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Bearing Dynamic Coefficients of Flexible-Pad Journal Bearings©

WEN JENG CHEN
Ingersoll-Rand Company
Centrifugal Compressor Division
Mayfield, Kentucky 42066

The application of flexible-pad journal bearings for high-speed rotating machinery has recently gained attention from equipment manufacturers for its simplicity of one-piece design which can eliminate the manufacturing tolerance stack-up and possible lower frictional power loss. This paper presents a general method for the calculation of bearing dynamic coefficients of flexible-pad journal bearings. These coefficients are critical to the rotor dynamics study. The flexibility of the support web and mass/inertia effects are included in this approach. The bearing coefficients of a conventional tilting-pad journal bearing can also be obtained from this method by adjusting the reduced pad dynamic flexibility matrix. The numerical results are in good correlation with the laboratory experiments.

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NOMENCLATURE

$\hat{A}$ = dynamic flexibility notation ($\hat{A} = \hat{K}^{-1}$)
$\mathbf{A}_p^*$ = reduced pad dynamic flexibility matrix, defined in Eq. [13]
C = damping
C_b = bearing radial clearance, assembled clearance
C_p = pad radial clearance, machined clearance
D_p = pivot diameter
D_h = pivot housing diameter
e_b = bearing eccentricity, distance from O_b to O_j
e_p = pad eccentricity, distance from O_p to O_j
E = Young's modulus
E_{cyc} = energy per cycle of motion
F = fluid film force
h_p = pivot film thickness
I_p = pad inertia
$\hat{I}_p$ = I_p/R_p^2
$\mathbf{I}$ = identity matrix
AE, EI = web cross-sectional properties, A = area, I = moment of inertia
j = $\sqrt{-1}$
K = stiffness
K_p = pivot stiffness
$\hat{K}$ = dynamic stiffness notation, $\hat{K}=K - \omega^2 m$
L = pivot axial length for line-contact tilting-pad bearing, web length
m = mass
m_p = pad mass
O_b = bearing geometric center
O_j = journal equilibrium center
O_p = pad center of curvature
O_{op} = pad center of curvature without movement
R = journal radius
R_p = distance from pad to O_{op}
W = bearing static load
W_p = static load on the individual pad
(x,y) = fixed global coordinates
(x,y) = vibrations in (x,y) coordinates
$(\bar{x}, \bar{y})$ = vibration amplitudes
Z = impedance notation, $Z = K + j\omega C$
$\mathbf{Z}$ = fluid film impedance matrix containing the fixed pad coefficients
$\mathbf{Z}^*$ = reduced effective impedance matrix, $\mathbf{Z}^* = (\mathbf{I} + \mathbf{Z}\mathbf{A}_p^*)^{-1}\mathbf{Z}$
(ξ,η) = local coordinate system for each pad, journal displacements
(ξ_{op},η_{op}) = displacements of pad center of curvature
(ξ_p,η_p,γ_p) = pad displacements, two translations and one rotation
ϕ_b = attitude angle
ω = frequency
θ = phase angle
ν = Poisson's ratio
ψ_p = pivot angular location measured from x to ξ in the direction of rotation

SUBSCRIPTS

b = bearing
(i,j) = indices
o = static equilibrium
p = pad
x,y,ξ,η = displacement directions

KEY WORDS

Fluid Film Bearings, Hydrodynamic Bearings, Tilting-Pad Bearings, Rotor Bearing Dynamics

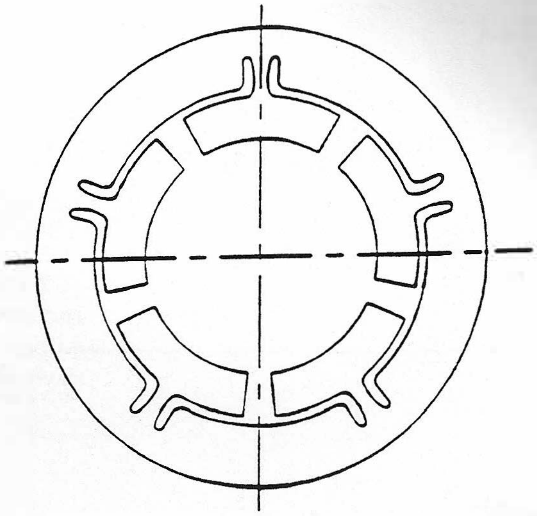
INTRODUCTION

In modern turbomachinery, there has been a marked increase in operating speed to improve the aerodynamic performance and decrease the shaft-bearing diameter to minimize frictional power loss. Typical bearing design requirements for integrally geared compressors include the capability of operating in the unloaded condition, near surge condition, a wide range of oil temperatures and variations in lubricant properties. The higher speed, flexible rotating assembly, and extreme operating conditions make bearing design very difficult and critical to the overall design process.

