# BALANCING OF MULTIMASS FLEXIBLE ROTORS

by

**Dr. E. J. Gunter**

**Professor of Mechanical Engineering**

**L. E. Barrett**

**Senior Research Engineer**

**Dr. P. E. Allaire**

**Assistant Professor of Mechanical Engineering**

**University of Virginia**

**Charlottesville, Virginia**

## PART I — THEORY

### 1.1 INTRODUCTION AND STATEMENT OF THE PROBLEM

It has been stated by various experts in field maintenance of rotating machinery such as Charles Jackson of Monsanto Corporation that over 90% of the field service problems involved with vibrations in turbomachinery have been associated with misalignment and unbalance of the equipment.

In modern petrochemical plants and utilities the cost of down time of a compressor or turbine due to the correction of vibration problems can amount to $50,000 to $100,000 a day. It is therefore highly desirable that field service engineers be able to quickly and accurately balance a large rotating unit on the site so as to minimize down time if the unit is suffering from a want of balance. With the new generation of rotating equipment being developed, it is often difficult and uneconomical to ship large turbo rotors back to the factory or to a specialized balancing facility to correct the rotor for unbalance. This is particularly true in the case where the equipment is in an inaccessible location such as an offshore drilling rig, or an Alaskan oil pumping station.

The problem of balancing a high speed multimass turborotor therefore reduces to the problem of reducing the rotor synchronous amplitude of motion and bearing forces transmitted to within acceptable limits throughout the operating range of the machine.

In this paper, we will consider only the problem of synchronous rotor response due to unbalance. There are numerous complex nonsynchronous vibrations that can occur in rotating machinery. Quite often the nonsynchronous motion can not be corrected merely by improving the rotor balance. The causes of this may be due to self-excited whirl motion caused by fluid film bearings, seals, balance pistons, aerodynamic effects, internal friction, shaft rubs, shaft asymmetry, or by external excitations through gear boxes, misaligned couplings, piping acoustics or transmitted foundation vibrations.

Therefore the first step in correcting a machine vibration is to identify the source of the problem to determine that the vibration encountered is not caused by self-excited vibrations or machine misalignment before balancing is attempted.

No piece of rotating equipment can ever be said to be perfectly balanced at all speeds as this is a physical impossibility. From an engineering viewpoint it is highly impractical to attempt to reduce the rotor amplitude to zero everywhere. What is desired, however, is to reduce the level of vibrations down to acceptable values. What is defined as an acceptable level of vibration is beyond the scope and the desire of the authors to explain. For example, what would have been considered an acceptable level of vibration in turbomachines 10 years ago is now often viewed as unacceptable due to the development of sophisticated electronic vibration monitoring equipment such as the noncontacting eddy current and capacitance probes and casing velocity measuring devices.

The balancing requirements on rotating equipment will obviously vary considerably according to the usage and applications. Therefore gas bearing gyroscopes or dental drills required to rotate at 200,000 RPM or high speed textile spindles will require closer tolerances than will steel rolling mills or low speed rotating machinery.

Therefore values of acceptable levels of vibration will vary widely with the class of turbomachinery and the specifications of balancing must often be based upon considerable field experience.

### 1.2 DISCUSSION OF UNBALANCE DISTRIBUTION

The total unbalance in a rotating machine may be due to a combination of several complex distributions of rotor mass as shown in Fig. 1.1. The first Fig. 1.1a, represents a continuous distribution of unbalance along the shaft. This may be caused by the fact that the journal bearing surfaces are not machined concentric with the main rotor. The second unbalance case, 1.1b, represents a series of localized radial point masses attached at a radius R, to a uniform shaft. Such a sys-

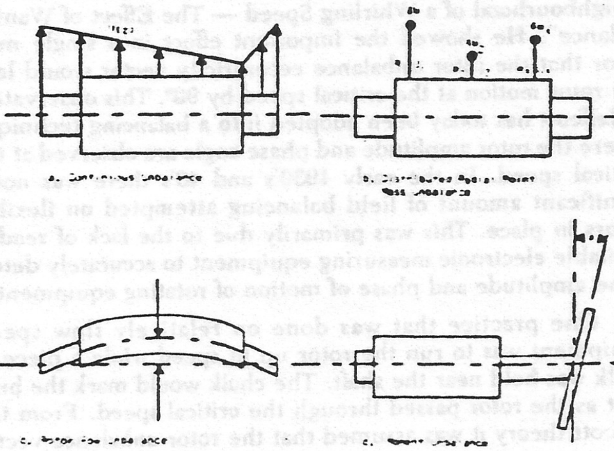

*Figure 1.1 Types of Unbalance Distribution*

The influence coefficient method for multimass rotors is based on the assumption that at a particular speed the rotor amplitudes are given by

$$[z] = [\alpha]\,[u] \qquad (1.3.1)$$

where z = complex amplitudes at N stations

$[\alpha]$ = N x N influence coefficient matrix

$[u]$ = Effective unbalance vector at N stations

In an actual rotor with run out, shaft bow and moment unbalance caused by skewed discs, the rotor amplitude is more closely represented by the vector equation

$$[z] = [\alpha]\,[u] + [\alpha_r]\,[z_r] + [\alpha_m]\,[u_m] + z_o \qquad (1.3.2)$$

where

$[z_r]$ = shaft bow vector

$[\alpha_r]$ = amplitude response matrix due to shaft bow

$[u_m]$ = moment unbalance due to local unbalance couples or disc skew

$[\alpha_m]$ = amplitude response matrix due to moment unbalance

$z_o$ = constant runout vector (mechanical or electrical)

The use of trial weights can only solve for the $[\alpha]$ influence coefficient matrix. Therefore, the direct application of the influence coefficient method without considering shaft run out or bow can result in a rotor apparently balanced at the speed at which the measurements are taken but unbalanced at other speeds.

In spite of these limitations it is felt by the authors that an adaptation of this method in conjuction with the modal method can be used to rapidly balance multimass flexible rotors with a minimum of measurement planes and trial runs.

*1.3.2. Modal Balancing Method*

The balancing problem has been approached by two different schools of thought; those who view the rotor as a series of point masses and those who treat the rotor as a continuous elastic body. The treatment of the rotor as a continuum has led to the modal concept pioneered by Bishop and Parkinson. In this method, the rotor amplitude is expressed as a power series function of the system undamped eigenvalues. Bishop shows that the general unbalance distribution may be expressed in terms of modal unbalance eccentricities. The rotor amplitude of motion near a critical speed is thus primarily affected by that particular modal unbalance distribution while the higher order modes have little influence on the lower critical speed response. They then state that the rotor should be balanced mode by mode by placing proper weights at the antinodes. Several excellent papers have been published by Kellenberger of Brown Boveri who has treated the balancing problem of both continuously distributed and local unbalances and discusses the problem of balancing in N or N + 2 planes.

The paper of Kellenberger on "Should a Flexible Rotor be Balanced in N or N + 2 Planes" was received with considerable criticism by Bishop and Parkinson who state that only N balance planes are required and that rigid rotor balancing is not necessary beforehand.

## 1.4 FLEXIBLE ROTOR BALANCING IN N + B PLANES

Den Hartog, in his paper on "The Balancing of Flexible Rotors" states that if a rotor consists of a straight weightless shaft with N concentrated masses along its length and supported in B bearings with an arbitrary unbalance distribution, then it can be balanced perfectly by placing small correction weights in N + B planes along the length of the rotor.

He also states that if the flexible rotor is balanced on rigid supports, then the balance of the rotor so obtained will not be a function of the bearing impedances. That is, the introduction of support flexibility and damping to the rotor system will not materially effect the rotor unbalance response if the N + B plane method of balancing is used.

It should be noted, however, that the inverse condition is not true. If a rotor is balanced on soft supports, such as with a standard flexible mount balancing machine, then the rotor may not necessarily be in balance when run in the actual machine in which the support stiffness values are substantially higher than the balance machine. Therefore it can be concluded that balancing on the hard bearing support balancing machines is preferable to balancing on soft support machines.

The N + B concept of Den Hartog is extended to multimass flexible rotors and he states that nearly perfect balance at all speeds can be obtained by balancing in N + B planes, where N now means the number of rotor critical speeds in the speed range from zero to four times the maximum service speed of the machine.

To demonstrate the N + B method of balancing consider Fig. 1.2 which represents a single mass rotor with an arbitrary unbalance of $u_k = m_k e_k$.

The equation of motion in y - z plane for the major mass stations can be obtained by the influence coefficient method as follows

$$y_l = \alpha_{11} P_l + \alpha_{lk} P_k \qquad (1.4.1)$$

where

$\alpha_{11}$ = deflection at station l due to a unit force at l

$\alpha_{lk}$ = deflection at station l due to a unit force at station k

$P_k$ = sum of external and inertial forces acting at station i

and

$$P_l = M\ddot{Y}_l = M\omega^2 y_l$$

for synchronous motion neglecting external damping or other external forces at the major mass stations

$$P_k = m_k(y_k + e_k)\omega^2 = \omega^2(u_k + m_k y_k)$$

The deflection at $y_l$ is given by

$$y_l = \omega^2[M\,y_l\,\alpha_{11} + (u_k + m_k y_k)\alpha_{lk}] \qquad (1.4.2)$$

where $u_k = m_k e_k$

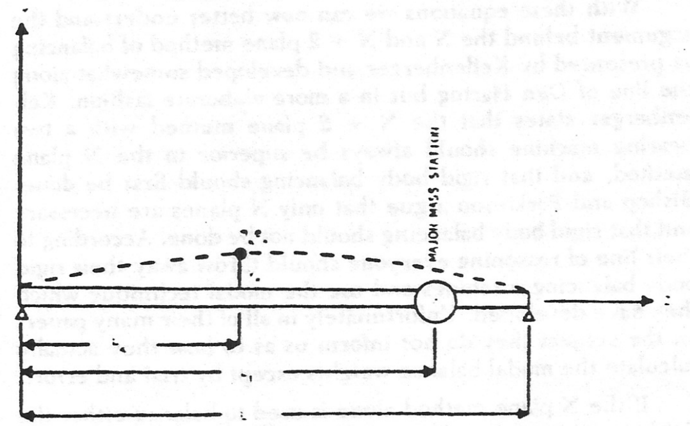


*Figure 1.2 Single Modal Mass Rotor With Arbitrary Point Mass Unbalance*

It is assumed that the balance correction weight $m_k$ is small in comparison to the major mass station.

