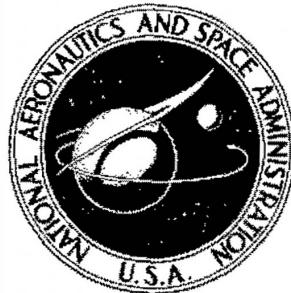


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DESIGN OF AN OIL SQUEEZE FILM DAMPER BEARING FOR A MULTIMASS FLEXIBLE-ROTOR BEARING SYSTEM

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16. Abstract A single-mass flexible-rotor analysis was used to optimize the stiffness and damping of a flexible support for a symmetric five-mass rotor. The flexible, damped support attenuates the amplitudes of motions and forces transmitted to the support bearings when the rotor operates through and above its first bending critical speed. An oil squeeze film damper was designed based on short bearing lubrication theory. The damper design was verified by an unbalance response computer program. Rotor amplitudes were reduced by a factor of 16 and loads reduced by a factor of 36 compared with the same rotor with rigid bearing supports.			
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SUMMARY

Results of a single-mass flexible-rotor analysis were used to optimize the stiffness and damping of a flexible, damped support for a symmetric five-mass rotor operating through and above its first bending critical speed. The flexible support attenuates the rotor amplitudes and forces transmitted to the ball bearings.

The stiffness and damping values obtained from the single-mass analysis were then used in the design of an oil squeeze-film damper and its centering spring. Short bearing lubrication theory was used. A cavitated oil film was assumed to exist in the nonrotating damper journal.

The damper design was verified by an unbalance response computer program. At the first critical speed, rotor amplitudes were reduced by a factor of 16 and bearing loads reduced by a factor of 36 compared with the same rotor with rigidly supported bearings. Amplitudes and forces at higher speeds were also reduced substantially.

INTRODUCTION

In many rotor design applications such as in turbojet engines, compressors, or turbines, the rotor experiences high vibrational amplitudes resulting in large forces being transmitted to the bearings and the support structure. These high vibrational responses may be due to several causes and may be roughly grouped under the headings synchronous and nonsynchronous response. The forces and amplitudes for synchronous response are usually associated with unbalance forces in the rotor. This unbalance may be a result of either the manufacturing process and/or the assembly of the components. Even if a rotor is well balanced initially, the balance degrades with use. Thermal gradients

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can cause the shaft to warp. Erosion of compressor and/or turbine blades can alter the balance of the rotor assembly. Therefore, in the design of a turbomachine provisions should be made so that the increase of unbalance with engine operation will not load the bearings excessively or cause large rotor amplitudes.

Another serious problem related to high-speed turbomachinery is the occurrence of nonsynchronous, self-excited whirl motion. This is commonly associated with fluid film bearings, but can also be caused by rotor internal friction (ref. 1) or variable aerodynamic loading (ref. 2).

Theoretical studies conducted by one of the authors (ref. 3) indicated that problems of both self-excited rotor instability and high vibrational (synchronous) response can often be greatly alleviated by a properly designed damping system at the rotor supports. Flexible damped rotor supports may be used to

- (1) Reduce the forces transmitted through the bearings and foundation
- (2) Reduce the amplitudes of motion of the rotor that could result in rubbing and excessive wear of close fitting components
- (3) Permit smooth operation through critical speeds
- (4) Protect the machine from sudden buildup of unbalance forces due to compressor or turbine blade loss
- (5) Protect the machine from potentially destructive, self-excited instability

This report will consider only steady, synchronous motion; that is, effects (1) to (3).

Damping may be achieved by various mechanisms, such as Coulomb friction, viscoelastic materials, and viscous dampers, which may utilize either compressible or incompressible fluids. This investigation will be concerned with the damping characteristics obtained for the incompressible-oil squeeze-film damper. Such a damper appears in figure 1, which shows a rolling-element bearing mounted in an oil squeeze-film damper. The annulus between the outside diameter of the ball bearing housing and the damper housing inside diameter is filled with oil. The orbital motion or precession of the ball bearing housing in the damping fluid generates a hydrodynamic pressure. This particular type of damper is now being used in several production aircraft turbojet engines and in other types of high speed turbomachinery.

This investigation was conducted to (1) examine the influence of flexible damped supports on rotor amplitudes and forces transmitted over a specific operating speed range, (2) show how single-mass rotor theory can be used to design a support system for a multimass rotor operating below the second bending critical speed, and (3) demonstrate design procedure for an oil squeeze-film damper.

SYMBOLS

A amplification factor at rotor critical speed, dimensionless

a_1	support amplitude, m (in.)
a_2	rotor amplitude, m (in.)
a_{cr}	amplitude at the critical speed, m (in.)
B	damping ratio, B_1/B_2 , dimensionless
B_b	bearing damping, N sec/m (lb sec/in.)
B_d	squeeze film damping, N sec/m (lb sec/in.)
\bar{B}_d	squeeze film damping coefficient, dimensionless
B_s	shaft damping, N sec/m (lb sec/in.)
B_1	support damping, N sec/m (lb sec/in.)
B_2	effective rotor-bearing system damping, N sec/m (lb sec/in.)
C	basic dynamic capacity of ball bearing, N (lb)
C'	adjusted dynamic capacity of ball bearing, N (lb)
c_r	one-half total clearance in damper bearing, also radial damper clearance, m (in.)
D_s	shaft diameter, m (in.)
E	elastic modulus, N/m^2 (lb/in. ²)
e	bearing eccentricity, m (in.)
e_μ	rotor mass eccentricity, m (in.)
F_b	bearing force, N (lb)
F_r, F_θ	radial and tangential bearing forces, N (lb)
F_1	force transmitted to foundation, N (lb)
g	gravitational constant, m/sec^2 (in./sec ²)
H	ball bearing life, hr
h	fluid film thickness, m (in.)
K	stiffness ratio = K_1/K_2 , dimensionless
K_b	bearing stiffness, N/m (lb/in.)
K_c	centering spring stiffness, N/m (lb/in.)
K_d	squeeze-film stiffness, N/m (lb/in.)
\bar{K}_d	squeeze-film stiffness, dimensionless
K_s	shaft stiffness, N/m (lb/in.)

