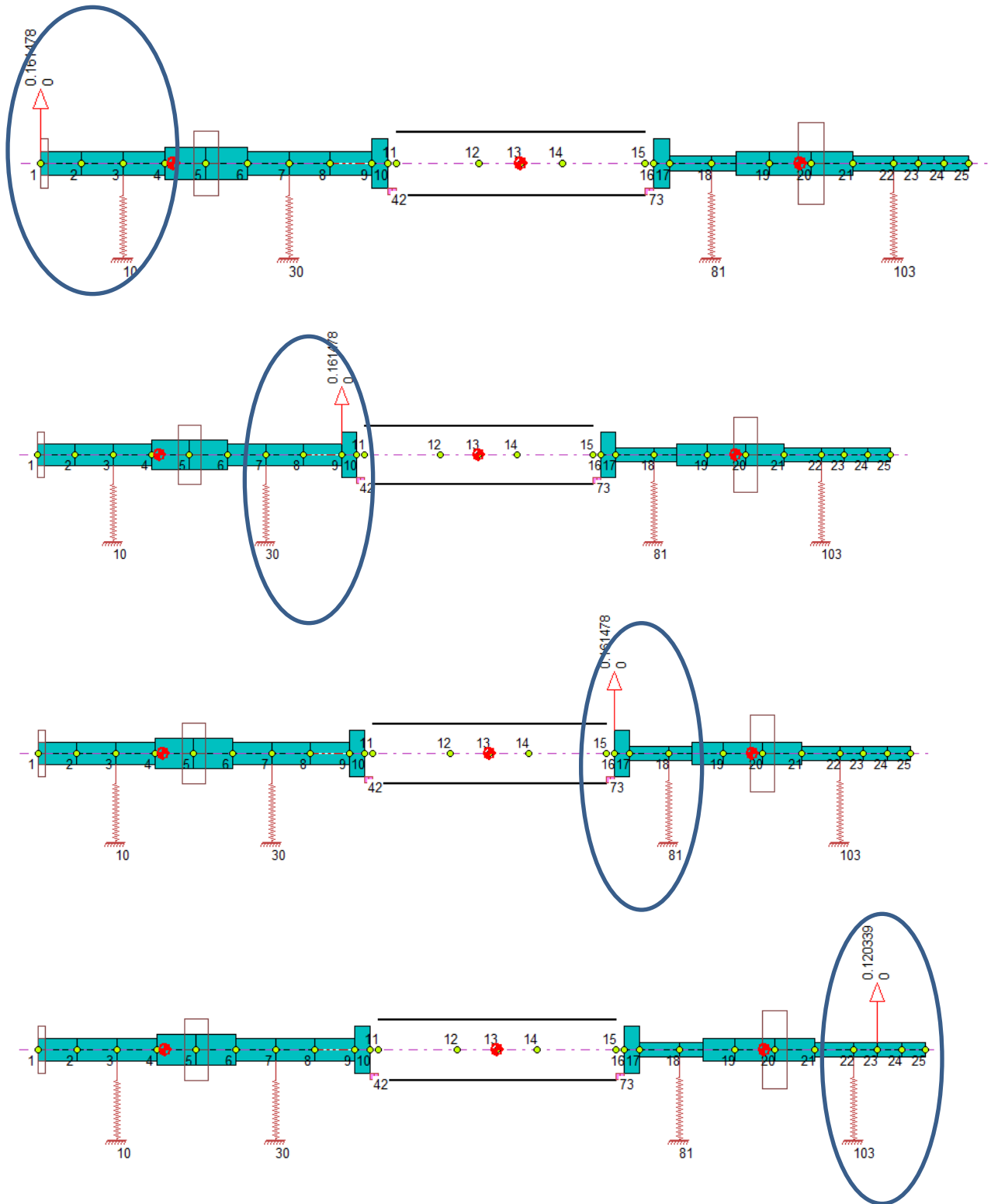
First, look at bearing four shaft end. Same issue of not enough elements in the overhang.