Hence $\frac{m_k\, y_k}{m\, y_1} \ll 1$

Solving for the deflection at the major mass station for a series of unbalances $u_k$

$$y_1 = \frac{\Sigma\, \alpha_{1k}\, u_k \omega^2}{1 - \omega^2 M \alpha_{11}} \qquad (1.4.3)$$

If the amplitude at the major mass station $y_1$ is to be zero at all speeds then it is apparent that the balancing requirement is

$$\Sigma\, \alpha_{1k}\, u_k = 0 \qquad (1.4.4)$$

In addition to reducing the amplitude of motion at the major mass station it is also desired to reduce the forces transmitted to the bearings. The bearing force reactions are given by

$$F_{b1} + F_{b2} = \omega^2[M\, y_1 + \Sigma\, m_k(y_k + e_k)] = 0 \qquad (1.4.5)$$

Since the unbalance masses $m_k$ are small in comparison to M and the shaft deflection $y_k$ is of the order of mils of deflection whereas the distance $e_k$ at which the unbalance masses are acting may be several inches, the vanishing of the bearing forces requires that

$$\omega^2[m\, y_1 + \Sigma\, m_k\, e_k] = 0 \qquad (1.4.6)$$

If the balance criterion of Eq. (1.4.4) is met so that the motion at the major mass station is zero, then the vanishing of the sum of the bearing forces requires that

$$\sum_{k=1}^{n} u_k = 0 \qquad (1.4.7)$$

This is recognized as simply the first requirement for rigid body balancing.

The third balancing requirement is obtained by summing moments about the first bearing

$$F_{b2} \cdot L = \omega^2[m\, y_1\, L_1 + \Sigma\, m_k\, L_k(y_k + e_k)] = 0 \qquad (1.4.8)$$

This reduces to the requirement that

$$\Sigma\, L_k u_k = 0 \qquad (1.4.9)$$

In summary the requirements for flexible rotor balancing may be stated as two equations of rigid body balance plus a flexible rotor balance requirement

$$\left.\begin{array}{ll} \text{a. } \Sigma\, u_k = 0 \\ \text{b. } \Sigma\, L_k\, u_k = 0 \\ \text{c. } \Sigma\, \alpha_{1k}\, u_k = 0 \end{array}\right\} \begin{array}{l}\text{rigid rotor}\\ \text{balance}\\ \text{flexible rotor}\end{array} \qquad (1.4.10)$$

With these equations we can now better understand the argument behind the N and N + 2 plane method of balancing as presented by Kellenberger and developed somewhat along the line of Den Hartog but in a more elaborate fashion. Kellenberger states that the N + 2 plane method with a two bearing machine should always be superior to the N plane method, and that rigid body balancing should first be done. Bishop and Parkinson argue that only N planes are necessary and that rigid body balancing should not be done. According to their line of reasoning everyone should throw away their rigid body balancing machines and use the modal technique which they have developed. Unfortunately in all of their many papers on the subject they do not inform us as to how they actually calculate the modal balance weights except by trial and error.

If the N plane method alone is used to balance either the single mass model of Fig. 1.2 or a multimass rotor to pass

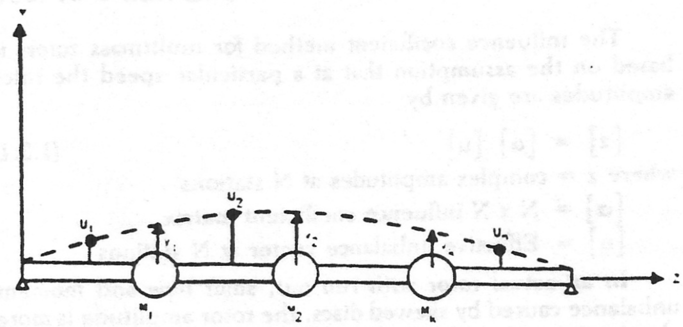

*Figure 1.3 Multimass Rotor With Distributed Point Mass Unbalance*

through the first critical speed, then only one balance correction weight is needed to reduce the amplitude at the major mass station or shaft antinode to zero. The balance correction $u_{b1}$ placed at the major mass station is given by

$$u_{b1} = -\frac{1}{\alpha_{11}} \Sigma\, \alpha_{1k}\, u_k \qquad (1.4.11)$$

Although the amplitude at the major mass station has been reduced to zero, the transmitted bearing forces are nonvanishing. In order to eliminate the transmitted bearing forces due to unbalance as well as reduce the rotor amplitude of motion while passing through the first critical speed, two additional balance planes are required. Let $u_{b2}$ and $u_{b3}$ be two additional balance correction weights placed on the rotor.

The balance correction weights are given by

$$\left.\begin{array}{ll} \text{a.} & u_{b1} + u_{b2} + u_{b3} = -\Sigma\, u_k = R_1 \\ \text{b.} & L_1 u_{b1} + L_2 u_{b2} + L_3 u_{b3} = -\Sigma\, L_k\, u_k = R_2 \\ \text{c.} & u_{b1} + \frac{\alpha_{12}}{\alpha_{11}} u_{b2} + \frac{\alpha_{13}}{\alpha_{11}} u_{b3} = -\frac{1}{\alpha_{11}} \Sigma \alpha_{1k} u_k = R_3 \end{array}\right\} \quad (1.4.12)$$

## 1.5 BALANCING IN A TEST FACILITY WITHOUT TRIAL WEIGHTS

The values of $u_k$ components are unknown can never be exactly calculated for a complex multimass system. However it should be noted that the first two quantities on the right hand side can be measured with a rigid rotor balancing machine.

In the simplified derivation presented, no damping effect of the bearings or on the rotor was taken into account. The influence of damping will alter the rotor amplitude and phase angle relationships from the simple expression given in eq. (1.4.3). However, several companies are developing elaborate spin pit test facilities in which they can run the rotor in a vacuum and mount the rotor on ball bearings. In this way the damping forces acting on the rotor are negligible and equations equivalent to eq. (1.4.3) are valid.

By placing a noncontacting probe near the center of the rotor which should correspond to the major balancing plane no. 1, the rotor may be balanced from the observation of the rotor amplitude of motion without the use of trial weights.

Eq. (1.4.3) may be rewritten in the form

$$y_1 = \frac{\Sigma\, \alpha_{1k}\, u_k \omega^2}{1 - f_1^2} \qquad (1.5.1)$$

where $f_1 = \frac{\omega}{\omega_{cr1}}$ = critical speed ratio on rigid supports

$\omega_{cr1}$ = rotor first critical speed on rigid supports

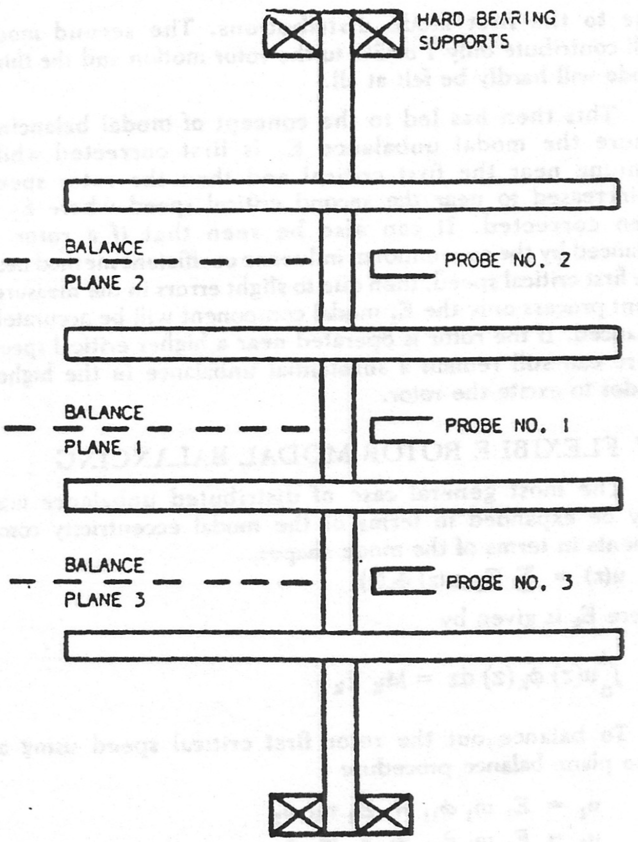


*Figure 1.4 Vertical Spin Pit Balance Facility*

The rotor first critical speed may be experimentally determined by placing the rotor on fixed end supports and electromagnetically exciting it to determine the resonance frequency. This works well on long thin rotors where gyroscopic effects due to shaft rotation do not materially change the synchronous critical speed.

The right hand side of eq. (1.4.12) for $R_3$ is given by

$$R_3 = -\frac{1}{\alpha_{11}} \Sigma\, \alpha_{1k}\, u_k = -M_1\, y_1\, \frac{(1-f_1^2)}{f_1^2} \qquad (1.5.2)$$

The value of $M_1$ for a multimass rotor is now interpreted as the modal mass and will be shown to be given by

$$M_1 = \int_0^L \rho\phi_1(z)^2\, dz = \Sigma\, m_i(z)\, \phi_{1i}(z)^2 \qquad (1.5.3)$$

where $\phi_{1i}(z)$ = mode of shape of rotor for first critical speed.

The balance correction weights for three plane balancing for the first critical speed is given by

$$\begin{bmatrix} u_{b1} \\ u_{b2} \\ u_{b3} \end{bmatrix} = -\begin{bmatrix} 1 & 1 & 1 \\ L_1 & L_2 & L_3 \\ 1 & \frac{\alpha_{12}}{\alpha_{11}} & \frac{\alpha_{13}}{\alpha_{11}} \end{bmatrix}^{-1} \begin{bmatrix} \Sigma\, u_k \\ \Sigma\, L_k U_k \\ \frac{w_1 y_1 (1-f_1^2)}{g f_1^2} \end{bmatrix} \qquad (1.5.4)$$

where $u_{b_i}$ are expressed in lb-in.

To summarize the three plane method without trial weights the following steps are taken

1. Determine $\Sigma\, u_k$ and $\Sigma\, L_k\, u_k$ by a standard soft mount bearing machine. However do not rigid body balance the rotor.
2. Calculate the rotor critical speed from a computer code with rigid supports or experimentally measure the rotor natural frequency by exciting it while placed on knife edge supports.
3. Place a static load at the major balance station 1 and measure the deflections at stations 2 and 3. From these deflections calculate the ratio of $\alpha_{12}/\alpha_{11}$ and $\alpha_{13}/\alpha_{11}$.
4. Calculate the rotor modal weight $W_1$ from the critical speed computer code. If this is not available take $W_1 = W_{total}/2$.
5. Place the rotor in the spin pit facility and record the rotor amplitude and phase $y_1$.
6. Calculate the balance correction weights $u_{b_i}$ and place on the rotor.
7. Rerun the rotor with the balance correction weights added and refine the balance by the influence coefficient method if the desired balance is not achieved.