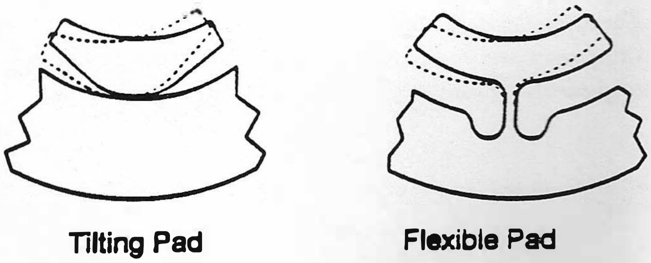
Fluid film bearings are available in a variety of configurations and are widely used in rotating machinery due to their favorable fatigue life and damping characteristics compared to rolling element bearings. In addition to carrying static loads, fluid-film journal bearings are often the major source of damping which can attenuate resonant response. However, the non-zero cross-coupling stiffness coefficients existing in the fixed-profile journal bearings can introduce a major destabilizing effect. This can cause the rotor system to be in a very destructive self-excited state. The tilting-pad bearings have been used in almost every rotating machine when the rotor operating speed is higher than twice the first critical speed due to their virtually inherent stability characteristics. However, the tilting-pad bearings with moving parts are mechanically complex and generally have lower damping and softer stiffness than fixed geometry bearings. Furthermore, the manufacturing tolerance stack-up existing in the tilting-pad bearing, as illustrated by Zeidan (*1*) and Chen et al. (*2*), can result in a wide range of bearing clearance and preload. This variance can be pronounced for relatively small-sized bearings which have been commonly used in high-speed application.

Recently, the flexible pad journal bearing, as shown in Fig. 1, has been gaining attention in bearing design for high-speed rotating machinery (*3*), (*4*). It is a one-piece design similar to that of the conventional tilting-pad bearing without the complexity of the moving parts. Therefore, the pivot wear, manufacturing tolerance stack-up, and the unloaded pad flutter problems associated with the conventional tilting-pad bearing can be eliminated. However, due to the flexibility of the support web, the pad is not free to pitch and the destabilizing tangential oil film force always exists, even when the pad inertia is neglected. The rotor-bearing stability must be carefully examined when the flexible-pad bearing is used. Hence, it is very important to effectively predict the bearing dynamic coefficients of flexible-pad bearings for use in the study of rotor-bearing dynamic characteristics.

The model of a fluid film bearing is complicated by its inherently nonlinear behavior. However, the linearized dynamic damping and stiffness coefficients are widely used in the analyses of critical speeds, steady-state unbalance response, and rotor stability. The use of linearized bearing dynamic coefficients in the analysis of complex flexible



(a)



(b)

Fig. 1—Bearing geometry.
(a) typical five-pads flexible-pad journal bearing
(b) motion of the pad

rotor-bearing systems is a common practice. The linearization of the bearing reaction forces also provides the advantage of decoupling the rotor assembly and bearing equations. Publications by Lund (*5*), (*6*) have reviewed the concepts of bearing dynamic coefficients and methods for computing them and their use in rotor dynamics calculations.

The pad assembly method for the calculation of tilting-pad journal-bearing dynamic coefficients was first proposed by Lund (*7*). Since then, numerous publications have been presented in the calculation of tilting-pad journal bearing coefficients with different numerical techniques (*8*) and inclusion of various effects (*9*)–(*13*). The pad assembly method provides an effective and fast means of calculating tilting-pad bearing coefficients by assembling single-pad data rather than a time-consuming iterative solution for all the pads. The single-pad data can be generated separately and stored for future assembly process and parametric study. The fast assembly process makes this method attractive and essential to the practical design engineers in the design parametric study. Isothermal and laminar flow with negligible fluid inertia is assumed in this analysis. A comprehensive literature review on the turbulence, fluid inertia, and thermal effects in fluid film bearings has been documented by Szeri (*14*). The computational method for turbulence and thermal effects has also been documented by Someya (*15*) and for inertia effect by Reinhardt and Lund (*16*). These effects on the bearing dynamic coefficients, in general, are less than the manufacturing tolerance errors and uncertainties from the other sources, e.g., lubricant properties and actual oil entry temperature. For some large bearings with lightly loaded and high-journal surface speed, the application of the thermohydrodynamic (THD) theory with turbulence and fluid inertia effects will be required to improve the accuracy of calculations (*14*), (*15*). However, this can be very time consuming during the initial bearing design stage. In the design process, the bearing coefficients at various operating conditions, such as minimum and maximum anticipated oil temperatures, possible ranges of clearance and preload due to manufacturing tolerance, and variation in the bearing static loads, are required for the rotordynamic study. Therefore, an effective, fast, and approximate solution is useful and indispensable.

Based on the pad assembly method, Armentrout and Paquette (*3*) have included the web rotational stiffness in the calculation of flexible-pad bearing coefficients. However, the pad radial and tangential movements under dynamic conditions were ignored. It is known that this additional two-translational flexibility can have a great influence on the bearing dynamic coefficients, especially the damping characteristics. The objective of this paper is to provide a general method for calculating the dynamic stiffness and damping coefficients of flexible-pad journal bearings to be readily used in the rotordynamics analysis. The effects of pad translations and rotation and the mass/inertia properties of the pad are included. The conventional tilting-pad bearing coefficients can be easily obtained by using the same procedure with elimination of the pad tangential motion. The proposed algorithm can be easily implemented into existing tilting-pad bearing programs based upon the pad assembly method to evaluate the flexure-pad bearing coefficients.