K_1	support stiffness, N/m (lb/in.)
K_2	effective rotor-bearing system stiffness, N/m (lb/in.)
L	bearing or shaft length, m (in.)
M	mass ratio = M_1/M_2
M_1	total support or bearing housing mass, kg (lb sec ² /in.)
M_2	rotor mass, kg (lb sec ² /in.)
N	rotor speed, rpm
p	pressure, N/m ² (psi)
R	damper bearing radius, m (in.)
t	time, sec
U	unbalance, kg m (oz in.)
u	circumferential journal velocity, m/sec (in./sec)
z	axial coordinate, m (in.)
ϵ	eccentricity ratio = e/c_r , dimensionless
θ, θ'	angular displacements, rad
μ	absolute viscosity, N sec/m ² (lb sec/in. ²)
$\dot{\phi}$	journal precession rate, rad/sec
ω	angular velocity, rad/sec
ω_{cr}	rigid support critical speed, rad/sec

INFLUENCE OF DAMPER SUPPORT ON SINGLE-MASS FLEXIBLE ROTOR ON ROLLING-ELEMENT BEARINGS

Successful operation of a high-speed rotor requires careful balancing to minimize the amplitude of motion and the forces transmitted through the bearings. Obviously, the magnitude of the forces transmitted will have a significant influence on the life of a rolling-element bearing.

Example 1 - Load, Life Relation for Rolling-Element Bearing

Consider a flexible rotor operating at a speed of 30 000 rpm and supported on rigidly mounted, 204 size, 20-millimeter bore, ball bearings. The basic load rating (see

ref. 4) of the bearing C is 9800 newtons (2210 lb). If the bearing is to have a reliability of 99 percent, the load reduction factor as given by Harris (ref. 5) is $C'/C = 0.61$. For a design life of 2000 hours the permissible bearing load is

$$F_b = \frac{\left(\frac{C'}{C}\right)C}{\left(\frac{N \cdot H}{10^6}\right)^{1/3}} = 1530 \text{ N} \quad (\text{or } 344 \text{ lb})$$

If the rotor has an unbalance of U acting at the bearing, the unbalance force transmitted is given by

$$F_b = U(\omega^2) = U \frac{\pi N_{\text{rpm}}}{30} \quad (1)$$

for $F = 1530$ newtons (344 lb), the unbalance U is 15.5 gram-centimeters (0.216 oz in.). Note that, because this is a flexible rotor, the effect of static unbalance may be magnified by rotor bending. Thus, the unbalance may need to be considerably lower than calculated to keep the bearing load to a permissible level. Figure 2, calculated from the equations in reference 3, shows the forces acting on the bearings over a range of speed. Whenever the rotor speed is less than 1.4 times the critical speed, the bearing force is greater than predicted by the simple equation (eq. (1)). At the critical speed, for this lightly damped rotor, the bearing force is 10 times that for a rigid rotor. If the unbalance is doubled, 31.0 gram-centimeters (0.432 oz in.), the calculated rotating load increases F to 3060 newtons (688 lb). The basic load rating equation shows that doubling the bearing load will cause a reduction of bearing life by a factor of 8. Thus

$$H = 250 \text{ hr}$$

From the preceding example, it can be seen that with a rigid support the unbalance can cause large forces to be transmitted through the bearing. If, however, the rotor is mounted on flexible, damped supports, then the forces transmitted through the bearings may be attenuated by the proper selection of the support damping value.

Figure 3 shows schematically a single-mass rotor mounted in elastic damped supports. Figure 4 (from ref. 3) represents the rotor amplitude against rotor speed for various values of support damping for this system. In figure 4 rotor speed has been normalized with respect to the rigid support critical speed. If the bearing supports have

no damping, the rotor then will have a very high response at the critical speed, which is 3/10 of the rigid support critical speed.

As the damping is increased in the flexible support, the rotor amplitude at the critical speed is diminished until, at the optimum value of dimensionless support damping ($B = 5.0$), the peak amplitude at the critical speed completely disappears. However, if the support damping value is increased beyond the optimum value, the amplitude of motion will increase at the rigid support critical speed. For example, a damping value of 50 is excessive and has caused the support to lock up. Thus the behavior approaches that of a rigidly supported rotor. Figure 4 has been plotted for an amplification factor A of 10. The amplification factor is the ratio of the rotor amplitude at the rotor critical speed to the rotor unbalance eccentricity, that is, $A = a_{cr}/e_\mu$. A value of 10 represents moderately light damping. Reference 3 points out that optimum support damping is virtually independent of A for $A \geq 10$; thus the information of reference 3 is applicable to a wide range of rotors.

The amplification factor (as shown in ref. 3) can also be expressed as

$$A = \frac{K_2}{\omega_{cr} B_2} \quad (2)$$

where B_2 is the equivalent rotor and bearing damping given by

$$B_2 = \frac{K_s^2 B_b}{(K_b + K_s)^2 + (\omega B_b)^2} + B_s \quad (3)$$

The critical speed ω_{cr} is calculated from

$$\omega_{cr} = \sqrt{\frac{K_2}{M_2}} \quad (4)$$

where K_2 is the equivalent rotor and bearing stiffness given by

$$K_2 = \frac{K_b K_s (K_s + K_b) + K_s (\omega B_b)^2}{(K_b + K_s)^2 + (\omega B_b)^2} \quad (5)$$

When the bearing damping is small (as for rolling-element bearings), the equivalent stiffness and damping reduce to

$$K_2 = \frac{K_b K_s}{K_b + K_s} \quad (6)$$

and

$$B_2 = \frac{K_s^2 B_b}{(K_b + K_s)^2} + B_s \quad (7)$$

The preceding equations (2) to (7) are given in reference 3.

The data in figure 4, which are for a low mass ratio system ($M = 0.1$), can be applied to the single-mass rotor shown in figure 5. Example 2 will illustrate how to determine the support stiffness and damping required to prevent the excitation of the rotor first bending critical speed.

Example 2 - Magnitude of Damping Required to Attenuate Rotor Amplitude at First Critical Speed for Low Support Stiffness and Mass Ratio

The rotor of figure 5 is modeled as a mass of 2.42 kilograms ($0.0138 \text{ lb-sec}^2/\text{in.}$) on a massless elastic shaft. The spring rate for this rotor is calculated from

$$K_s = \frac{3\pi E D_s^4}{4L^3} \quad (8)$$

where

$$E = 2.1 \times 10^{11} \text{ N/m}^2 \quad (\text{or } 30 \times 10^6 \text{ psi})$$

$$D_s = 0.0254 \text{ m} \quad (\text{or } 1 \text{ in.})$$

$$L = 0.48 \text{ m} \quad (\text{or } 19 \text{ in.})$$

Thus

$$K_s = 18.2 \times 10^5 \text{ N/m} \quad (\text{or } 10,300 \text{ lb/in.})$$

The rotor is mounted in ball bearings which are assumed to have a much higher stiffness than that calculated for the shaft. Thus the critical speed on rigid supports can be calculated from

$$\omega_{cr} = \sqrt{\frac{K_2}{M_2}} \approx \sqrt{\frac{K_s}{M_2}}$$

$$\omega_{cr} = 8280 \text{ rpm} \quad (867 \text{ rad/sec})$$

The amplification factor A for this rotor on rigid supports is assumed to be 10. Solving equation (2) for B_2 results in

$$B_2 = \frac{K_2}{\omega_{cr} A} = 210 \text{ N} \cdot \text{sec/m} \quad (\text{or } 1.20 \text{ lb} \cdot \text{sec/in.})$$

The optimum damping ratio B for a minimum response over the speed range was 5 (see fig. 4). Solving for the required support damping results in

$$B_1 = B \times B_2 = 1050 \text{ N sec/m} \quad (\text{or } 6.00 \text{ lb sec/in.})$$

The corresponding support stiffness for a stiffness ratio K of 0.1 is

$$K_1 = K \times K_2 = K \times K_s = 0.1 K_s = 1.82 \times 10^5 \text{ N/m} \quad (\text{or } 1030 \text{ lb/in.})$$

In an actual design a support with such a soft spring rate would be susceptible to shock and self-excited whirl instabilities (ref. 3). A support having a higher stiffness may be desirable; therefore, a different value of damping would be necessary to minimize the rotor amplitudes and forces transmitted.