## 1.6 FLEXIBLE ROTOR MODAL RESPONSE

The dynamical equations of motion of the multimass rotor may be written in matrix form where the rotor is composed of N mass stations

$$[m]_N\, [\ddot{y}]_N + [c]_N\, [\dot{y}]_N + [k]_N\, [y]_N = \omega^2\, [u]\, e^{i\omega t} \qquad (1.6.1)$$

The N mass station system has N critical speeds, however, we are only interested in the first j critical speed values such that

$4\, \omega_{max} < \omega_{cj}$

For each value of critical speed there is a corresponding mode shape

$\phi_j(y_j)$ or $\phi_{ji}$

It is assumed that the rotor amplitude can be expressed as a series in terms of the critical speed mode shapes as follows

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_N \end{bmatrix} = q_1 \begin{bmatrix} \phi_{11} \\ \phi_{12} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{1N} \end{bmatrix} + q_2 \begin{bmatrix} \phi_{21} \\ \phi_{22} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{2N} \end{bmatrix} + \ldots + q_j \begin{bmatrix} \phi_{j1} \\ \phi_{j2} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{jN} \end{bmatrix} \qquad (1.6.2)$$

The displacements at any station k are given by

$$y_k = \sum_{i=1}^{j} q_i\, \phi_{ik} \qquad (1.6.3)$$

The equations of motion may be expressed in the series form by

$$\sum_{i=1}^{j} \ddot{q}_i\, [M]\, [\phi_i] + \sum_{i=1}^{j} \dot{q}_i\, [c]\, [\phi_i] + \Sigma\, q_i\, [k]\, [\phi_i] = \omega^2\, [u] \qquad (1.6.4)$$

The equations may be uncoupled by multiplying by the kth mode and using the principal of orthogonality of the modes. It is also assumed that damping may also be approximately treated if the damping is either small, or is proportional to the $[k]$ or $[M]$ matrix, or if the complete complex mode shapes are used.

Multiplying by the transpose vector $[\phi_j]^T$ we obtain

$$[\phi_k]^T [M] [\phi_i] \ddot{q}_i + [\phi_k]^T [c] [\phi_i] \dot{q}_i + [\phi_k]^T [k] [\phi_i] q_i = \omega^2 [\phi_k]^T [u] \quad (1.6.5)$$

From orthogonality we obtain

$$[\phi_k]^T [M] [\phi_i] = \delta_{ik} M_k$$

$m_k$ = modal mass for the kth natural frequency

We now obtain j uncoupled equations of the form

$$m_k \ddot{q}_k + c_k \dot{q}_k + K_k q_k = \omega^2 [\phi_k]^T [u] e^{i\omega t} \quad (1.6.6)$$

By dividing the equation by the modal mass $M_k$, the resulting equation can be obtained

$$\ddot{q}_k + 2\omega_k \xi_k \dot{q}_k + \omega_k^2 q_k = E_k e^{i\omega t} \quad (1.6.7)$$

where

$w_k$ = kth natural frequency

$\xi_k$ = kth mode damping $= \dfrac{[\phi_k]^T [c] [\phi_k]}{2 M_k \omega_k}$

$E_k$ = modal unbalance eccentricity $= \dfrac{[\phi_k]^T [u]}{M_k}$

The solution of $q_k$ is given by

$$q_k = \frac{\omega^2 E_k}{\omega^2 - \omega_k^2 + 2 i \omega_k \xi_k \omega} \quad (1.6.8)$$

Let

$$f_k = \frac{\omega}{\omega_k}$$

then

$$q_k = \frac{E_k f_k^2}{1 - f_k^2 + 2 i \xi_k f_k} = e_k A_k (f_k) \quad (1.6.9)$$

Notice now that the modal multiplication factor $q_k$ appears to be in similar form to the amplitude equation for the single mass rotor.

The motion at any station y is given by

$$[y] = E_1 A_1 [\phi_1] + E_2 A_2 [\phi_2] + \cdots + E_j A_j [\phi_j] \quad (1.6.10)$$

Examination of the amplitude response of the rotor at first glance would appear to be expressed as an infinite sum of all of the various modal components. However most rotating machinery usually operates only through several critical speeds. Examination of the modal amplification factors shows that the higher modes become vanishingly small and do not have to be considered.

For example consider the modal amplification factors for a uniform rotor operating at 95% of the first critical speed and $\omega_2/\omega_1 = 4$, $\omega_3/\omega_1 = 9$. The modal amplification factors are given by (neglecting damping)

$$A_1 = \frac{.95^2}{1 - .95^2} = 9.26$$

$$A_2 = \frac{\left(\frac{.95}{4}\right)^2}{1 - \left(\frac{.95}{9}\right)^2} = 0.06; \quad \frac{A_2}{A_1} = 0.006$$

$$A_3 = \frac{\left(\frac{.95}{9}\right)^2}{1 - \left(\frac{.95}{4}\right)^2} = 0.01; \quad \frac{A_3}{A_1} = 0.0001$$

Therefore if the rotor is operated close to the first critical speed, the amplitude of motion will be predominately due to the first mode distributions. The second mode will contribute only 1 or 2% to the rotor motion and the third mode will hardly be felt at all.

This then has led to the concept of modal balancing where the modal unbalance $E_1$ is first corrected while running near the first critical and then the rotor speed is increased to near the second critical speed where $E_2$ is then corrected. It can also be seen that if a rotor is balanced by the conventional influence coefficient method near the first critical speed, then due to slight errors in the measurement process only the $E_1$ modal component will be accurately balanced. If the rotor is operated near a higher critical speed there can still remain a substantial unbalance in the higher modes to excite the rotor.

## 1.7 FLEXIBLE ROTOR MODAL BALANCING

The most general case of distributed unbalance u(z) may be expanded in terms of the modal eccentricity components in terms of the mode shapes

$$u(z) = \sum E_k m(z) \phi_k(z)$$

where $E_k$ is given by

$$\int_0^L u(z) \phi_k(z)\, dz = M_k E_k$$

To balance out the rotor first critical speed using a three plane balance procedure

$$u_1 = E_1 m_1 \phi_{11} + E_2 m_2 \phi_{21}$$
$$u_2 = E_1 m_2 \phi_{12} + E_2 m_2 \phi_{22}$$
$$u_3 = E_1 m_3 \phi_{13} + E_2 m_2 \phi_{23}$$

Where $m_i$ are the effect masses at the balance stations. The effective masses may be computed from the critical speed mode shape and values of modal mass by

$$M_1 = m_1 \phi_{11}^2 + m_2 \phi_{12}^2 + m_3 \phi_{13}^2$$
$$M_2 = m_1 \phi_{21}^2 + m_2 \phi_{22}^2 + m_3 \phi_{23}^2$$

The effective mass stations should also satisfy the rotor orthogonality conditions

$$0 = m_1 \phi_{11} \phi_{21} + m_2 \phi_{12} \phi_{22} + m_3 \phi_{13} \phi_{23}$$

The unbalance can now be expressed as the sum of the two modal components

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}_1 + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}_2$$

The first set of unbalances will excite the rotor first mode and the second set will only excite the second critical speed.

In the combined modal — influence coefficient method of balancing near the first or higher order critical speed, a modal unbalance distribution is used rather than a single value. The trial unbalance distribution for the first mode is given by

$$[u_1]_T = u_T \begin{bmatrix} 1 \\ \dfrac{m_2 \phi_{12}}{m_1 \phi_{11}} \\ \dfrac{m_3 \phi_{13}}{m_1 \phi_{11}} \end{bmatrix} \quad (1.7.4)$$

This trial weight distribution is placed on the shaft and the rotor amplitude is measured. The trial unbalance eccentricity is given by

$$E_{1t} = \frac{u_T}{M_1}\left[\phi_{11} + \frac{m_2\,\phi_{12}^2}{m_1\,\phi_{11}} + \frac{m_3\,\phi_{13}^2}{m_1\,\phi_{11}}\right] \qquad (1.7.5)$$

The rotor amplitude of motion at any point along the rotor is given by

$$[y]_T = (E_1 + E_{1T})\,A_1[\phi_1] + E_2 A_2[\phi_2] + \ldots + E_j A_j[\phi_j] \qquad (1.7.6)$$

Subtracting the original measured amplitude at station k from the trial run we obtain

$$y_{Tk} - y_k = E_{1T}\,A_1\,\phi_{k1} \qquad (1.7.7)$$

The complex first modal amplification factor is determined by

$$A_1 = \frac{1}{N}\sum_{k=1}^{N}\left(\frac{y_{Tk} - y_k}{E_{1T}\phi_{ik}}\right) \qquad (1.7.8)$$

Where N is the number of stations at which measurements are made. If the first probe is assumed to be near the maximum rotor amplitude then the modal unbalance is given by

$$E_1 = \frac{y_1}{A_1\,\phi_{11}} = \left(\frac{y_1}{y_{T1} - y_1}\right)E_{1T} \qquad (1.7.9)$$

The modal balance correction $E_{1b}$ is placed opposite the unbalance eccentricity

$$E_{1b} = -E_1$$

The correction balance weights for the first critical speed are simply given by ratio of rotor amplitude at the center station 1 by

$$\begin{bmatrix} u_{11} \\ u_{21} \\ u_{31} \end{bmatrix}_b = \left(\frac{y_1}{y_1 - y_{T1}}\right)\begin{bmatrix} u_{11} \\ u_{21} \\ u_{31} \end{bmatrix}_T \qquad (1.7.10)$$

## 1.8 SUMMARY

a) A large fraction of all vibration problems in turbomachinery are due to unbalance. The real cost of these problems is usually measured in lost production rather than in the cost of balancing procedures.

b) Unbalances may be generally categorized in four ways: continuous unbalance distributions, point masses, shaft bows, and moment unbalances. Turbomachinery is usually subject to some combination of all four types.

c) Much of recent balancing literature is devoted to the influence coefficient method. For multi-mass flexible rotors, this method is successful where it is possible to place several noncontacting probes along the rotor and to have a computer available for inversion of the influence coefficient matrix. In many applications, the placement of probes along the rotor may not be possible for either technical or financial reasons.

d) The influence coefficient method can be relatively easily modified to include the effects of shaft bow, moment unbalance due to local unbalance couples or disc skew, and mechanical or electrical runout in the shaft.

e) Requirements for balancing of multi-mass flexible rotors may be stated as two equations describing a rigid body balance plus a flexible rotor balance requirement.

f) Modal balancing has been approached in two separate ways: considering the rotor as a series of point masses and treating it as a continuous rotor. This has resulted in the discussion of balancing in N + 2 planes or N planes. In order to balance a flexible single mass rotor on flexible supports, three balance planes are required to reduce the rotor amplitude at the mass as well as the transmitted bearing forces to zero. This concept may be extended to multi-mass rotors. When the bearing stiffnesses are large compared to the shaft stiffness, N plane balancing in sufficient. For cases where the bearing stiffnesses are of the same order of magnitude or smaller than the shaft stiffness, N + 2 plane balancing is required.

g) A method of balancing without trial weights is presented.

h) Accurate modal balancing will result in balance weight distributions which do not excite other modes. Small residual errors in the influence coefficient method may result in excitation of modes not directly balanced.

i) A combination modal-influence coefficient method of applying a modal trial weight distribution has been presented. This may result in an optimum technique for flexible multimass rotors.