MATHEMATICAL MODEL

The static equilibrium of the journal center is described by bearing eccentricity (e_b) and attitude angle (ϕ_b), both referenced from the bearing center. Under dynamic conditions, the journal oscillates at small displacements (x,y) around its static equilibrium position, shown in Fig. 2. For small vibration, the linearized fluid-film forces acting on the journal are obtained by summing over all the pads:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \sum_p \begin{Bmatrix} F_x \\ F_y \end{Bmatrix}_p = \sum_p \begin{Bmatrix} F_{xo} + dF_x \\ F_{yo} + dF_y \end{Bmatrix}_p$$
$$= \begin{Bmatrix} F_{xo} \\ F_{yo} \end{Bmatrix} - \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} - \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} \quad [1]$$

The static equilibrium force, F_{xo}, should be equal to the static load W in the opposite direction and F_{yo} is zero. The linearized dynamic forces contain the stiffness and damping matrices. The stiffness and damping matrices can be further decomposed into conservative, i.e, elastic and gyroscopic, and non-conservative, i.e., circulatory and dissipative matrices. The physical insights of these bearing forces and their fundamental properties have been reviewed by several publications (*5*), (*6*), (*17*), (*18*). For a harmonic motion with a frequency of ω, the energy removed from the rotor system per cycle of motion is:

$$E_{cyc} = \pi\omega[C_{xx}\bar{x}^2 + C_{yy}\bar{y}^2 - (C_{xy} + C_{yx})\bar{x}\bar{y}\sin(\theta_x - \theta_y)] \quad [2]$$
$$- \pi(K_{xy} - K_{yx})\bar{x}\bar{y}\cos(\theta_x - \theta_y)$$

where $\bar{x}$, $\bar{y}$ are the vibration amplitudes and θ_x, θ_y are the phase angles. The first term is the energy removed by the damping coefficients. The second term is the product of the difference of cross-coupling stiffness coefficients times the whirl orbit area, i.e, $(K_{xy} - K_{yx})$ · area of whirl orbit. With the positive $(K_{xy} - K_{yx})$, the cross-coupling stiffness coefficients can actually add energy to the motion and destabilize the rotor system in the linear sense.

Single Pad Analysis

In order to calculate the dynamic coefficients of a complete bearing, it is necessary to understand the dynamic behavior of the individual pad. A fixed local coordinate system (ξ,η) is defined for each pad with its origin in the bearing center. The ξ-axis passes through the pivot point and the η-axis is perpendicular to the ξ-axis. The ξ-axis is also the static loading line for each pad. For a given journal position, the corresponding pad Sommerfeld number and fixed pad coefficients are calculated by treating the individual pad as a partial arc bearing. The fixed pad coefficients are tabulated as a function of nondimensional pivot film thickness to be readily used in the bearing assembly process. For small vibration and under dynamic loading, the journal center oscillates with amplitudes of (ξ, η) and the pad center of curvature oscillates with amplitudes of (ξ_{op},η_{op}). The pad also moves in the ξ-η plane with two translational (ξ_p,η_p) and one rotational (γ_p) degrees of freedom due to the flexibility of the support web. The dynamic forces acting on the journal center, expressed in the fixed local coordinates, are:

$$\begin{Bmatrix} dF_\xi \\ dF_\eta \end{Bmatrix} = - \begin{bmatrix} K_{\xi\xi} & K_{\xi\eta} \\ K_{\eta\xi} & K_{\eta\eta} \end{bmatrix} \begin{Bmatrix} \xi - \xi_{op} \\ \eta - \eta_{op} \end{Bmatrix} \quad [3]$$
$$- \begin{bmatrix} C_{\xi\xi} & C_{\xi\eta} \\ C_{\eta\xi} & C_{\eta\eta} \end{bmatrix} \begin{Bmatrix} \dot{\xi} - \dot{\xi}_{op} \\ \dot{\eta} - \dot{\eta}_{op} \end{Bmatrix}$$

The motion of the pad can be described by the following equations of motion:

$$\begin{bmatrix} m_{\xi\xi} & m_{\xi\eta} & m_{\xi\gamma} \\ m_{\eta\xi} & m_{\eta\eta} & m_{\eta\gamma} \\ m_{\gamma\xi} & m_{\gamma\eta} & m_{\gamma\gamma} \end{bmatrix}_p \begin{Bmatrix} \ddot{\xi}_p \\ \ddot{\eta}_p \\ \ddot{\gamma}_p \end{Bmatrix} + \begin{bmatrix} K_{\xi\xi} & K_{\xi\eta} & K_{\xi\gamma} \\ K_{\eta\xi} & K_{\eta\eta} & K_{\eta\gamma} \\ K_{\gamma\xi} & K_{\gamma\eta} & K_{\gamma\gamma} \end{bmatrix}_p \quad [4]$$
$$\begin{Bmatrix} \xi_p \\ \eta_p \\ \gamma_p \end{Bmatrix} = \begin{Bmatrix} -dF_\xi \\ -dF_\eta \\ -R_p dF_\eta \end{Bmatrix}$$

where the pad mass/inertia and stiffness matrices can be obtained from any finite element method and commercial software by using condensation techniques, e.g., the Guyan Reduction Method, to reduce the degrees of freedom and retaining only three pad degrees of freedom-two translations and one rotation. A simplified approach using beam

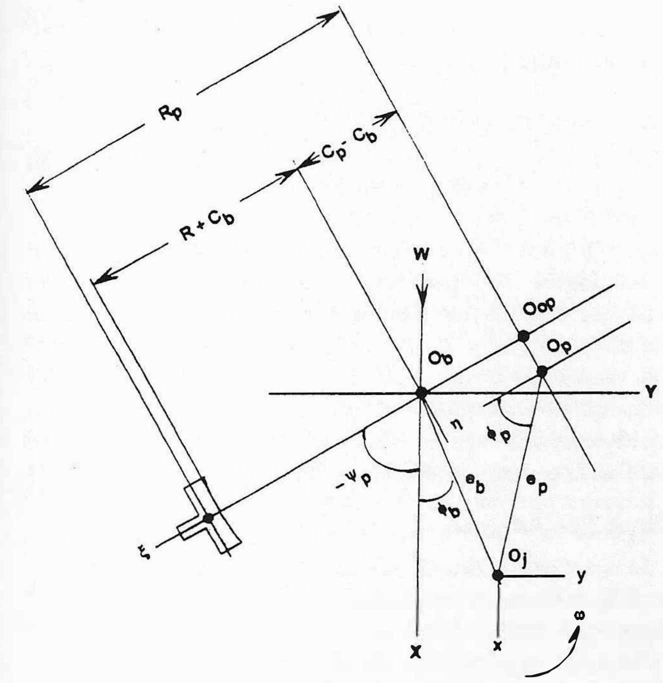


Fig. 2—Coordinate systems.