This is not a problem for this shaft end but be careful to make a good model.

The change made for this end can be seen below.

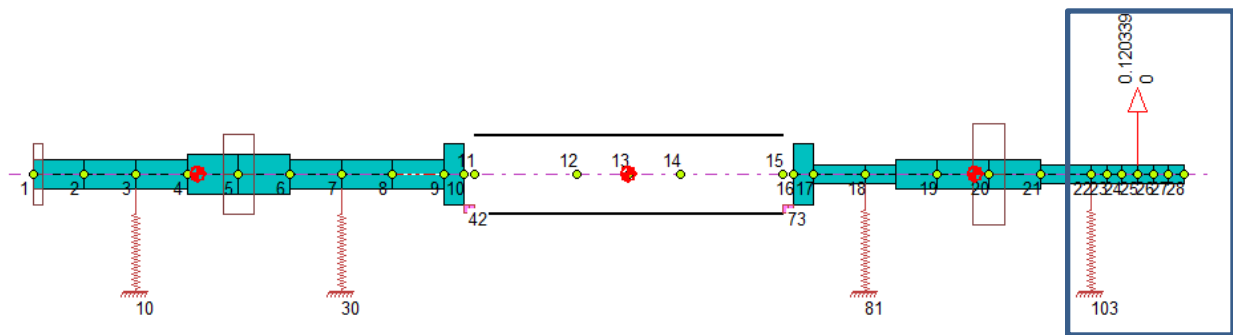
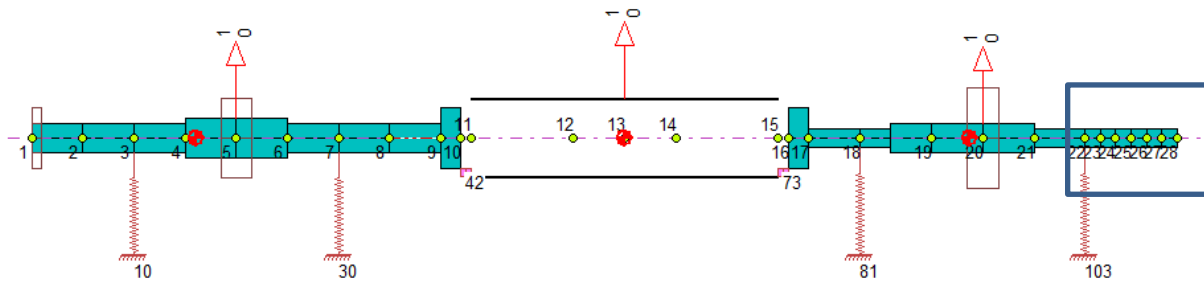
Still not an issue.

Look at all of the Morton models to be sure the cg is acceptable for Case 4.

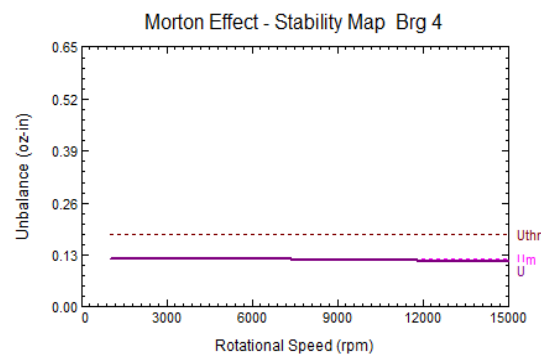
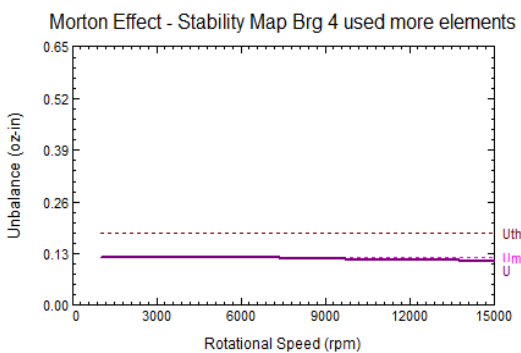


The brg 4 model seems to be off in the cg location.