## REFERENCES


1. Rankine, W. J., "On the Centrifugal Force of Rotating Shafts," Engineer, Vol. 27, 1869, p 249.
2. Dunkerley, S., "On the Whirling and Vibration of Shafts." Phil. Trans., Vol. 185, 1894.
3. Chree, C., "The Whirling and Transverse Vibration of Rotating Shafts," Phil. Mag., Vol. 7, 1904.
4. Jeffcott, H. H., "The Lateral Vibration of Loaded Shafts in the Neighborhood of a Whirling Speed — The Effect of Want of Balance," Phil. Mag., Vol. 37, 1919, p. 304.
5. Stodola, A., "Kritische Welienstrorung Infolge Der Nachgie — Bigkeit Des Oelpolsters Im Lager," Schweizerische Bauzeitung, Vol. 85, 1925, p 265.
6. Thearle, E. L., "Dynamic Balancing of Rotating Machinery in the Field," Trans. A.S.M.E., Vol. 56, 1934, pp. 745-753.
7. Bishop, R. E. D., "The Vibration of Rotating Shafts." J. Mech. Eng. Sci., Vol. 1, No. 1, 1959, pp. 50-64.
8. Bishop, R. E. D., "Unbalanced and Initially Bent Shafts," Engineering, Nov. 25, 1960, pp 735-736.
9. Bishop, R. E. D., and Gladwell, G. M. L., "The Vibration and Balancing of an Unbalanced Flexible Rotor," J. Mech. Engineering Sci., Vol. 1, No. 1, 1959, pp. 66-77.
10. Parkinson, A. G., Jackson, K. L., and Bishop, R. E. D., "Some Experiments on the Balancing of Small Flexible Rotors: Part I — Theory," J. Mech. Engineering Sci. Vol. 5, No. 1, 1963, pp. 114-128.
11. Dimentberg, F. M. *Flexural Vibrations of Rotating Shafts*, Butterworth and Co., Ltd., London, 1961.
12. Goodman, T. P., "A Least-Squares Method for Computing Balance Corrections," J. of Engineering for Ind., Trans A.S.M.E., Series B, Vol. 86, 1964, pp. 273-279.
13. Dimentberg, F. M., "Theory of Balancing Flexible Rotors," Russ. Eng. J., 1964, p. 11.
14. Rieger, N. F. "Rotor-Bearing Dynamics Design Technology, Part 1: State-of-the-Art." Technical Report AFAPL-TR-65-45, May, 1965.

15. Thomson, W. T., *Vibration Theory and Applications*, Prentice-Hall, N.J., 1965.

16. Loewy, R. G., and Piarullit, V. J., *Dynamics of Rotating Shafts*, Shock and Vibration Monograph, No. 4, Naval Research Laboratory Washington, D.C.

17. Gunter, E. J., "Dynamic Stability of Rotor-Bearing Systems," NASA Report SP-113, 1966.

18. Lund, J. W. "Self-Excited Stationary Whirl Orbits of a Journal in a Sleeve Bearing," Ph.D. Thesis, R. P. I., 1966.

19. Wilcox, J. B., *Dynamic Balancing of Rotating Machinery*, Pitman, London, 1967.

20. Kellenberger, W., "Balancing Flexible Rotors on Two Generally Flexible Bearings," Brown Boveri Review, Sept., 1967.

21. Wilcox, J. B., *Dynanic Balancing of Rotating Machinery*, Pitman and Sons, Ltd., London, 1967.

22. Bishop, R. E. D., "Vibration and Balancing of Flexible Shafts," Applied Mechanics Reviews, Vol. 21, No. 5, May 1968, p. 439.

23. Little, R. M., "Current State of the Art of Flexible Rotor Balancing Technology," Ph.D. Thesis Bibliography, U. of Virginia, Charlottesville, Va., 1971.

24. Le Grow, J. W., "Multiplane Balancing of Flexible Rotors — A Method of Calculating Correction Weights," A.S.M.E. paper No. 71 — Vibr. 52.

25. Lund, J. W. and Jonnesen, J., "Analysis and Experiments on Multi-Plane Balancing of a Flexible Rotor," A.S.M.E. paper No. 71 — Vibr. 74.

26. Tessarzik, J. M., Badgley, R. H., and Anderson, W. J., "Flexible Rotor Balancing by the Exact Point-Speed Influence Coefficient Method," A.S.M.E. paper No. 71 — Vibr. 91.

27. Bishop, R. E. D. and Parkinson, A. G., "On the Use of Balancing Machines for Flexible Rotors," J. of Engineering for Ind., Trans. A.S.M.E., Series B, Vol. 94, 1972, pp. 561-576.

28. Kellenberger, W., "Should a Flexible Rotor Be Balanced in N or (N + 2) Planes?" J. of Engineering for Ind., Trans. A.S.M.E., Series B, Vol. 94, 1972, pp. 548-560.

29. Kirk, R. G., and Gunter, E. J. "Nonlinear Transient Analysis of Multimass Flexible Rotors — Theory and Applications," NASA CR-2300, 1973.

30. Dostal, M., "Vibration Control of Rotor Bearing Systems," Ph.D. Thesis, U. of Sussex, 1974.

31. Nicholas, J. C., Gunter, E. J., and Allaire, P. E., "Effect of Residual Shaft Bow on Unbalance Response and Balancing of a Single Mass Flexible Rotor: Part I — Unbalance Response," *Journal of Engineering for Power*, Series A, Trans. A.S.M.E., Vol. 98, No. 2, April, 1976, pp. 171-181.

32. Den Hartog, J. P., "The Balancing of Flexible Rotors," Air. Space and Instrumentation (Draper Anniv. Vol), McGraw-Hill, New York, 1963.

# PART II — EXPERIMENTAL RESULTS

## 2.1 SINGLE MASS ROTOR RESPONSE — RIGID SUPPORT

Fig. 2.1.1 represents the amplitude of motion of a single mass rotor before and after balancing. Before balancing, the rotor has a slight residual bow of 1 mil and reaches a rotor amplitude of 16. The rotor was balanced to zero amplitude at 1,800 RPM, which was just below the rotor critical speed. Note that although the rotor amplitude was balanced to zero, there is still a substantial amplitude response at the first critical speed of over 6 mils. This amplitude response at the first critical speed is due to the fact that the rotor bow was not taken into consideration in the balancing.

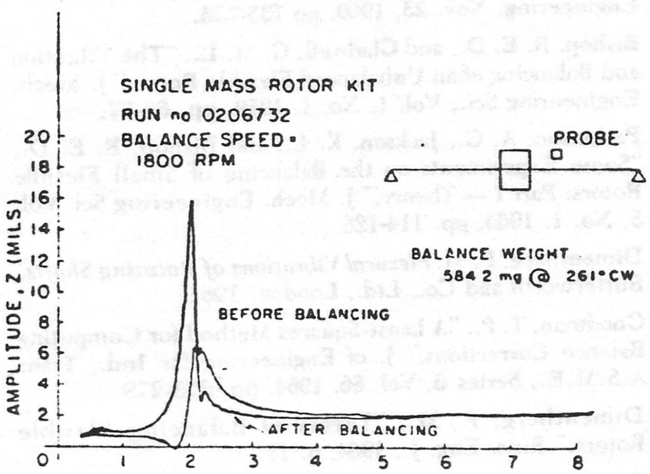


*Figure 2.1.1 Amplitude vs. RPM*

In Fig. 2.1.2, the rotor was rebalanced at 1,800 RPM considering the rotor bow. It can be seen that the rotor amplitude almost goes to zero at the critical speed of 2,000 RPM, and the amplitude is quite uniform above the critical speed. Note that the difference in balancing weights between the two cases is only 24 mg. shifted through an angle of 20°. The total balancing weight is 560 mg. Therefore it can be seen that the additional correction required for the shaft bow is extremely small in comparison to the total rotor unbalance.

Fig. 2.1.3 represents another single mass rotor with a smaller disc attached to it. With the lighter weight disc, the critical speed is higher and is approximately at 4,000 RPM. The three curves represent a trial and error balancing of the rotor until the low rotor amplitude of curve 3 was obtained. Notice

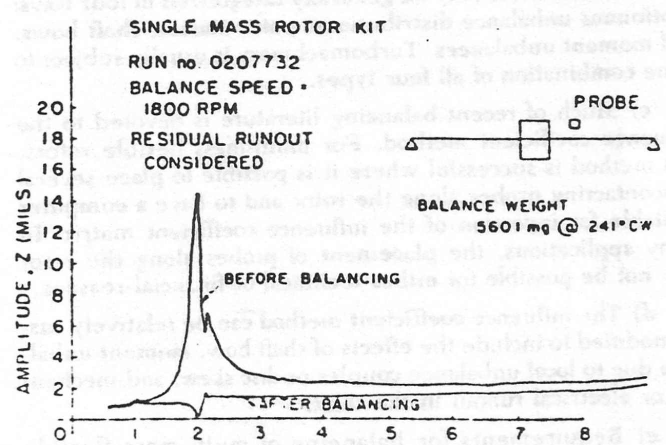


*Figure 2.1.2 Amplitude vs. RPM*

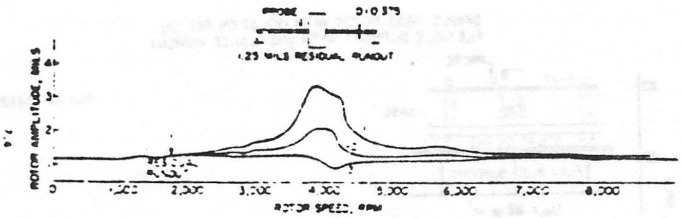


*Figure 2.1.3 Balancing of a Single Mass Rotor With a Bowed Shaft*

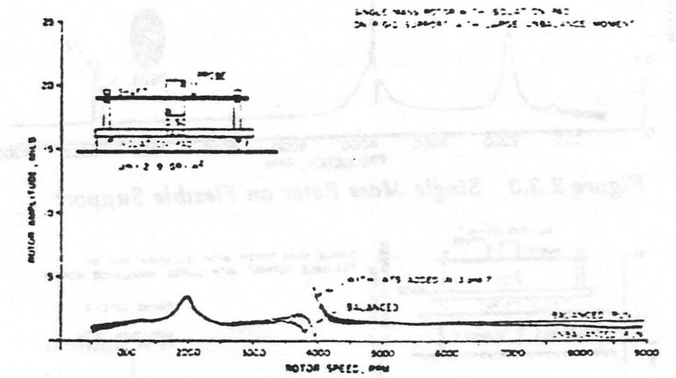


*Figure 2.2.1 Single Mass Rotor Amplitude Mounted on an Isolation Pad with a Large Unbalanced Moment*

with curve 3 that there is a slight dip in the rotor amplitude at the critical speed. This is characteristic of a rotor with a shaft bow.