A further consideration in the choice of support stiffness is the mass ratio M , which is the ratio of bearing housing mass M_1 to rotor mass M_2 . Figure 6 shows that the mass ratio M , if one has a choice of stiffness ratios, should be as low as possible to minimize the rotor amplitude. For a given mass ratio, a tuned system produces near-minimum amplitude. A tuned system is a support system for which the stiffness ratio K equals the mass ratio M . Since for some rotor-bearing systems it is difficult to make the bearing and its housing much lighter than the rotor, the following example will use mass and stiffness ratios of unity, that is, $M = K = 1$.

Example 3 - Damping Required to Minimize Rotor Amplitude When $K = M = 1$

Figure 7 shows optimum damping ratios for a tuned system and the resultant minimum rotor amplitudes. For a value of $M = K = 1$, the optimum damping ratio B is 14. Rotor amplitudes over a speed range for several damping ratios are shown in figure 8. The rotor-bearing damping B_2 was previously calculated as 210 newton-seconds per meter (1.20 lb-sec/m). Thus the support damping is given by

$$B_1 = B \times B_2 = 2940 \text{ N} \cdot \text{sec}/\text{m} \quad (\text{or } 16.8 \text{ lb} \cdot \text{sec/in.})$$

From figures 7 and 8 for $K = M = 1$ and the optimum damping ratio of 14, the maximum rotor amplitude will be 1.6 times the unbalance eccentricity. This represents a reduction in amplitude to one-sixth that of a rotor running on rigid supports that has an amplification factor of 10.

In the rigidly mounted bearing model considered in the sample ball bearing calculation (example 1), the ratio of the transmitted force to the rotating unbalance force ranged up to 10. In the proper design of a flexibly mounted rotor with damping, the dynamic transmissibility and the bearing forces should be considerably less. With insufficient damping the dynamic transmissibility may exceed the rigid support value at the rotor critical speed (see fig. 9 for $B = 0.01$). In this instance the force transmitted through the bearings also exceeds the rigid support value (fig. 10). The problem to be considered now will be the selection of a value of damping to use in the support to minimize bearing forces and forces transmitted to the foundation. Figure 9 is a plot of the dynamic transmissibility over a speed range for various values of damping. The damping value chosen to minimize rotor amplitude ($B = 14$) is near-optimum for minimizing transmissibility, although $B = 10$ results in a slightly lower maximum foundation force. Figure 10 shows that $B = 14$ minimizes the force transmitted through the bearings. When the rotor is operating above 1.4 times the rigid support critical speed, support damping decreases the forces transmitted through the bearings, although increasing the damping at high speeds ($> 1.8 \omega_{cr}$) will cause increased loads to be transmitted to the foundation. The proper design of the damper then must take into consideration the forces and amplitudes of motion to be permitted throughout the operating speed range. In addition to these steady-state conditions, the proper damper design must also take into consideration the damping necessary to insure stability and transient operation due to shock and other suddenly applied loads (such as those that might occur due to the loss of a blade in the compressor or turbine). For the rotor considered here a damping ratio B of 10 to 14 should be satisfactory. Since the rotor-bearing damping B_2 is 210 newton-seconds per meter (1.20 lb sec/in.), the required support damping is 10 to 14 times this, or 2100 to 2940 newton-seconds per meter (12.0 to 16.8 lb-sec/in.).

**DAMPER SUPPORT REQUIRED FOR A MULTIMASS ROTOR OPERATING
BELOW THE SECOND BENDING CRITICAL SPEED**

In contrast to the singlemass rotor, a multimass rotor has more than one bending critical speed. Thus, in general, singlemass rotor data are not adequate for the design of a damper support for a multimass rotor. However, if the maximum service speed is below the second bending critical speed, singlemass theory may be entirely adequate, as will now be shown.

The rotor shown in figure 11 is designed to simulate the flexible rotor of a small, lightweight turbocompressor which is required to operate above its first bending critical speed and below the second bending critical speed. Oil squeeze film dampers are to be used at the bearing supports to attenuate rotor motion. Two single-row, deep-groove ball bearings, series 204, will be used to support the rotor. Each bearing support housing has a mass of 1.21 kilograms ($0.0069 \text{ lb sec/in.}^2$). A cantilevered centering spring supports the ball bearing housing; the spring rate can be chosen to complement that of the squeeze film. Oil is supplied to the damper from a circumferential groove. Two piston rings located in circumferential grooves along with metering orifices control the flow of damping oil from the bearing ends. These features are shown schematically in figure 1.

Critical speeds were calculated for this rotor on rigidly supported bearings by the critical speed computer program of reference 6. A bearing stiffness of 65.5 meganewtons per square meter (375 000 lb/in.) was assumed. The first three critical speeds and associated mode shapes are shown in figure 12. The first bending critical speed was calculated as 8280 rpm (867 rad/sec).

The shaft and bearing span of this five-mass rotor are identical to those of the single-mass rotor of example 2. Thus, the five-mass rotor will have the same stiffness as the single-mass rotor. For calculation purposes, the five masses may be replaced with a single mass at the rotor center, which results in the same critical speed. This equivalent single mass may be calculated from

$$M_2 = \frac{K_2}{\omega_{cr}^2} = \frac{1.82 \text{ MN/m}^2}{(867 \text{ rad/sec})^2} = 2.42 \text{ kg} \quad (\text{or } 0.0138 \text{ lb} \cdot \text{sec}^2/\text{in.})$$

The actual rotor mass is 5.68 kilograms ($0.0325 \text{ lb sec}^2/\text{in.}$).

The equivalent mass is identical to that of the single mass rotor. The ratio of support mass to rotor mass is

$$M = \frac{M_1}{M_2} = \frac{2(1.21)}{2.42} = 1$$

The supports will be designed for a tuned system, that is, $K = M$. This being the case, the rotor properties, support stiffness, and mass ratios are the same as those for the single-mass rotor, and all of the previously calculated values may be used without change. Thus the required damping ratio B is 10 to 14, and the support damping B_1 is 2100 to 2940 newton-seconds per meter (12.0 to 16.8 lb sec/in.) or one-half of this for each of the two supports.