theory is presented in the later context. For small vibration, the displacements are related by the following expressions:

$$\xi_{op} = \xi_p \qquad [5]$$

$$\eta_{op} = \eta_p + R_p\gamma_p \qquad [6]$$

Consider harmonic motions with a frequency of ω. The assumed frequency ω is used for the calculation of reduced bearing dynamic coefficients. Normally, the shaft rotational speed is selected to be the reduction frequency. Thus, the reduced bearing coefficients are called the synchronously reduced coefficients. It is convenient and desirable to introduce the impedance notation in complex form as:

$$Z = K + j\omega C \qquad [7]$$

Substitution of Eqs. [5] – [7] into Eq. [3], the dynamic forces can be expressed as:

$$\begin{aligned} \begin{Bmatrix} dF_\xi \\ dF_\eta \end{Bmatrix} &= -\begin{bmatrix} Z_{\xi\xi} & Z_{\xi\eta} \\ Z_{\eta\xi} & Z_{\eta\eta} \end{bmatrix} \begin{Bmatrix} \xi - \xi_p \\ \eta - \eta_p - R_p\gamma_p \end{Bmatrix} \\ &= -\begin{bmatrix} Z_{\xi\xi} & Z_{\xi\eta} \\ Z_{\eta\xi} & Z_{\eta\eta} \end{bmatrix} \begin{Bmatrix} \xi \\ \eta \end{Bmatrix} + \begin{bmatrix} Z_{\xi\xi} & Z_{\xi\eta} \\ Z_{\eta\xi} & Z_{\eta\eta} \end{bmatrix} \begin{Bmatrix} \xi_p \\ \eta_p + R_p\gamma_p \end{Bmatrix} \end{aligned} \qquad [8]$$

By introducing the dynamic stiffness notation $\hat{K} = K - \omega^2 m$, the pad equations of motion, Eq. [4], can be written as:

$$\begin{bmatrix} \hat{K}_{\xi\xi} & \hat{K}_{\xi\eta} & \hat{K}_{\xi\gamma} \\ \hat{K}_{\eta\xi} & \hat{K}_{\eta\eta} & \hat{K}_{\eta\gamma} \\ \hat{K}_{\gamma\xi} & \hat{K}_{\gamma\eta} & \hat{K}_{\gamma\gamma} \end{bmatrix}_p \begin{Bmatrix} \xi_p \\ \eta_p \\ \gamma_p \end{Bmatrix} = \hat{\mathbf{K}}_p \begin{Bmatrix} \xi_p \\ \eta_p \\ \gamma_p \end{Bmatrix} = \begin{Bmatrix} -dF_\xi \\ -dF_\eta \\ -R_p dF_\eta \end{Bmatrix} \qquad [9]$$

or

$$\begin{Bmatrix} \xi_p \\ \eta_p \\ \gamma_p \end{Bmatrix} = \begin{bmatrix} \hat{A}_{\xi\xi} & \hat{A}_{\xi\eta} & \hat{A}_{\xi\gamma} \\ \hat{A}_{\eta\xi} & \hat{A}_{\eta\eta} & \hat{A}_{\eta\gamma} \\ \hat{A}_{\gamma\xi} & \hat{A}_{\gamma\eta} & \hat{A}_{\gamma\gamma} \end{bmatrix}_p \begin{Bmatrix} -dF_\xi \\ -dF_\eta \\ -R_p dF_\eta \end{Bmatrix} = \hat{\mathbf{A}}_p \begin{Bmatrix} -dF_\xi \\ -dF_\eta \\ -R_p dF_\eta \end{Bmatrix} \qquad [10]$$

where

$\hat{\mathbf{A}}_p = \hat{\mathbf{K}}_p^{-1}$ is defined as the pad dynamic flexibility matrix.

Introducing two transformation matrices provides:

$$\begin{Bmatrix} \xi_p \\ \eta_p + R_p\gamma_p \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & R_p \end{bmatrix} \begin{Bmatrix} \xi_p \\ \eta_p \\ \gamma_p \end{Bmatrix} \qquad [11]$$

and

$$\begin{Bmatrix} -dF_\xi \\ -dF_\eta \\ -R_p dF_\eta \end{Bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & -R_p \end{bmatrix} \begin{Bmatrix} dF_\xi \\ dF_\eta \end{Bmatrix} \qquad [12]$$

Substitution of Eqs. [11] and [12] into Eq. [10] gives the following relationship:

$$\begin{aligned} &\begin{Bmatrix} \xi_p \\ \eta_p + R_p\gamma_p \end{Bmatrix} = \\ &-\begin{bmatrix} \hat{A}_{\xi\xi} & \hat{A}_{\xi\eta} + R_p\hat{A}_{\xi\gamma} \\ \hat{A}_{\eta\xi} + R_p\hat{A}_{\gamma\xi} & \hat{A}\,\mathrm{ei}\eta\eta + R_p\,(\hat{A}_{\eta\gamma} + \hat{A}_{\gamma\eta}) + R_p^2\,\hat{A}_{\gamma\gamma} \end{bmatrix}_p \\ &\begin{Bmatrix} dF_\xi \\ dF_\eta \end{Bmatrix} = -\mathbf{A}_p^* \begin{Bmatrix} dF_\xi \\ dF_\eta \end{Bmatrix} \end{aligned} \qquad [13]$$

where $\mathbf{A}_p^*$ is a (2×2) matrix referred to as the reduced pad dynamic flexibility matrix which relates the pad motion and the fluid-film dynamic forces.