## 2.2 INFLUENCE OF ISOLATION PAD ON SINGLE MASS ROTOR RESPONSE

Fig. 2.2.1 represents the rotor motion of the single mass system on an isolation pad with a large unbalance moment on the disc. Note that the critical speed response at 4,000 RPM is quite low with the unbalance added. This is because of the attenuation characteristics of the isolation pad. When the rotor is balanced, there is again observed a reduction of the rotor amplitude at the critical speed due to the slight shaft bow. It is of interest to note that there is a response of the rotor system at approximately one-half of the rotor critical speed at 2,000 RPM. Note that the balance of the rotor does not appear to effect this rotor response. This appears to be a secondary or gravitational critical speed caused by the rotor weight or shaft asymmetry. This rotor response cannot be predicted by linear theory and is due to nonlinear effects in the system.

Fig. 2.2.2 represents the balanced single mass rotor with and without the isolation pad. When the rotor is operating with the isolation pad on the support system, the rotor seems to be extremely well balanced, and there is very little response at the first critical speed. However when the rotor is clamped to a rigid support to eliminate the influence of the isolation pad, there is an increase in the secondary critical speed response at 1,800 RPM, and the unbalance response at the critical speed is so high that it is impossible to run the rotor through the critical speed region without damaging the system. This figure then represents a dramatic example of the influence of an isolation pad which contains damping of the foundation to reduce the rotor critical speed response.

Fig. 2.2.3 represents the rotor on the isolation pad after balancing and with weights added to it. In the unbalance condition there is a very small response on the isolation pad. Again when the rotor assembly is clamped down, there is a violent unbalance response at the first critical speed.

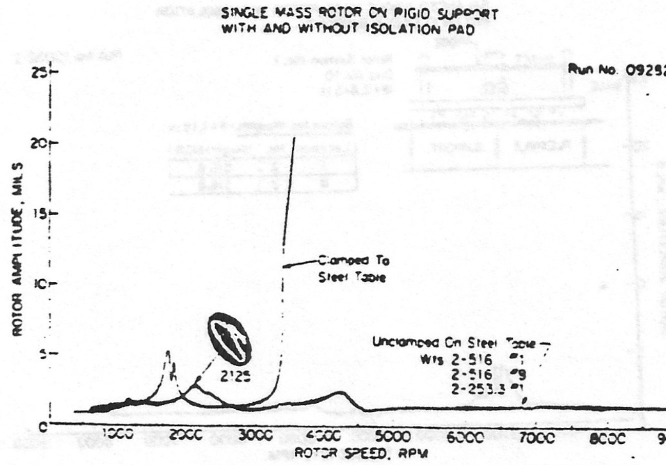


*Figure 2.2.2 Single Mass Rotor or Rigid Support*

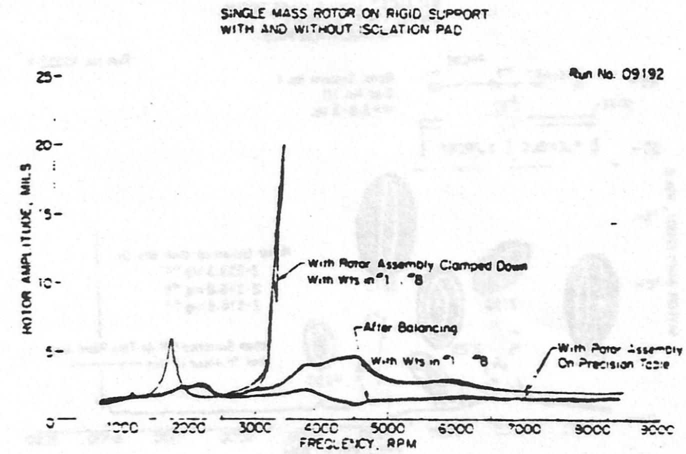


*Figure 2.2.3 Single Mass Rotor on Rigid Support*

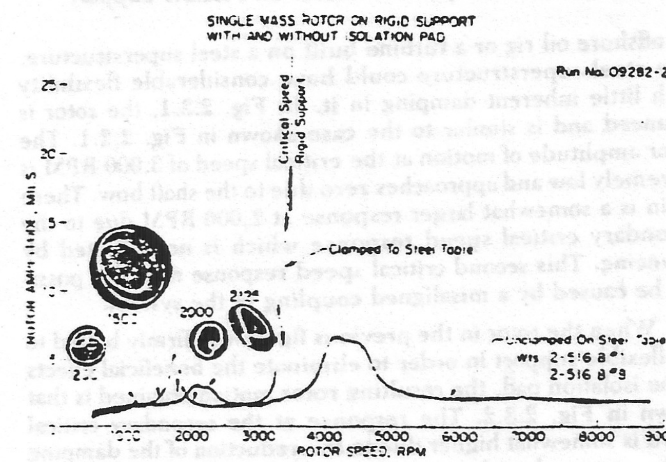


*Figure 2.2.4 Single Mass Rotor on Rigid Support*

Fig. 2.2.4 is similar to the previous figure with a slightly different unbalance weight. Shown on the plots are traces of the rotor motion obtained from the oscilloscope pictures of the shaft motion.

## 2.3 INFLUENCE OF A FLEXIBLE SUPPORT ON ROTOR RESPONSE WITH AND WITHOUT ISOLATION PAD

Fig. 2.3.1 represents the balanced rotor with an isolation pad under it on a flexible support. This type of condition could occur in practice, for example, with a compressor or turbine on

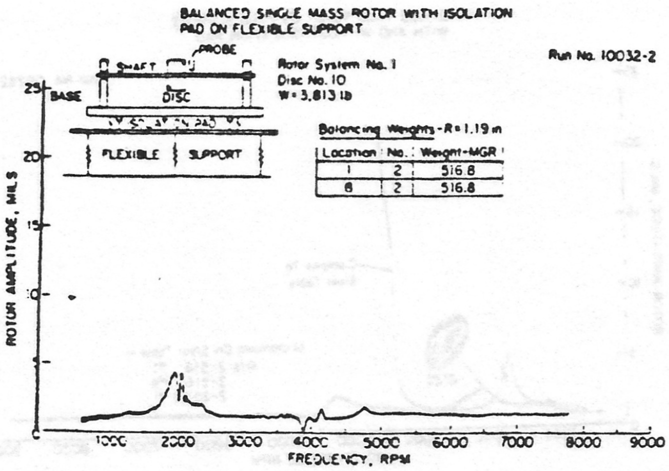


Figure 2.3.1 Single Mass Rotor on Flexible Support

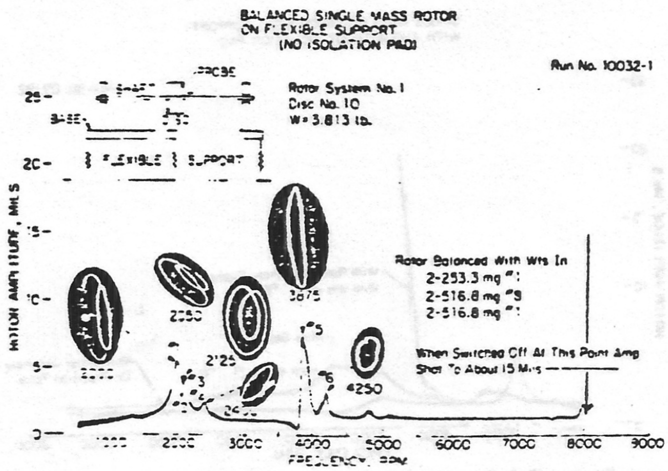


Figure 2.3.2 Single Mass Rotor on Flexible Support

an offshore oil rig or a turbine built on a steel superstructure. The steel superstructure could have considerable flexibility with little inherent damping in it. In Fig. 2.3.1, the rotor is balanced and is similar to the case shown in Fig. 2.2.1. The rotor amplitude of motion at the critical speed of 3,900 RPM is extremely low and approaches zero due to the shaft bow. There again is a somewhat larger response at 2,000 RPM due to the secondary critical speed response which is not affected by balancing. This second critical speed response may also possibly be caused by a misaligned coupling in the system.

When the rotor in the previous figure was firmly bolted to the flexible support in order to eliminate the beneficial effects of the isolation pad, the resulting rotor motion obtained is that shown in Fig. 2.3.2. The response at the secondary critical speed is somewhat higher due to the reduction of the damping in the system, but the response at the critical speed at 3,800 RPM is now extremely pronounced. The orbit obtained from the oscilloscope shows a predominantly vertical response of the system in the direction of the flexible support. When the rotor speed was increased to approximately 8,000 RPM, the amplitude of motion became extremely severe with an excitation of the first critical speed. The amplitude of motion abruptly shot up to 15 mils. Therefore it can be seen that a machine could be adequately balanced on a test stand, and when it is installed in an installation which has a low stiffness, it is possible that severe vibrational amplitudes may result in the rotor system even though the level of unbalance in the rotor is quite low. This effect has been caused by the flexible support reducing the effective damping of the isolation pad and the

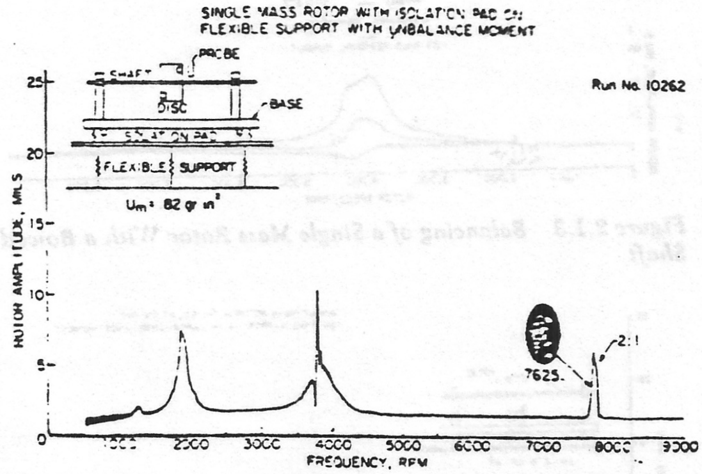


Figure 2.3.3 Single Mass Rotor on Flexible Support

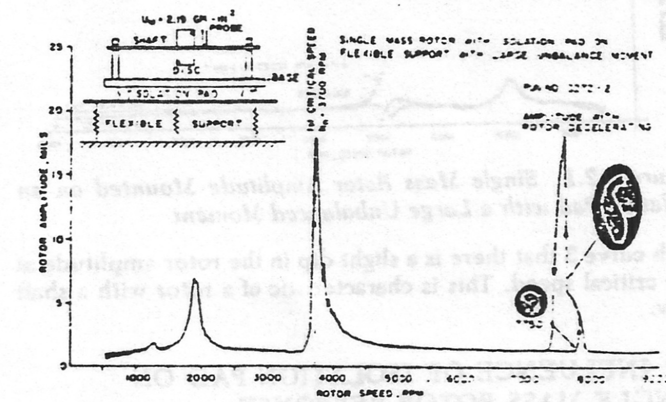


Figure 2.3.4 Single Mass Rotor Amplitude with Isolation Pad on Flexible Support with Large Unbalanced Moment

bearings. The rotor has now been changed from a moderately low amplification factor rotor to one of extremely high amplification. Therefore the rotor critical speed is severely excited and also nonsynchronous motion occurring at twice the running speed is encountered. If this type of motion were encountered with an operational machine, then disastrous effects could result. The amplitude of motion at the 8,000 RPM range occurs so rapidly that it would be impossible to control this motion in an actual rotor before destruction occurred.