CHARACTERISTICS OF THE SQUEEZE-FILM DAMPER

A squeeze-film damper is shown schematically in figure 13. It consists of a cylindrical journal, which is prevented from rotating in a cylindrical bearing. The journal center is assumed to make a circular orbit about the bearing center.

The Reynolds lubrication equation for incompressible flow is given in reference 7 as

$$\frac{1}{R^2} \frac{\partial}{\partial \theta} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = \frac{6}{R} \frac{\partial(hu)}{\partial \theta} + 12 \frac{\partial h}{\partial t} \quad (9)$$

If the bearing is considered to be very short relative to its diameter, the pressure-induced flow $(h^3/\mu)(\partial p/\partial \theta)$ in the circumferential direction is negligible compared with the shear flow in the circumferential direction. It is convenient at this point to introduce rotating coordinates according to

$$\theta' = \theta - \omega t \quad (10)$$

where ω is the rate of precession.

The Reynolds equation now becomes

$$\frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = -12\omega \frac{\partial h}{\partial \theta'} \quad (11)$$

Assuming that the bearing and journal axes are parallel, the film thickness h is given by

$$h = c_p(1 + \epsilon \cos \theta') \quad (12)$$

where the eccentricity ratio ϵ is defined by

$$\epsilon = \frac{e}{c_r} \quad (13)$$

Since h does not depend on z , the Reynolds equation may be integrated directly with the boundary conditions $P(\theta', 0) = P(\theta', L) = 0$ to give

$$P(\theta', z) = \frac{6\mu\omega}{h^3} \frac{\partial h}{\partial \theta'} (Lz - z^2) \quad (14)$$

It is common practice in lubrication analyses to neglect negative pressures since, in general, a fluid cannot sustain a tensile stress. If it is assumed that cavitation will occur, the preceding expression may be integrated over the area of positive pressure to give a radial restoring force F_r and a tangential force F_θ that opposes the journal motion. The results, which are derived in detail in reference 8, are

$$F_r = \frac{2\mu RL^3\omega\epsilon^2}{c_r^2(1-\epsilon^2)^2} \quad (15)$$

$$F_\theta = \frac{\pi\mu RL^3\omega\epsilon}{2c_r^2(1-\epsilon^2)^{3/2}} \quad (16)$$

These results can also be given in terms of stiffness and damping coefficients:

$$K_d = \frac{F_r}{e} = \frac{2\mu RL^3\epsilon\omega}{c_r^3(1-\epsilon^2)^2} \quad (17)$$

and

$$B_d = \frac{F_\theta}{\omega e} = \frac{\pi\mu RL^3}{2c_r^3(1-\epsilon^2)^{3/2}} \quad (18)$$

These coefficients may be expressed in dimensionless form as

$$\bar{K}_d = \frac{K_d c_r^3}{\mu \omega R L^3} = \frac{2\epsilon}{(1 - \epsilon^2)^2} \quad (19)$$

and

$$\bar{B}_d = \frac{B_d c_r^3}{\mu L^3 R} = \frac{\pi}{2(1 - \epsilon^2)^{3/2}} \quad (20)$$

Both \bar{K}_d and \bar{B}_d are functions only of the eccentricity ratio and are plotted as such in figure 14.

DESIGN OF SQUEEZE-FILM DAMPER BEARING FOR MULTIMASS FLEXIBLE ROTOR TO OPERATE THROUGH THE FIRST BENDING CRITICAL SPEED

The magnitude of damping required has been determined, and it is now necessary to design the damper bearing to produce this amount of damping. The expression for B_d (eq. (18)) shows that damping is a function of damper clearance, length, radius, eccentricity ratio, and oil viscosity. Generally, the diameter of the damper housing is dictated by the ball bearing outside diameter. The same oil is usually used for both the damper and the ball bearing; thus, the viscosity of damper oil is fixed. It remains, however, to select values of the radial clearance c_r and length L .

Since film pressure and consequently film stiffness increase rapidly with increasing eccentricity ratios, it is not desirable to operate at very large eccentricity ratios because the film stiffness will make the overall support stiffness too large. A maximum eccentricity ratio of $\epsilon = 0.4$ at the first critical speed was chosen for this damper design. For that value a dimensionless damping \bar{B}_d of 2.04 is obtained from figure 14.

The next parameter to determine is the damper clearance c_r . Figure 15 (from ref. 3) shows that, for an optimally damped system ($B = 10$ to 20), the maximum support amplitude is about equal to the mass eccentricity of the rotor, that is, the displacement of the rotor center of gravity from the geometrical center. The five-mass rotor of figure 11 is expected to have a maximum unbalance of 29 gram-centimeters (0.4 oz in.) distributed fairly uniformly over the five masses. This corresponds to a mass eccentricity of 0.05 millimeter (0.002 in.). If the maximum damper eccentricity ratio is 0.4, the damper clearance must be $0.05/0.4 = 0.13$ millimeter ($0.002/0.04 = 0.005$ in.).

It remains only to determine the damper length. The design of figure 1, with a circumferential oil supply groove divides the damper into two equal halves. Each half functions separately and thus provides one-fourth of the total required support damping. The total support damping needed was previously determined to be 2100 to 2940 newton seconds per meter (12.0 to 16.8 lb sec/in.). The rotor is expected to operate with considerably less than the maximum unbalance most of the time; the maximum of 29 gram-centimeters (0.4 oz-in.) represents a degraded value after considerable service time. Less unbalance means lower amplitudes and, thus, lower stiffness of the squeeze film (eq. (17)). To maintain squeeze-film stiffness as much as possible at lower eccentricities, the damper will be sized for damping near the top of the range for an eccentricity ratio of 0.4. Thus, the design value will be 2800 newton-seconds per meter (16 lb sec/in.). Damper length can now be determined from

$$L = c_r \sqrt[3]{\frac{B_1}{4\bar{B}_d \mu R}}$$

where

$$c_r = 0.13 \text{ mm} \quad (\text{or } 0.005 \text{ in.})$$

$$\mu_{140^\circ F} = 0.0119 \text{ N sec/m}^2 \quad (\text{or } 1.73 \times 10^{-6} \text{ lb sec/in.}^2)$$

$$R = 39.6 \text{ mm} \quad (\text{or } 1.56 \text{ in.})$$

$$B_1 = 2800 \text{ N-sec/m} \quad (\text{or } 16 \text{ lb-sec/in.})$$

$$\bar{B}_d = 2.04$$

$$L = 11.4 \text{ mm} \quad (\text{or } 0.45 \text{ in.})$$

The net radial stiffness of the cavitated squeeze film can now be calculated. For $\epsilon = 0.4$ a value of dimensionless stiffness $\bar{K}_d = 1.13$ is obtained from figure 14. A value of film stiffness can now be calculated from