Substituting Eq. [13] into Eq. [8] and rearranging it, the dynamic forces become:

$$\begin{Bmatrix} dF_\xi \\ dF_\eta \end{Bmatrix} = -(\mathbf{I} + \mathbf{Z}\mathbf{A}_p^*)^{-1}\,\mathbf{Z}\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = -\mathbf{Z}^*\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} \qquad [14]$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{Z}$ is the fluid-film impedance matrix containing the fixed pad coefficients, and $\mathbf{A}_p^*$ is the reduced pad dynamic flexibility matrix containing only the structural properties of the flexible pad. The reduced effective impedance matrix is of the form:

$$\mathbf{Z}^* = \begin{bmatrix} Z^*_{\xi\xi} & Z^*_{\xi\eta} \\ Z^*_{\eta\xi} & Z^*_{\eta\eta} \end{bmatrix} = \begin{bmatrix} K^*_{\xi\xi} & K^*_{\xi\eta} \\ K^*_{\eta\xi} & K^*_{\eta\eta} \end{bmatrix} + j\omega\begin{bmatrix} C^*_{\xi\xi} & C^*_{\xi\eta} \\ C^*_{\eta\xi} & C^*_{\eta\eta} \end{bmatrix} \qquad [15]$$

The reduced effective stiffness and damping coefficients for a single flexible pad are determined from the real and imaginary parts of the reduced effective impedance coefficients. By adjusting the reduced pad dynamic flexibility matrix, $\mathbf{A}_p^*$, several special cases are discussed in the following sections.

Tilting-Pad Journal Bearing with Rigid Pivot

In this case, the pad is free to tilt and no translational motion is allowed (*7*), (*8*). The dynamic behavior of the pad can be expressed in mathematical form:

$$\xi_p = 0;\ \eta_p = 0;\ \text{and}\ (K_{\gamma\gamma})_p = 0 \qquad [16]$$

The reduced pad dynamic flexibility matrix becomes:

$$\mathbf{A}_p^* = \begin{bmatrix} 0 & 0 \\ 0 & 1/(-\omega^2 \bar{I}_p) \end{bmatrix} \qquad [17]$$

where $\bar{I}_p = \frac{I_p}{R_p^2}$ and I_p is the pad inertia. The reduced effective impedance of the pad can be easily calculated from Eq. [14]:

$$\mathbf{Z}^* = (\mathbf{I} + \mathbf{Z}\mathbf{A}_p^*)^{-1}\mathbf{Z} =$$

$$\begin{bmatrix} Z_{\xi\xi} - \dfrac{Z_{\xi\eta} Z_{\eta\xi}}{Z_{\eta\eta} - \omega^2 \bar{I}_p} & Z_{\xi\eta} - \dfrac{Z_{\xi\eta} Z_{\eta\eta}}{Z_{\eta\eta} - \omega^2 \bar{I}_p} \\ \dfrac{-\omega^2 \bar{I}_p Z_{\eta\xi}}{Z_{\eta\eta} - \omega^2 \bar{I}_p} & \dfrac{-\omega^2 \bar{I}_p Z_{\eta\eta}}{Z_{\eta\eta} - \omega^2 \bar{I}_p} \end{bmatrix} \qquad [18]$$

For small bearings, the pad inertia is negligible and the reduced effective impedance matrix becomes:

$$Z^*_{\xi\xi} = Z_{\xi\xi} - \frac{Z_{\xi\eta} Z_{\eta\xi}}{Z_{\eta\eta}}\ ;\quad Z^*_{\xi\eta} = Z^*_{\eta\xi} = Z^*_{\eta\eta} = 0 \qquad [19]$$

The dynamic force from the pad is acting along the ξ-axis and the tangential force vanishes, which implies inherent stability of the bearing.

Tilting-Pad Journal Bearing with Flexible Pivot

The effect of pivot flexibility on the dynamic coefficients has been investigated in several publications (*9*) – (*11*) and the results have shown that the total damping can be reduced by the effect of the pivot stiffness. In this case, the pad tangential movement is fixed and the radial movement is constrained by a pivot stiffness. The pad is free to tilt with zero angular stiffness:

$$\eta_p = 0;\ \text{and}\ (K_{\gamma\gamma})_p = K_p \qquad [20]$$

The reduced pad dynamic flexibility matrix becomes:

$$\mathbf{A}_p^* = \begin{bmatrix} \dfrac{1}{(K_p - \omega^2 m_p)} & 0 \\ 0 & \dfrac{1}{(-\omega^2 \bar{I}_p)} \end{bmatrix} \qquad [21]$$