Fig. 2.3.3 represents the rotor motion with the isolation pad on the flexible support with unbalance. Here the unbalance does cause a response at the critical speed and a substantial secondary critical speed amplitude. There is a small component at twice the first critical speed at 7,625 RPM.

Fig. 2.3.4 represents the system similar to the previous figure except that the unbalance has been increased from .82 gm-in² moment to 2.19. There is little increase in the secondary critical speed at 2,000 RPM, but the first critical speed response at 3,700 RPM increases dramatically. Note the nonlinear rapid increase of amplitude upon the increase in speed. The first critical speed response occurs extremely rapidly. Upon the increase in speed, there was a small response at 7,750 RPM. Upon decelerating the rotor there was an extremely rapid unbalance response with an amplitude of the same order of magnitude as the first critical speed response. This excitation at two times the critical speed could cause severe machine problems upon rotor deceleration.

Fig. 2.3.5 is similar to the previous figure except that the rotor amplitudes were taken with rapid acceleration and deceleration rates. With balancing weights in stations 7 and 3, no

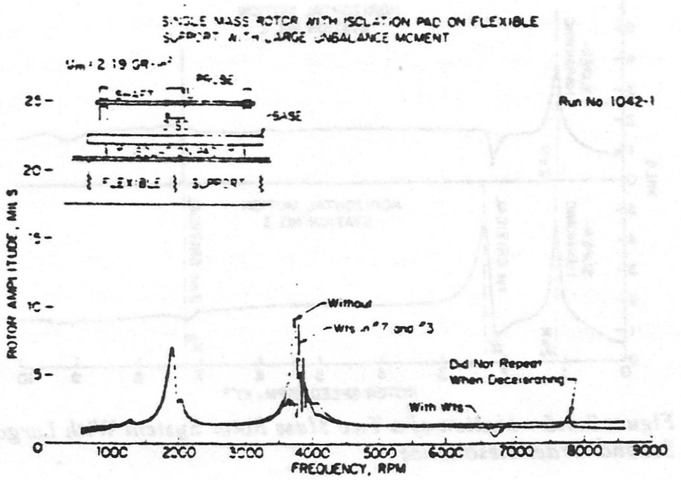


*Figure 2.3.5 Single Mass Rotor Amplitude with isolation pad on flexible support with large unbalanced moment*

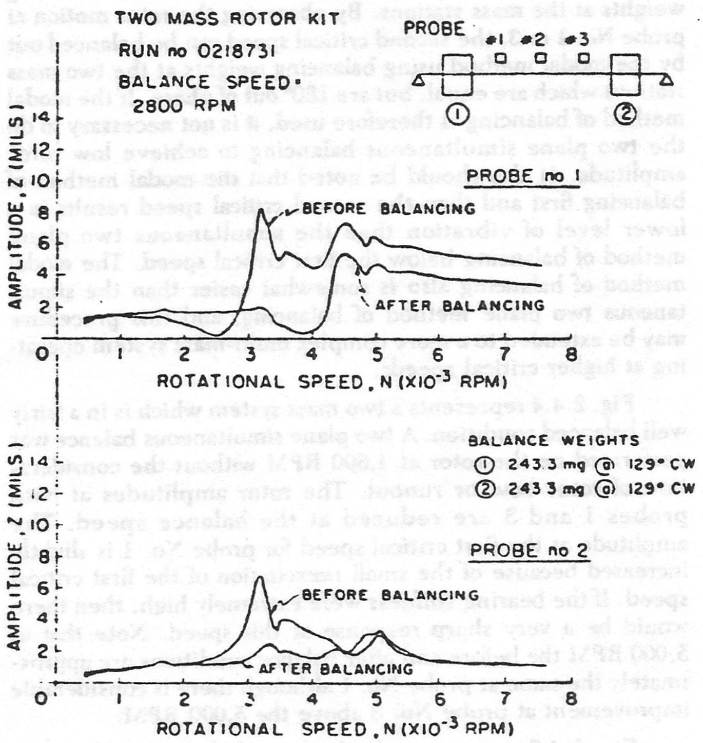


*Figure 2.4.1 Two Mass Rotor Kit*

high response was obtained at twice the rotor critical speed. With the high unbalance and rapid acceleration, the rotor critical speed was greatly reduced. If the rotor is rapidly decelerated then the high response at twice the rotor critical speed was not excited.

## 2.4 TWO MASS ROTOR MOTION

Fig. 2.4.1 represents the motion of a two mass rotor system in which the two masses are placed symmetrically inboard of the bearing location. The shaft motion was monitored by three noncontacting probes. The probe No. 2 was located at the rotor center, and the probes 1 and 3 were located near the major mass stations. The rotor system has two critical speeds; one at 3,200 RPM and the other at 4,700 RPM. At the first critical speed, the mode shape is at a maximum at the rotor center, and the two mass stations are in phase. In the second critical speed, the mass stations are out of phase, and the probe No. 2 is a nodal point. Before balancing, probe No. 1 shows a substantial amplitude at the first and the second critical speed.

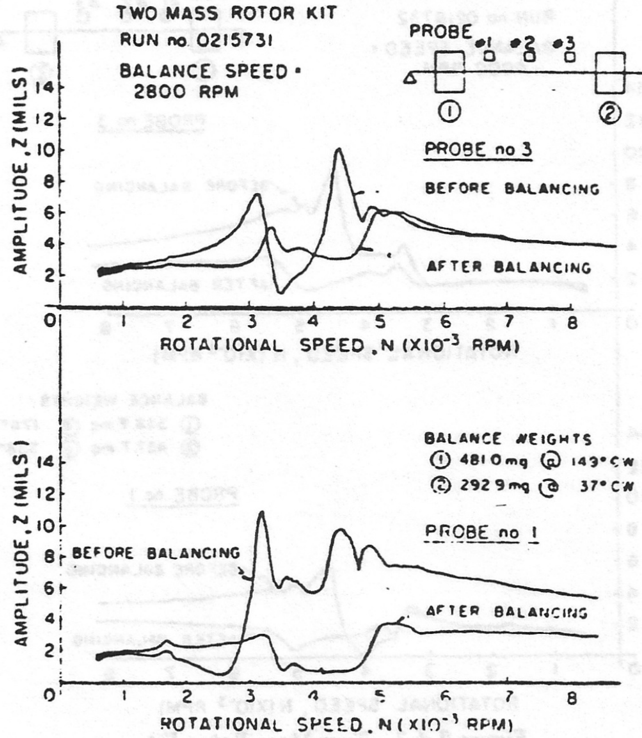


*Figure 2.4.2 Two Mass Rotor Kit*

Probe No. 2, which is at the rotor center, only shows a sizable peak at the first critical speed. Probe No. 2, which is at the rotor center, only shows a sizable peak at the first critical speed as would be expected. The balance speed that was used for this run was 2,800 RPM, which is below the first critical speed. After the rotor first critical speed was balanced by the application of two weights in phase, the first critical speed amplitude is reduced considerably for both probes, 1 and 2, and a slight reduction at the second critical speed is achieved at probe No. 1. The balancing condition achieved on this run is equivalent to a modal balancing procedure since the balancing weights used at the masses are identical and are of the same phase relationship. The center probe was used for the balance readings and the two balance weights were calculated by the single plane balance theory.

In Fig. 2.4.2 the rotor system was similar to the run shown in the previous figure except there was somewhat more shaft bow and unbalance in the mass stations. From probe No. 3 there can be seen two distinct resonance frequencies. The resonance frequency at the second mode for probe No. 3 is the highest because the second mode is a conical mode. For probe No. 1 the maximum amplitude occurs at the first critical speed, and this motion is in phase to the peak at station No. 3. A two plane simultaneous balance was performed on the rotor at the 2,000 RPM and the resulting rotor amplitude is shown in the figure. There is a considerable reduction in amplitude at the probe station No. 1 and somewhat of a reduction at probe No. 3. The amplitudes of the vibration were not reduced to a very low level because in the balance calculations, rotor bow was not taken into consideration.

Fig. 2.4.3 represents the two mass system in which the first mode has been balanced out. As can be seen, there exist a substantial second mode amplitude at probe No. 3. The balancing speed selected was 4,000 RPM, which is below the second critical speed. At this speed, the amplitude at the probe locations 1 and 3 are approximately 180° out of phase. The balance correction weights were calculated for these two speeds using the two plane method without the consideration of rotor bow.