$$K_d = \frac{\bar{K}_d \mu \omega R L^3}{c_r^3}$$

where ω is the damper angular precession speed. At the first critical speed of 867 rad/sec (8280 rpm), $K_d = 338$ kilonewtons per meter (1930 lb/in.). The total damper spring rate is four times this value or 1350 kilonewtons per meter (7720 lb/in.). As was stated before, the overall support stiffness is the sum of the centering spring stiffness and the damper stiffness since the two springs act in parallel. The centering spring stiffness may now be chosen to provide optimum stiffness at the first critical speed. For $K = 1$ total support stiffness $K_1 = 1820$ kilonewton-meters (10 300 lb/in.). Thus, the stiffness K_C of each centering spring must be $1/2(1820 - 1350) = 235$ kilonewton-meters (1290 lb/in.). This is a very soft spring and will need to be pre-loaded to center the rotor at low speeds.

To determine how effective this damped flexible support is in attenuating the rotor amplitude and bearing forces, an unbalance response computer program was used to produce figures 16 to 18. The program is that of reference 9 which treats nonaxisymmetric rotor supports and nonlinear stiffness and damping in the squeeze film.

Figure 16 shows rotor amplitude at midspan, and figure 17 shows forces that would be transmitted to the ball bearing for both a rigid and a flexible damped support. The long-dash curves of these figures represent the rotor operating with good balance; the total unbalance is 7 gram-centimeters (0.1 oz in.). The corresponding mass eccentricity is 0.013 millimeter (0.0005 in.). The first critical speed has shifted upward to about 9000 rpm. Figure 16 shows that the rotor amplitude is about three times the mass eccentricity at the first critical speed. The amplitude then drops with increasing speed and remains low out to 30 000 rpm. Amplitude then increases with speed, reaching a maximum of four times the mass eccentricity at 39 000 rpm. Figure 17 shows that the bearing experiences virtually no force buildup due to the critical speed. At 9000 rpm the bearing force is only 14 newtons (3 lb). This is only one-half the force that would be experienced by a bearing on a rigidly supported rigid rotor (for which $F_b = (1/2)M_2 e \mu \omega^2$). Bearing force generally rises with speed to 39 000 rpm and then drops off.

Now consider the case where the initial balance at assembly has degraded for any one or a combination of the reasons mentioned in the INTRODUCTION. Instead of a total unbalance of 7 gram-centimeters (0.1 oz-in.), let us assume the unbalance now is 29 gram-centimeters (0.4 oz-in.) resulting in a mass eccentricity of 0.05 millimeter (0.002 in.). The solid curves of figures 16 and 17 show results for the rotor on squeeze-film supports, and the short-dash curves show results for the rotor on rigid supports. At the first critical speed the center amplitude of the flexibly supported rotor is about three times the mass eccentricity, as with the lesser unbalance. Over the entire speed range the amplitude changes nearly the same as for the low unbalance. However, the peak amplitude speed has shifted from 39 000 to 43 000 rpm due to greater damping in the squeeze film at the higher eccentricity. The rigidly supported rotor, in

contrast, has an amplitude of 50 times the mass eccentricity at the first critical speed. Thus the squeeze-film support reduces the rotor amplitude by a factor of 16.

Figure 17 shows that bearing force at the first critical speed is only 51 newtons (11 lb) with the flexible support and that with a rigid support it is 1900 newtons (420 lb), or 36 times greater. Similar load reductions occur at the higher critical speeds. At 28 000 rpm the load for the rigidly supported bearing is 26 000 newtons (5800 lb). This far exceeds the load rating of 1530 newtons (344 lb) for the 204 series bearing. With the squeeze-film support, however, the load is only 480 newtons (108 lb), well within the bearing capacity.

Figure 18 shows the resultant damper amplitudes for the flexibly supported rotor. As predicted for a single-mass rotor, damper amplitudes are approximately equal to the mass eccentricity up to twice the first critical speed. At higher speeds, damper eccentricities increase. For a large unbalance the increase is about 70 percent; for the small unbalance it is nearly a factor of 5 greater.

Though the damper was sized only for the first critical speed, the results show that amplitudes and forces at the higher critical speeds are also reduced substantially. Thus single-mass rotor data are useful, not only for multimass rotors operating below the second critical speed, but for this rotors also at higher speeds. However, for rotors operating through several critical speeds, the authors recommend that a rotor response analysis be used after the design of the damper to determine damper and rotor performance at the higher critical speeds.

Rotor response was also calculated with the unbalance concentrated at the center mass, rather than distributed over the five masses. This resulted in a much larger unbalance loading than with distributed unbalance. Consequently, the squeeze-film damper was overloaded; rotor amplitudes and bearing loads were approximately double those for rigidly supported bearings. The point to be noted is that an improperly designed squeeze-film damper (inadequate clearance, etc.) can be worse than a rigid bearing support. Reference 10 also illustrates this phenomenon.

SUMMARY OF DESIGN PROCEDURE

- (1) For a symmetric multimass rotor, determine an equivalent single mass M_2 from

$$M_2 = \frac{K_2}{\omega_{cr}^2}$$

where K_2 is the rotor-bearing stiffness at midspan (given by eq. (5)), and ω_{cr} is the first bending critical speed for the multimass rotor on rigidly supported bearings.

(2) Calculate the mass ratio $M = M_1/M_2$ for the total bearing mass M_1 to be used and the equivalent rotor mass M_2 .

(3) From figure 8 determine the optimum damping ratio B . Determine the absolute support damping B required from $B_1 = B \times B_2$. For lightly damped bearings, the value of effective bearing system damping B_2 may be estimated from $K_2/10\omega_{cr}$.

(4) Determine the absolute support stiffness K from

$$K_1 = K \times K_2$$

For a tuned system, which produces near-minimum rotor amplitude, $K = M$. Figure 8 and the design examples of this report assume that a tuned system is to be used.

(5) Assume a damper eccentricity ratio ϵ that will not make the overall stiffness of the support too large, and from figure 14 determine values of dimensionless stiffness \bar{K}_d and damping \bar{B}_d . Generally, the maximum value of ϵ should be less than 0.4.

(6) From the actual rotor mass and the maximum anticipated unbalance, calculate the mass eccentricity from

$$e_\mu = \frac{U}{M_2}$$

For an optimally damped system (as in fig. 15), the maximum support amplitude is approximately equal to the mass eccentricity, that is, $a_1/e_\mu = 1$.