Increase the number of stations keep same length of overhang.



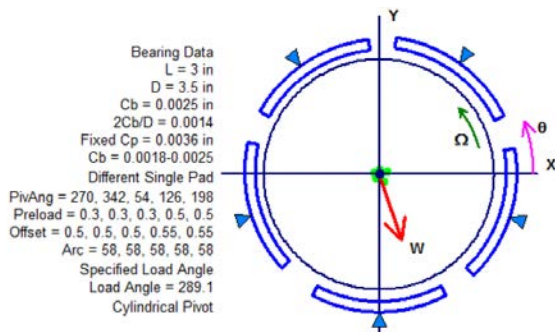
Here the cg has moved over just a little. Not an issue this time. Always be sure the model is acceptable.



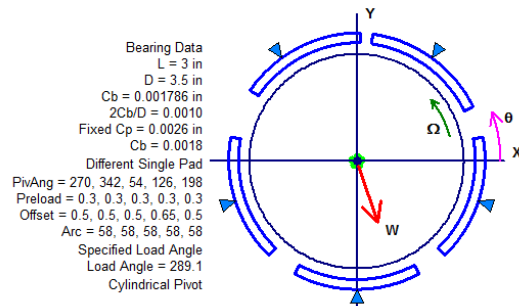
Bearing 4 is actually having almost no effect and seems to be reducing the imbalance in fact.

Closing remarks

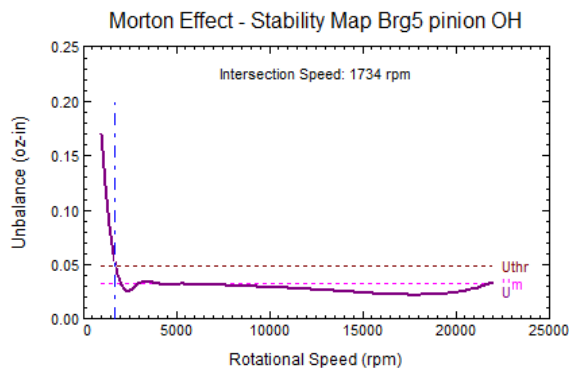
It should be known that all bearings may not converge for some conditions. For a low preload bearing running at very low speed or a heavy load for a gear and pinion or any rotor with a large load, the upper pads may become unloaded. In the TMAP (Thermal Morton Analysis Program) this causes the solution for that speed to fail to converge. Usually, it is just the upper pads or the pads opposite a large gear load, then the low preload pads need to be given a larger pad clearance such that the pad will create a positive load. Another possible solution would be to offset the unloaded pads if you cannot get the pad clearance to solve the problem. To increase the preload of all pads may work but currently that is not a good solution since it changes the bearing too much. It is possible to investigate in BePerf the pads being at no load, but BePerf currently has a constant pad clearance constraint which makes it hard to use the increased pad clearance to solve the failure of TMAP. In addition, the TMAP solution has the rotor bearing journal making an orbit and all pads must remain loaded. Shown below are more details of the bearings and stability plots, very interesting in fact. (just skip lower speeds)



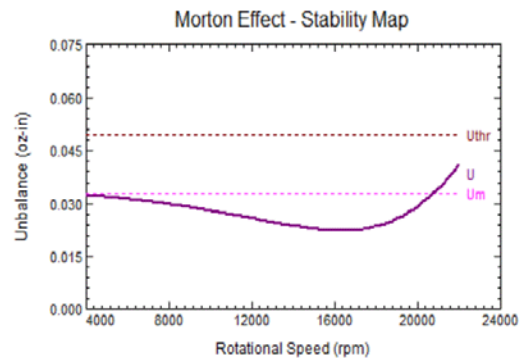
-a- This will run in TMAP



-b- This will run in TMAP



-c- Brg as shown in (a)



-d- Run with offset on loaded pad (b) start at 4000 rpm Load 2389 lb

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