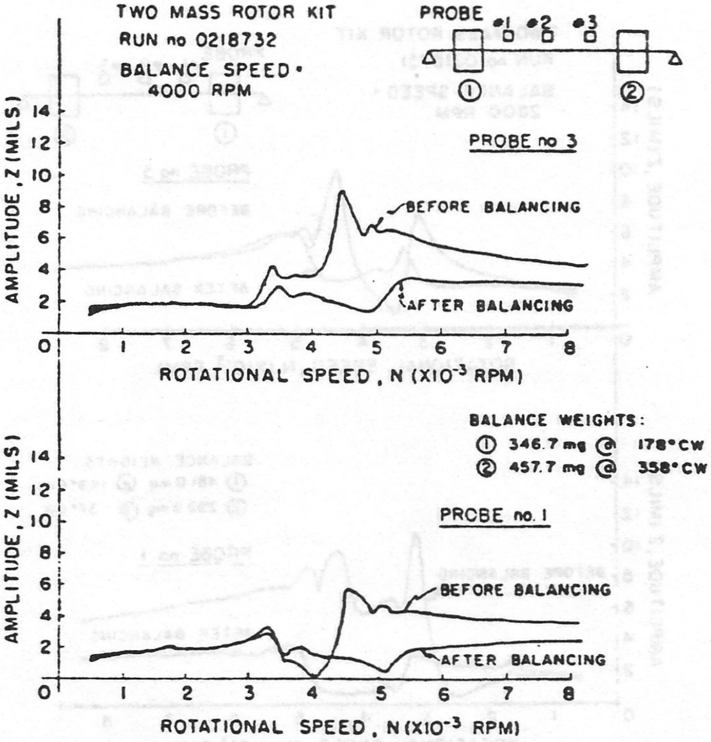


*Figure 2.4.3 Two Mass Rotor Kit*

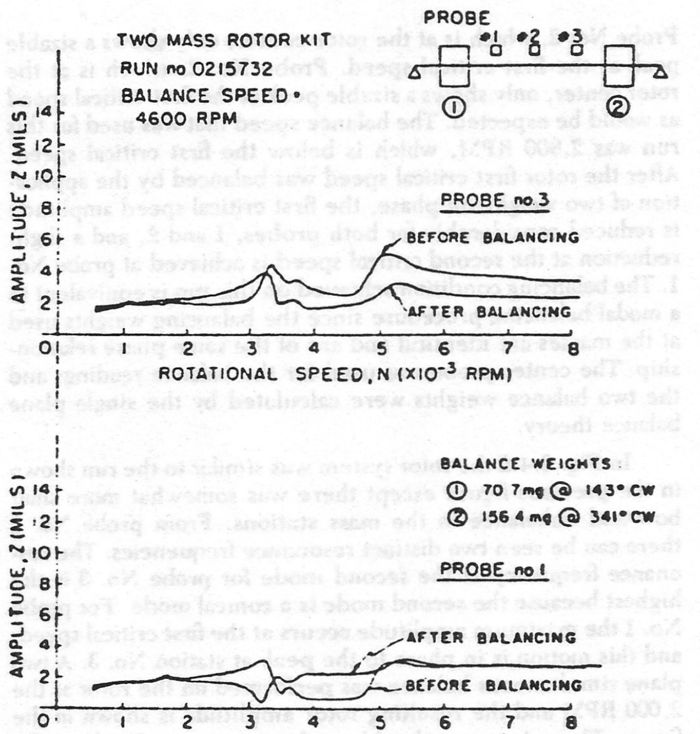


*Figure 2.4.4 Two Mass Rotor Kit*

The motion after balancing has caused a considerable reduction of the rotor amplitude. Note that the application of the balancing weights causes little excitation of the first critical speed. Upon close examination of the balance correction weights, the components are almost of the same magnitude and are exactly 180° out of phase. Therefore these balance weights would represent a second mode correction factor. The rotor could have been balanced for the first and second critical speeds by the use of probe No. 1 and No. 2 only. For example, by the use of probe No. 2 monitoring the rotor center, the first critical speed can be balanced out by the use of in phase modal balancing weights at the mass stations. By observing the rotor motion at probe No. 1 or 3, the second critical speed can be balanced out by the modal method using balancing weights at the two mass stations which are equal, but are 180° out of phase. If the modal method of balancing is therefore used, it is not necessary to do the two plane simultaneous balancing to achieve low rotor amplitude. It also should be noted that the modal method of balancing first and then the second critical speed results in a lower level of vibration than the simultaneous two plane method of balancing below the first critical speed. The modal method of balancing also is somewhat easier than the simultaneous two plane method of balancing, and this procedure may be extended to a more complex multi-mass system operating at higher critical speeds.

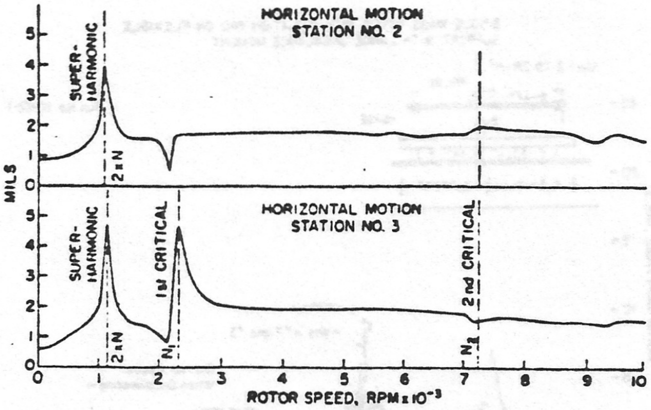


*Figure 2.4.5 Motion of a Two Mass Rotor System With Large Second Order Resonance*

Fig. 2.4.4 represents a two mass system which is in a fairly well balanced condition. A two plane simultaneous balance was performed on the rotor at 4,600 RPM without the consideration of rotor bow or runout. The rotor amplitudes at both probes 1 and 3 are reduced at the balance speed. The amplitude at the first critical speed for probe No. 1 is slightly increased because of the small reexcitation of the first critical speed. If the bearing stiffness were extremely high, then there would be a very sharp response at this speed. Note that at 5,000 RPM the before and after balance conditions are approximately the same at probe No. 1 although there is considerable improvement at probe No. 3 above the 5,000 RPM.

Fig. 2.4.5 represents the horizontal motion at the rotor center at station No. 2 and at the major mass station, station No. 3, for a well balanced two mass system. Note that at the center station there is a considerable amplitude drop at the first critical speed indicating a bowed shaft. At one-half of the first critical speed, there is a large superharmonic excitation. At the second critical speed, there is no excitation. At station No. 3, there is a first critical speed response and a superharmonic excitation when the rotor is operating at one-half the critical speed. The superharmonic oscillation is caused by nonlinearities in the system and may be caused by a misaligned shaft and coupling.

## 2.5 THREE MASS ROTOR RESPONSE

Fig. 2.5.1 represents the unbalance response before and after balancing the first critical speed of a three mass rotor system. It is seen from the figure that there is a considerable first mode excitation as observed by probes 1 and 2 at the first critical speed at 3,000 RPM. The maximum amplitude at probe location 1 was approximately 13 mils and 12 mils at probe

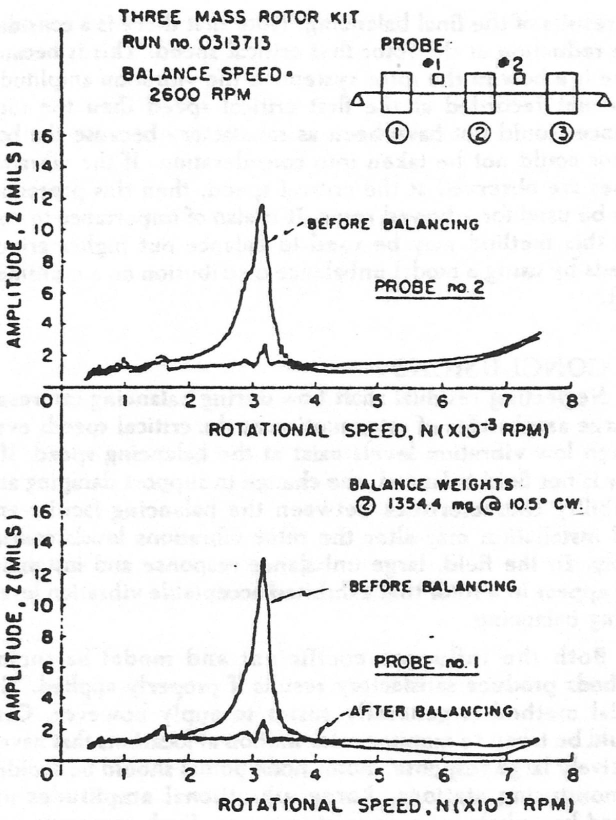


*Figure 2.5.1 Three Mass Rotor Kit*

location No. 2. These amplitudes were in phase. The rotor was balanced by the single plane correction at the mass station No. 2 to balance out the first mode excitation. A balance weight of 1,364 mg. at a phase angle of 10.5° was placed on the second disc. It can be seen that the first critical speed has been well balanced out. This particular run is an example of how a multimass rotor can be balanced out in its first critical speed by the application of modal balancing. In the case of the three mass system, only one balancing station at the rotor center is required to balance out the first critical speed. As the speed increases to 7,000 RPM, note that the amplitude at probe station No. 2 increases. This is because the rotor is approaching a second critical speed. Additional probes placed at the bearing locations showed that there was a sizable amplitude at the 7,000 RPM range.

Fig. 2.5.2 represents a three mass system which has been balanced for two critical speeds by means of a modal method. The rotor first critical speed at 2,300 RPM has an extremely high response of 13 mils. A heavier mass was placed at station No. 2 than was used in the run shown in Fig. 2.5.1 in order to reduce the critical speeds so that two would occur in the operating range. It is also of interest to note that there is a slight residual runout, and that also small components of the second and third order harmonics were excited in the rotor. These secondary harmonics however were not affected by the balance level in the rotor. In the first balancing run (curve No. 2 on the figure) unbalance weights were placed in phase at the three discs. As it can be seen from the response curve, the first critical speed amplitude was greatly reduced. However when the rotor speed was increased beyond 4,500 RPM such that the second critical speed was approached, it was not possible to operate through the second critical speed.