(7) The damper clearance can now be calculated from

$$c_r = \frac{e_\mu}{\epsilon}$$

(8) For a circumferentially grooved damper, the damper half length is determined from the following:

$$L = c_r \sqrt[3]{\frac{B_1}{4B_d \mu R}}$$

The values of total support damping B_1 , the squeeze damping \bar{B}_d , and the clearance C_r all have been previously determined. The damper radius R will usually be dictated by the size of the rolling or sliding bearing to be used. The viscosity μ is that of the bearing lubricating oil.

(9) Calculate the stiffness of each centering spring from

$$K_c = 1/2 \left(K_1 - 4 \bar{K}_d \mu \omega_{cr} R L^3 / c_r^3 \right)$$

SUMMARY OF RESULTS

Theoretical data for a single-mass rotor were used to determine flexible support properties (stiffness and damping) to attenuate rotor amplitudes and bearing loads for a multimass rotor operating through the first bending critical speed. An equivalent single mass for the multimass rotor was calculated from the rotor first critical speed (determined from a critical speed computer program) and the rotor shaft stiffness. A squeeze-film damper support was then designed to provide the required damping at the assumed unbalance conditions. Analytical rotor response results showed that:

1. The squeeze-film damper successfully attenuated rotor amplitudes and bearing loads at the first critical speed. Rotor midspan amplitude was reduced by a factor of 16, and bearing load was reduced by a factor of 36 compared with an identical rotor with rigidly supported bearings.
2. Amplitudes and forces at higher critical speeds were also reduced substantially.
3. With unbalance less than the design value, amplitude and forces were also well controlled. However, with unbalances much greater than the design value, amplitudes and forces were larger than with rigidly supported bearings.
4. Bearing loads are well under permissible values for the flexibly supported rotor. With rigid supports, bearing forces are very high near rotor critical speeds, resulting in drastically shortened bearing lives.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 7, 1974,
505-04.

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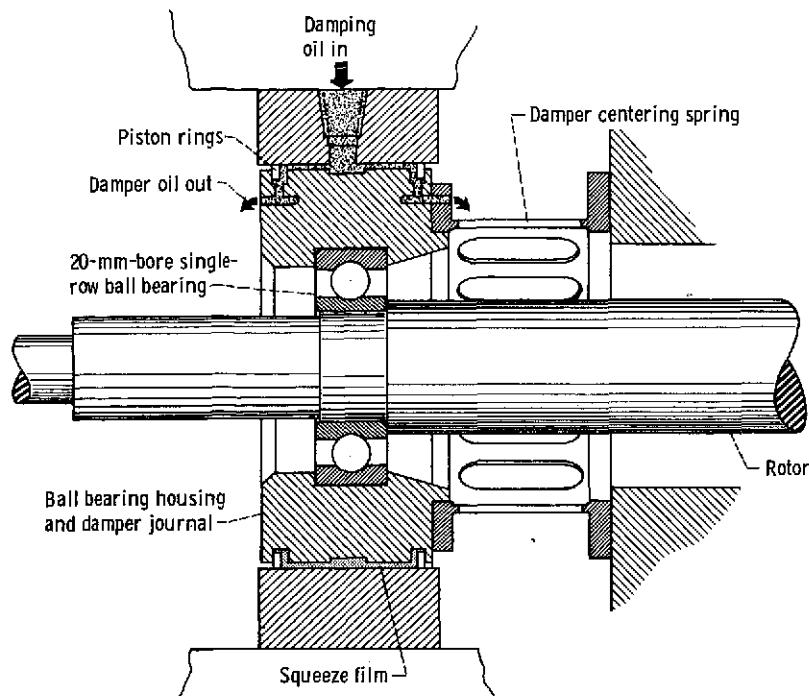


Figure 1. - Squeeze-film damper with central feed groove.

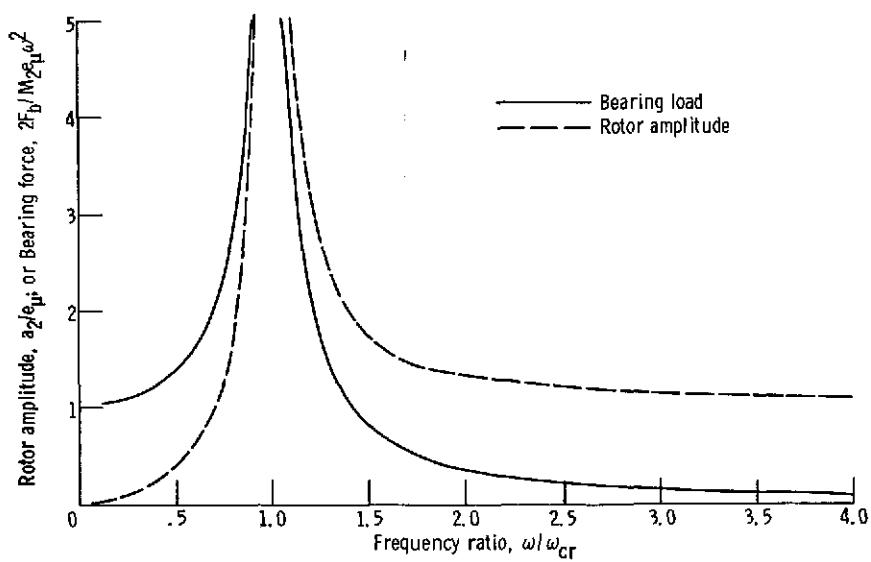


Figure 2. - Bearing force and rotor amplitude for single-mass flexible rotor in rigidly mounted bearings. Amplification factor, 10.

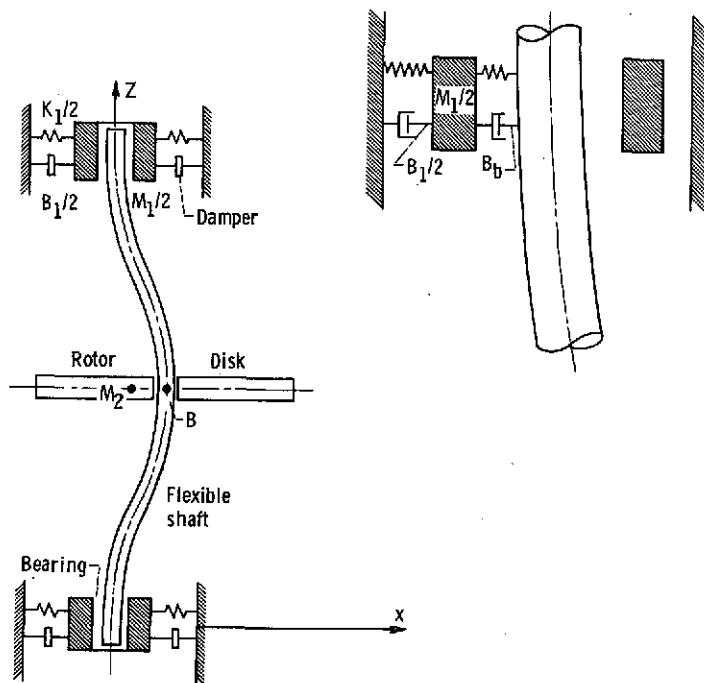


Figure 3. - Single-mass rotor on damped elastic supports.