The reduced effective impedance of the pad can be easily calculated from Eq. [14]:

$$\mathbf{Z}^* = \frac{1}{\Delta}\begin{bmatrix} (K_p - \omega^2 m_p)(Z_{\eta\eta} - \omega^2 \bar{I}_p) & (K_p - \omega^2 m_p)(-Z_{\xi\eta}) \\ (-\omega^2 \bar{I}_p)(-Z_{\eta\xi}) & (-\omega^2 \bar{I}_p)(Z_{\xi\xi} + K_p - \omega^2 m_p) \end{bmatrix} \begin{bmatrix} Z_{\xi\xi} & Z_{\xi\eta} \\ Z_{\eta\xi} & Z_{\eta\eta} \end{bmatrix} \qquad [22]$$

where

$$\Delta = (Z_{\xi\xi} + K_p - \omega^2 m_p)(Z_{\eta\eta} - \omega^2 \bar{I}_p) - (Z_{\xi\eta} Z_{\eta\xi}) \qquad [23]$$

If the pad inertia is negligible ($m_p = I_p = 0$), the effective impedances are:

$$Z^*_{\xi\xi} = \frac{K_p(Z_{\xi\xi} Z_{\eta\eta} - Z_{\xi\eta} Z_{\eta\xi})}{(Z_{\xi\xi} + K_p)(Z_{\eta\eta}) - (Z_{\xi\eta} Z_{\eta\xi})};\ Z^*_{\xi\eta} = Z^*_{\eta\xi} = Z^*_{\eta\eta} = 0 \qquad [24]$$

Again, the tangential dynamic force is zero and the pad has only radial stiffness and damping. The reduced effective impedance can be considered as the oil film impedance in series with the pivot stiffness (*10*). The pivot stiffness for a variety of pivot configurations presented by Kirk and Reedy (*11*) is listed below for reference.

For a sphere on a flat plat, i.e., point contact:

$$K_p = 0.968\ (E^2 D_p W_p)^{1/3} \qquad [25]$$

For a sphere in a sphere, i.e., point contact:

$$K_p = 0.968 \left(\frac{E^2 D_h D_p W_p}{D_h - D_p}\right)^{1/3} \qquad [26]$$

For a line contact:

$$K_p = \frac{\pi E L}{2(1 - v^2)\left[-\frac{1}{3} + \ln\left(\frac{4EL(D_h - D_p)}{2.15^2\, W_p}\right)\right]} \qquad [27]$$

where Poisson's ratio is $v_h = v_p = v = 0.3$, and Young's modulus is $E_h = E_p = E$. The D_p and D_h are the pivot diameter and pivot housing diameters, respectively. W_p is the static load on the pad and must be determined after the journal equilibrium position has been found. The pivot stiffness is the slope of the load-deflection curve and is a function of W_p. For the unloaded pad, the pivot stiffness is zero.

Simplified Flexible-Pad Bearing

For a simplified flexible-pad bearing, the pad can be treated as a lumped inertia in the free end of a cantilever beam as shown in Fig. 3. The pad dynamic stiffness matrix is then:

$$\hat{\mathbf{K}}_p = \begin{bmatrix} \hat{K}_{\xi\xi} & 0 & 0 \\ 0 & \hat{K}_{\eta\eta} & \hat{K}_{\eta\gamma} \\ 0 & \hat{K}_{\gamma\eta} & \hat{K}_{\gamma\gamma} \end{bmatrix}_p \qquad [28]$$

where:

$$(\hat{K}_{\xi\xi})_p = (AE/L) - \omega^2 m_p \qquad [29]$$

$$(\hat{K}_{\eta\eta})_p = \left(\frac{12EI}{L^3} - \frac{6W_p}{5L}\right) - \omega^2 m_p \qquad [30]$$

$$(\hat{K}_{\gamma\eta})_p = (\hat{K}_{\eta\gamma})_p = \left(\frac{-6EI}{L^2} - \frac{W_p}{10}\right) \qquad [31]$$

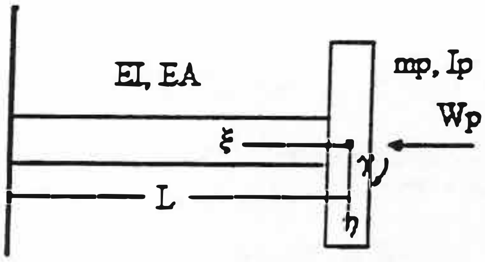


Fig. 3—Cantilever beam model.

$$(\hat{K}_{\gamma\gamma})_p = \left(\frac{4EI}{L} - \frac{2W_pL}{15}\right) - \omega^2 I_p \qquad [32]$$

The geometric stiffness caused by the axial load on the pad is included in the above equation. The reduced pad dynamic flexibility matrix calculated from Eq. [14] gives:

$$\mathbf{A}_p^* = \begin{bmatrix} \frac{1}{\hat{K}_{\xi\xi}} & 0 \\ 0 & \frac{1}{\Delta_p}(\hat{K}_{rr} - 2R_p\hat{K}_{\eta\gamma} + R_p^2\hat{K}_{\eta\eta}) \end{bmatrix} \qquad [33]$$

where:

$$\Delta_p = \hat{K}_{\eta\eta}\hat{K}_{\gamma\gamma} - \hat{K}_{\eta\gamma}^2 \qquad [34]$$

Pad Assembly

The pad assembly is a straightforward process. The single-pad data described in the previous section are calculated using the fixed local coordinates (ξ,η) for each individual pad. However, the complete bearing dynamic characteristics are usually described using the fixed global coordinates (x,y). To assemble the pads, the single-pad data are transformed into the global coordinate system by the following coordinate transformation:

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} \cos\psi_p & -\sin\psi_p \\ \sin\psi_p & \cos\psi_p \end{bmatrix}_p \begin{Bmatrix} \xi \\ \eta \end{Bmatrix}_p \qquad [35]$$

where ψ_p is the angle measured from x to ξ in the direction of rotation. Therefore, the complete bearing static equilibrium forces are:

$$\begin{Bmatrix} F_{xo} \\ F_{yo} \end{Bmatrix} = \begin{Bmatrix} -W \\ 0 \end{Bmatrix} = \sum_p \begin{bmatrix} \cos\psi_p & -\sin\psi_p \\ \sin\psi_p & \cos\psi_p \end{bmatrix}_p \begin{Bmatrix} -W_p \\ 0 \end{Bmatrix} = \sum_p \begin{Bmatrix} -W_p & \cos\psi_p \\ -W_p & \sin\psi_p \end{Bmatrix} \qquad [36]$$

and the complete bearing dynamic coefficients in the global coordinates are:

$$\begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} = \sum_p \begin{bmatrix} \cos\psi_p & -\sin\psi_p \\ \sin\psi_p & \cos\psi_p \end{bmatrix}_p \begin{bmatrix} Z_{\xi\xi}^* & Z_{\xi\eta}^* \\ Z_{\eta\xi}^* & Z_{\eta\eta}^* \end{bmatrix} \begin{bmatrix} \cos\psi_p & \sin\psi_p \\ -\sin\psi_p & \cos\psi_p \end{bmatrix}_p \qquad [37]$$

Then, the complete bearing stiffness and damping coefficients used in the rotor dynamics calculation are determined from the real and imaginary parts of the impedance coefficients:

$$Z_{i,j} = K_{i,j} + j\omega C_{i,j} \quad (i = x,y;\ j = x,y) \qquad [38]$$

In order to assemble the complete bearing, an important geometric relationship between the bearing eccentricity and pad eccentricity is required. At static conditions, the pad equilibrium position in the ξ coordinate is obtained from Fig. 2 and is normalized with respect to the pad radial clearance C_p:

$$\frac{e_p}{C_p}\cos\varphi_p = \frac{e_b}{C_p}\cos(2\pi - \psi_p + \varphi_b) + \left(\frac{C_p - C_b}{C_p}\right) \qquad [39]$$

where the left-hand side of Eq. [39] can also be expressed as a function of the nondimensional pivot film thickness;

$$\frac{e_p}{C_p}\cos\varphi_p = 1 - \left(\frac{h_p}{C_p}\right) \qquad [40]$$

and the second term in the right-hand side of Eq. [39] is the definition of the pad preload.

NUMERICAL ALGORITHM

For a given bearing geometry and associated operating conditions, the bearing dynamic performance can be obtained by solving the two nonlinear static equilibrium forces, Eq. [36], for two unknowns, i.e., static equilibrium coordinates. Once the equilibrium position has been determined, the bearing dynamic coefficients can be easily calculated using the procedure outlined above. The computational algorithm is summarized as follows:

1. Input the bearing geometry and operating parameters.
2. Form the single-pad data for a range of nondimensional pivot film thickness. The single-pad data which are normally expressed in nondimensional format are calculated by treating the single pad as a partial-arc journal bearing. The nondimensional-pad data, i.e., pad Sommerfeld number, fixed pad coefficients, oil flow, etc., are tabulated as a function of the nondimensional pivot film thickness. These nondimensional data can be stored and reused for future computation. If the single-pad data are available, this step can be skipped.
3. Solve for the journal equilibrium position. This is an iterative process, and a nonlinear equation solver, such as the Newton-Raphson numerical scheme, must be employed. First, an initial guess of (e_b, φ_b) is assumed. Then, a computational loop for each pad must be performed. For each pad, the nondimensional pivot film thickness is determined from Eqs. [39] and [40]. The pad Sommerfeld number and load on each pad can then be interpolated from the single-pad data. Once the load on each pad has been determined, the total load on the bearing is calculated from Eq. [36]. If the

calculated bearing load does not equal the applied bearing load, i.e., Eq. [36] is not satisfied, a new set of (e_b, φ_b) is updated from the nonlinear equation solver. The iterative process repeats until the journal equilibrium position is located and the static equilibrium conditions are achieved.

4. Assemble the effective bearing dynamic coefficients. Once the equilibrium position has been determined, the reduced pad dynamic flexibility matrix, $\mathbf{A}_p^*$, for each pad can then be formed from Eqs. [13], [17], [21] and [23] for a variety of pivot configurations. The reduced pad effective impedance matrix from Eq. [14] can then be easily evaluated. The complete bearing impedance matrix is assembled from Eq. [37]. The reduced effective bearing stiffness and damping coefficients for a complete bearing are determined from the real and imaginary parts of the reduced effective impedance coefficients.

RESULTS

Two actual bearings are employed to illustrate the calculation procedure and the results are discussed. The first bearing is a conventional tilting-pad journal bearing and the bearings have been successfully operated in high-speed industrial compressors since 1987. The second bearing is a flexible-pad journal bearing with the same bearing parameters. A single journal-bearing system is first utilized as a parametric study to understand the effects of the pivot flexibility and pad inertia on the bearing dynamic coefficients and their influence on the system stability characteristics. The bearing parameters are listed in Table 1. Four cases have been studied and their bearing dynamic coefficients and stability characteristics are compared and listed in Table 2. It should be noted that the logarithm decrement, shown in last column of Table 2, was calculated by assuming a journal-bearing two degrees of freedom system. This logarithm decrement may be considered as a measure of sensitivity to instability and can be used for comparison purposes only. The system stability must be determined from the complete rotor-bearing dynamics analysis. As illustrated in the industrial compressor, the rotor flexibility and aerodynamic cross-couplings can make the system unstable even when the logarithm decrement is positive in this single journal-bearing system.