In order to balance out the second critical speed a second mode balance distribution was used on the rotor in which weights of 253 mg. were placed at the end mass stations 1 and

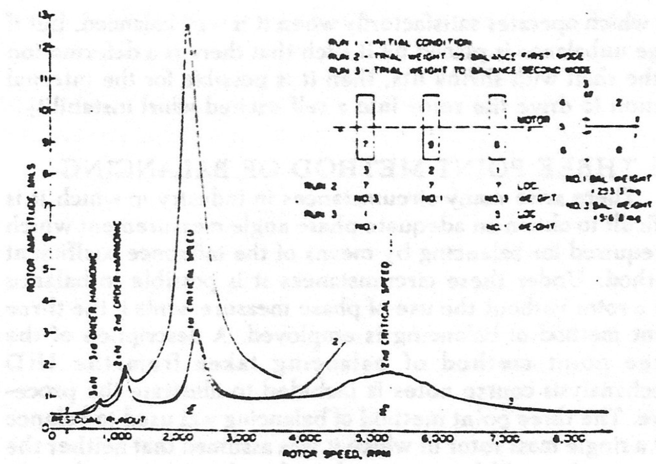


*Figure 2.5.2 Modal Balancing of a Three Mass Rotor*

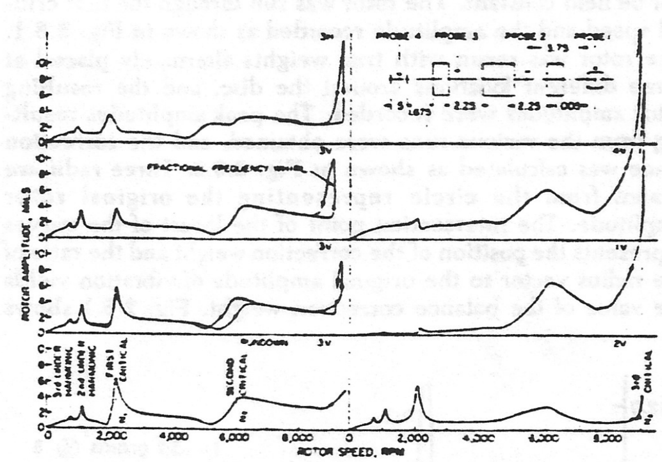


*Figure 2.5.3 Unbalance Response and Self-Excited Instability With a Three-Mass Rotor System*

3,180° out of phase. These unbalance distributions are second modal components and hence will not excite the rotor first critical speed. In response run No. 3, it can be seen that there is no additional excitation of the first critical speed, and the second critical speed has been satisfactorily balanced. Therefore by using modal distribution of balance weights for the first and second critical speeds the rotor can be balanced through two critical speeds by using single plane balancing theory based on influence coefficients with one probe observing the rotor motion between discs 1, 2, and 3.

Fig. 2.5.3 represents the motion of a three mass rotor at which three critical speeds are excited at 2,000, 6,000, and 9,000 RPM. In this particular run, an unbalance distribution was placed on the rotor to excite the third mode. Unbalances at the end discs were placed in phase and unbalance at the center disc was placed out of phase. The third mode unbalance distribution on the rotor caused a large synchronous vibration to develop at the end probe positions of 1 and 4 of over 20 mils. When this occurred, the center of the rotor at 9,000 RPM jumped abruptly into a fractional frequence whirl motion. This whirl motion was eliminated by reducing the rotor speed. The self excited whirl motion in the rotor was only observed when there was sufficient unbalance in the rotor to cause a considerable amount of bending of the shaft. It is believed that the bending of the shaft initiated internal friction between the shaft and the disc causing a self excited whirl motion. It has been reported in the literature by Newkirk and others that often a large shock or excitation is required to initiate shaft whirl due to internal friction. Therefore it is possible to have a compressor which operates satisfactorily when it is well balanced, but if large unbalance is placed on it such that there is a deformation of the shaft with shrink fits, then it is possible for the internal friction to drive the rotor into a self excited whirl instability.

## 2.6 THREE POINT METHOD OF BALANCING

There arise many circumstances in industry in which it is difficult to obtain an adequate phase angle measurement which is required for balancing by means of the influence coefficient method. Under these circumstances it is possible to balance out a rotor without the use of phase measurements if the three point method of balancing is employed. A description of the three point method of balancing taken from the IRD Mechanalysis course notes is included to illustrate the procedure. The three point method of balancing was used to balance out a single mass rotor in which it was assumed that neither the phase angle could be measured or that the rotor speed could not be held constant. The rotor was run through the first critical speed and the amplitude recorded as shown in Fig. 2.6.1. The rotor was rerun with trial weights alternately placed at three different locations around the disc, and the resulting rotor amplitudes were recorded. The peak amplitudes resulting from the various runs were obtained, and the correction plane was calculated as shown in Fig. 2.6.2. Three radii are drawn from the circle representing the original rotor amplitude. The intersection point of the locust of the curves represents the position of the correction weight and the ratio of the radius vector to the original amplitude of vibration yields the value of the balance correction weight. Fig. 2.6.1 shows the results of the final balancing. Note that there is a considerable reduction at the rotor first critical speed. This is because there is a bow in the rotor system. If the vibration amplitudes were not recorded at the first critical speed then the rotor balance would not have been as satisfactory because the bow bector could not be taken into consideration. If the vibration values are observed at the critical speed, then this procedure may be used for a bowed rotor. It is also of importance to note that this method may be used to balance out higher critical speeds by using a modal unbalance distribution on a multimass shaft.

## 2.7 CONCLUSIONS

Neglecting residual shaft bow during balancing can result in large amplitudes of rotor motion at the critical speeds even though low vibration levels exist at the balancing speed. If a rotor is not field balanced, the change in support damping and flexibility characteristics between the balancing facility and field installation may alter the rotor vibrations levels considerably. In the field, large unbalance response and instability may appear in a rotor that exhibited acceptable vibration levels during balancing.

Both the influence coefficient and modal balancing methods produce satisfactory results if properly applied. The modal method is generally easier to apply however. Care should be taken to monitor rotor motion at locations that have a relatively large response. Rotor node points should be avoided as monitoring stations. Large vibratiional amplitudes not caused by unbalance may exist in rotors. Such responses cannot be altered or removed by balancing although they can be affected by rotor acceleration and support changes.

Satisfactory balancing in the field can be achieved without phase angle information using a three point balancing technique. The method can be applied as both a single plane correction or as a modal balance correction. Care must be taken to select proper balancing speeds and to take into account permanent rotor bow.

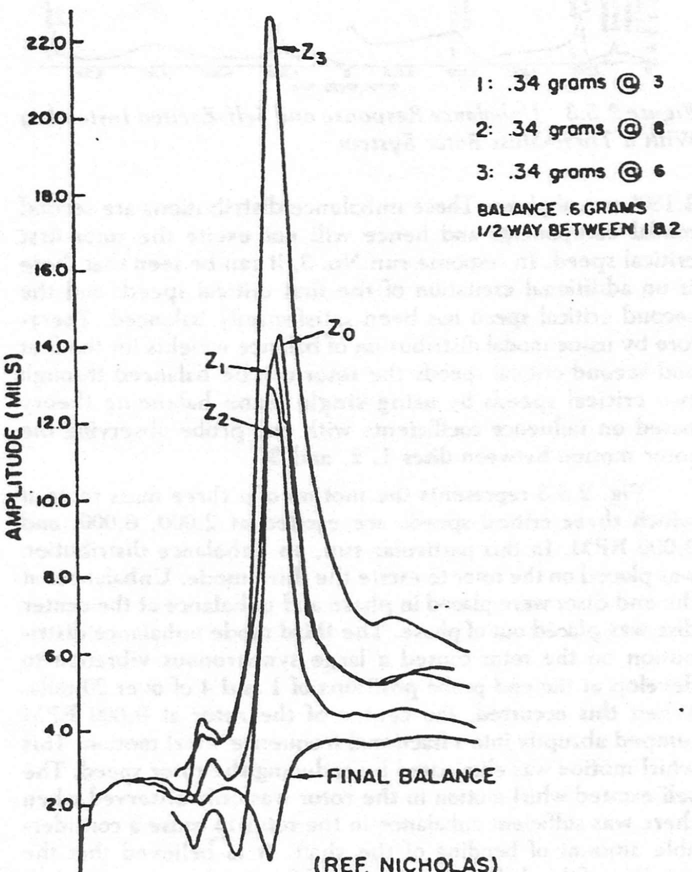


*Figure 2.6.1 Amplitude vs. RPM*

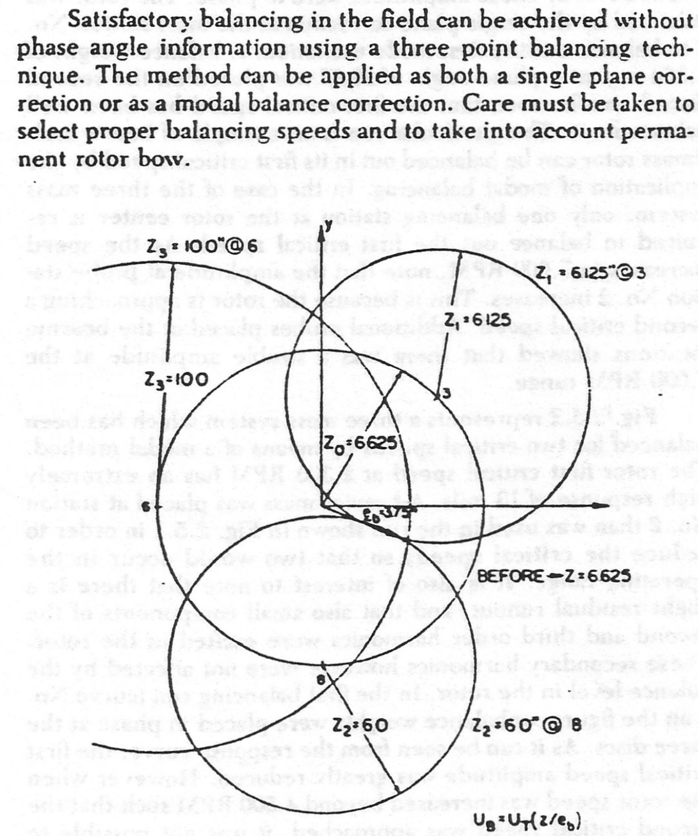


*Figure 2.6.2 Calculation of Correction Plane*

## REFERENCES


1. DeLaval, G., "Diary Sketches and Notes from DeLaval Memorial Lecture,"Gustaf DeLaval. The High Speeds and the Gear," Stal — Laval Turbine AB Stockholm, 1968.
2. Eshelman, R., "Development of Methods and Equipment for Balancing Flexible Rotors." Armour Research Foundation. I. I. T. Final Report NOBS Contract 78753, May, 1962.
3. Parkinson, A. G., Jackson, K. L., and Bishop, R. E. D., "Some Experiments on the Balancing of Small Flexible Rotors: Part II — Experiments," J. Mech. Engineering Sci., Vol. 5, No. 2, 1963, pp. 133-145.
4. Tondl, A., *Some Problems of Rotor Dynamics*, Publishing House, Czechoslavak Academy of Sciences, Prague, 1965.
5. Sternlicht, B. and Lewis, P., "Vibration Problems with High Speed Turbomachinery," J. Eng. for Ind., Trans. A.S.M.E., Feb., 1968.
6. Jackson, C., "Using the Orbit to Balance," Mechanical Engineering, Feb., 1971, pp. 28-32.
7. Nicholas, J. C., Gunter, E. J., and Allaire, P. E., "Effect of Residual Shaft Bow on Unbalance Response and Balancing of a Single Mass Rotor: Part II — Balancing," *Journal of Engineering for Power*, Series A, Trans. A.S.M.E., Vol. 98, No. 2, April, 1976, pp. 182-189.