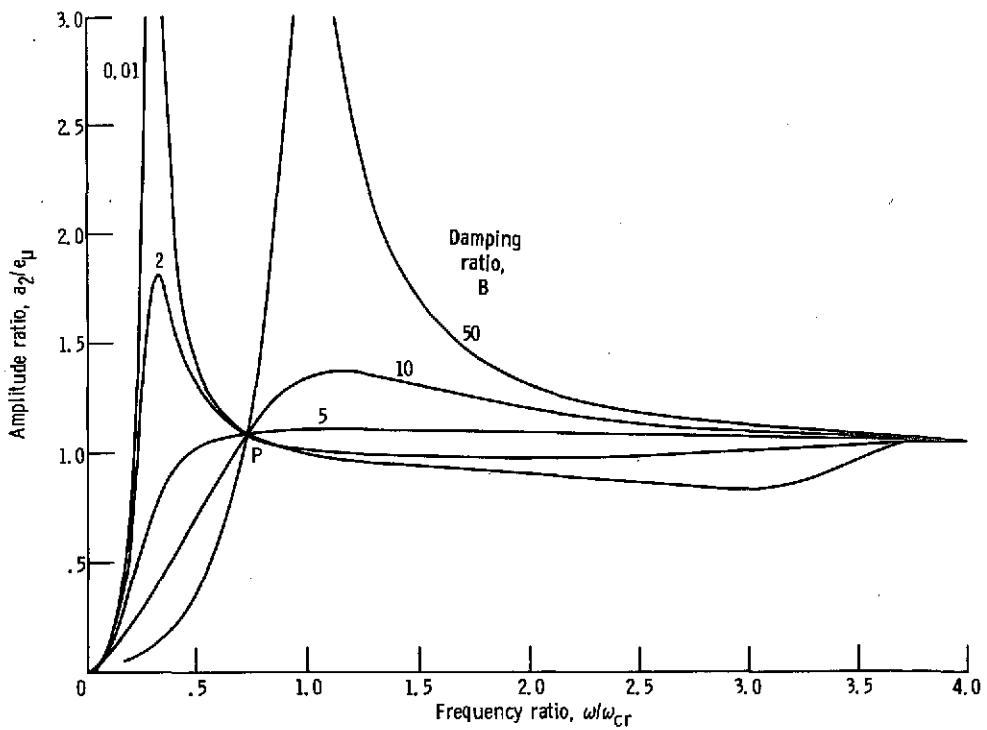


Figure 4. - Rotor amplitude as function of speed for low mass ratio tuned support system for various values of support damping (from ref. 3). Stiffness ratio, 0.1; mass ratio, 0.1; amplification factor, 10.

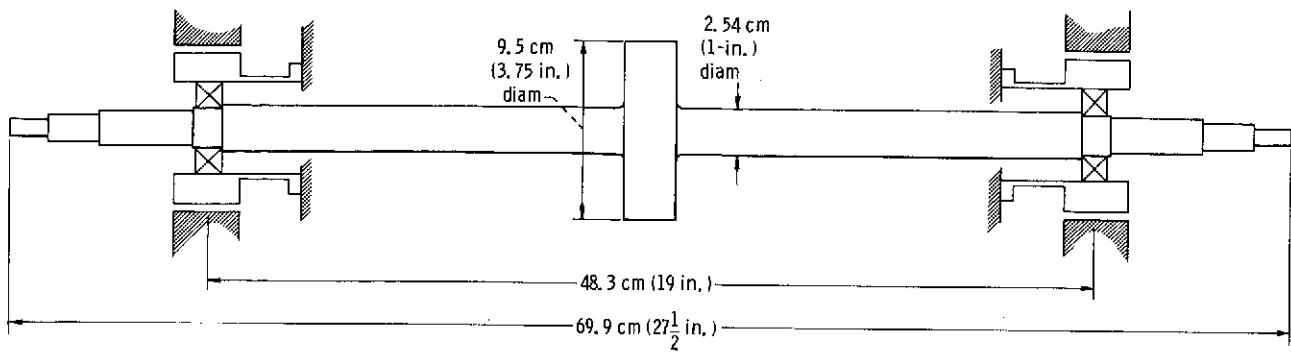


Figure 5. - Single-mass flexible rotor.

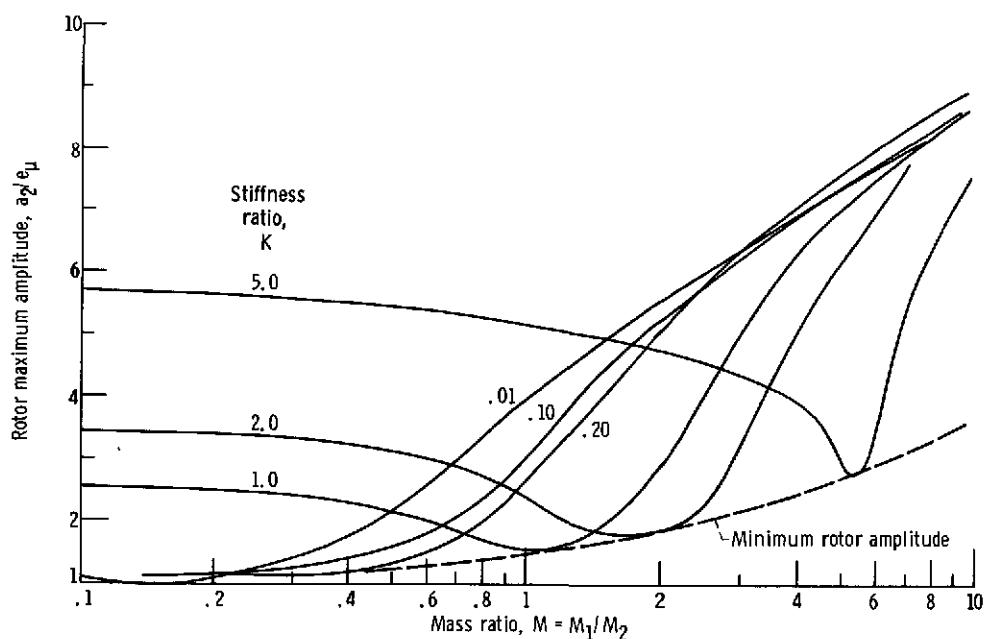


Figure 6. - Rotor maximum amplitude for various values of stiffness and mass ratio with optimum support damping (from ref. 3). Amplification factor, 10.