For a small bearing, the pad inertia has negligible effect on the bearing coefficients as shown in Case 2. With the pivot flexibility, the damping coefficients have decreased by about eight percent and stiffness coefficients have decreased by about three percent, as shown in Case 3. This effect has also been illustrated by Kirk and Reedy (*11*). By using the flexible-pad bearing, the damping coefficients have been further decreased. Also, significant cross-coupling stiffness coefficients have been found in Case 4 due to the flexibility of the support web and the resistance to the free rotation, i.e., tilt.

A high-speed, double-overhung rotor assembly supported by two identical conventional tilting-pad bearings was employed as a test vehicle for the flexible-pad bearings. The first three rigid bearing critical speeds were calculated to be approximately 472, 695, and 3030 Hz. The rotor operating speed was 1275 Hz, which was above the first two rigid bearing critical speeds. An unstable sub-synchronous vibration with a whirl frequency of 370 Hz was observed in

TABLE 1—SUMMARY OF BEARING PARAMETERS

Five pads: 57-degree arc; Load between pads
Length = 20.8 mm, Diameter = 20.8 mm, Radial clearance = 0.059 mm,
Preload = 0.25, Load = 890 N, Speed = 1275 Hz,
Pad mass = 0.007 kg, Pad inertia = 1.11 E-7 Kg-m^2,
Viscosity = 4 Centipoise

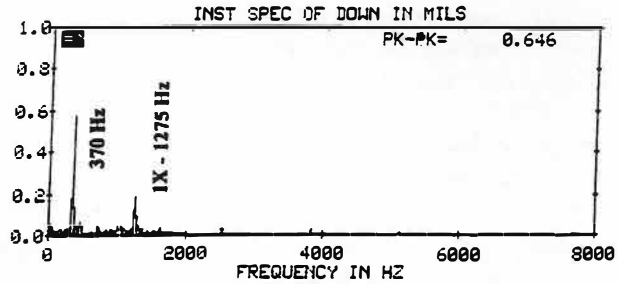
Tilting-pad bearing with line contact pivot configuration:
Pivot diameter = 3.81 mm, Pivot housing diameter = 3.84 mm;
Flexible-pad bearing with the following web data:
Area = 15.87 mm^2, Moment of inertia = 0.77 mm^4, Length = 3.81 mm

TABLE 2—SUMMARY OF BEARING PERFORMANCE

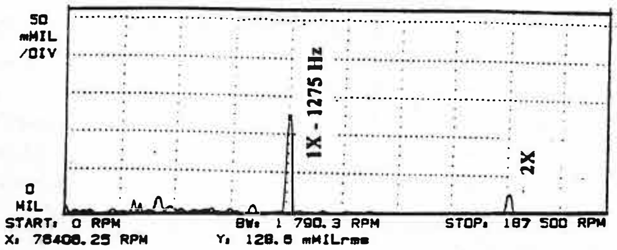
	STIFFNESS (N/m × 10^{-6})				DAMPING (N-s/m × 10^{-3})				
	Kxx	Kxy	Kyx	Kyy	Cxx	Cxy	Cyx	Cyy	Log. Decr.
Case 1	90.39	0	0	47.94	7.81	0	0	4.53	0.271
Case 2	89.88	−0.29	−0.26	46.67	7.78	0.04	−0.25	4.60	0.271
Case 3	87.63	−0.26	−0.28	45.49	7.11	0.04	−0.25	4.25	0.251
Case 4	87.47	1.38	−2.46	47.24	6.79	−0.07	0.11	4.12	0.240

Case 1: Tilting-pad journal bearing with rigid pivot and zero pad inertia
Case 2: Tilting-pad journal bearing with rigid pivot and pad inertia
Case 3: Tilting-pad journal bearing with flexible pivot and pad inertia
Case 4: Flexible-pad journal bearing with simplified cantilever beam model

the laboratory when replacing the tilting-pad bearings with the flexible-pad bearings. Negative logarithm decrement was found when performing the complete rotor-bearing system stability calculation by using these flexible-pad bearings. A new set of flexible-pad bearings with four pads and smaller bearing clearance and 0.6 offset were designed and tested. Details of the bearing design iteration and test results are presented by Chen et al. (*2*). The unstable subsynchronous vibration was eliminated and the machine has been continuously operated above 1000 hours without any mechanical problems. The spectra taken for both tests are shown in Fig. 4.



(a)



(b)

Fig. 4—Spectrums.
(a) flexible-pad bearing No. 1—five pads
(b) flexible-pad bearing No. 2—four pads

CONCLUSIONS

A systematic approach for calculating the bearing dynamic coefficients of flexible-pad journal bearings has been presented. The effects of the flexibility of support web and the mass/inertia properties of the pad are included in the reduced pad dynamic flexibility matrix. The three degrees of freedom of the pad motion are eliminated by assuming a harmonic motion with an assumed reduction frequency. The bearing dynamic coefficients of conventional tilting-pad journal bearings can be calculated by using the same procedure.

The flexibility of the support web can lower the damping coefficients and can also generate the destabilizing cross-coupling stiffness coefficients. The support web must be thick enough to carry the applied load and be free from fatigue damage, while also providing the rotational flexibility to imitate the conventional tilting-pad bearing. The method presented in this paper is of practical importance in the bearing design to meet the stability requirement of the rotor system.

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