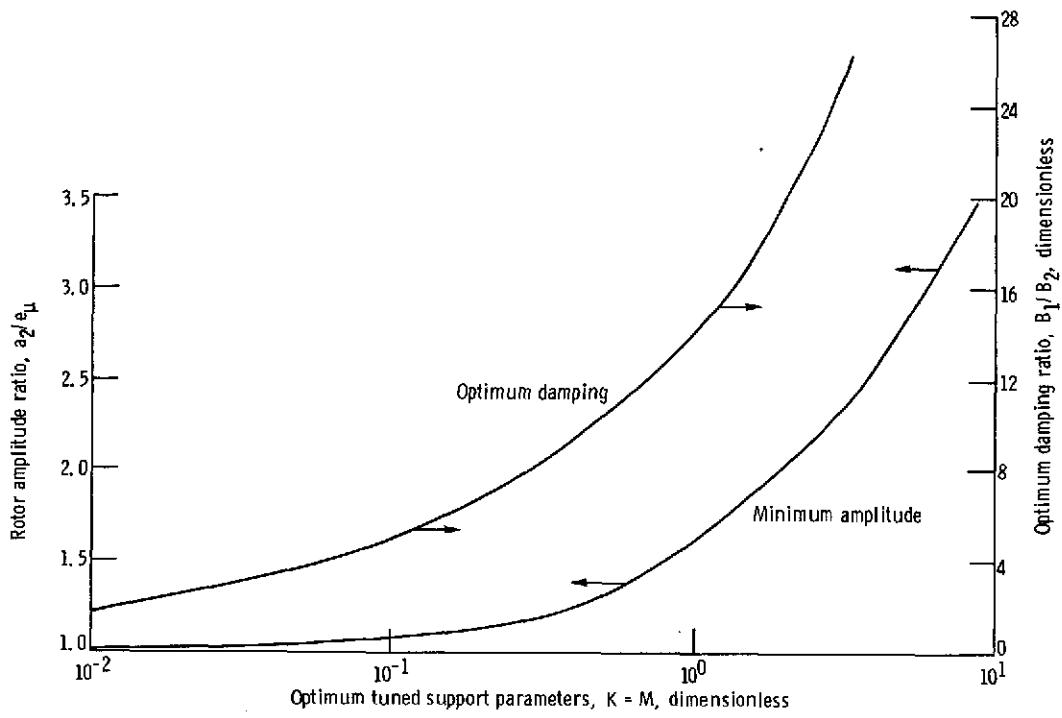


Figure 7. - Optimum support damping and maximum rotor amplitude as function of mass ratio (from ref. 3).
Amplification factor, 10.

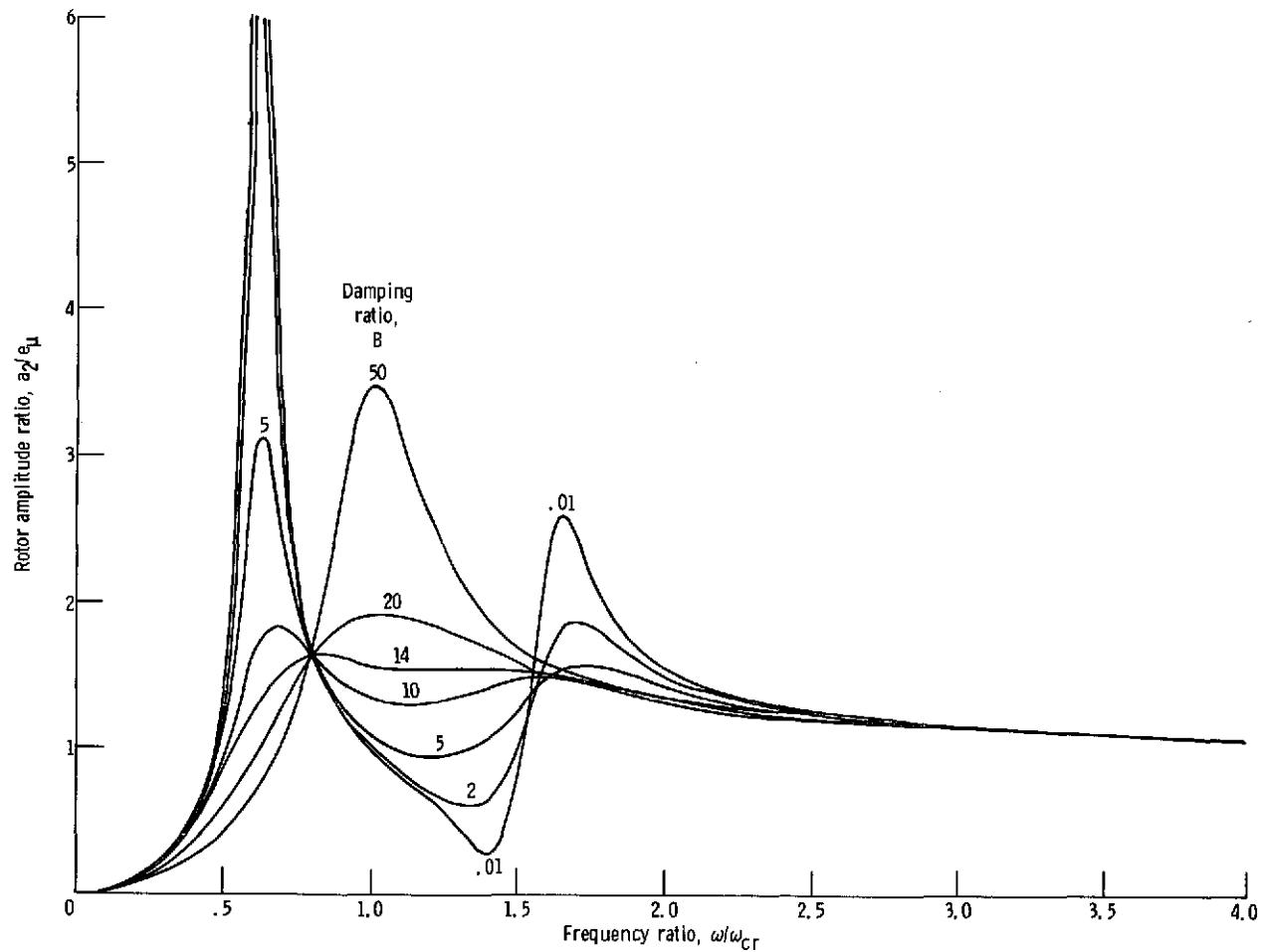


Figure 8. - Absolute rotor motion with tuned support system for various values of support damping (from ref. 3). Stiffness ratio, 1; mass ratio, 1; amplification factor, 10.

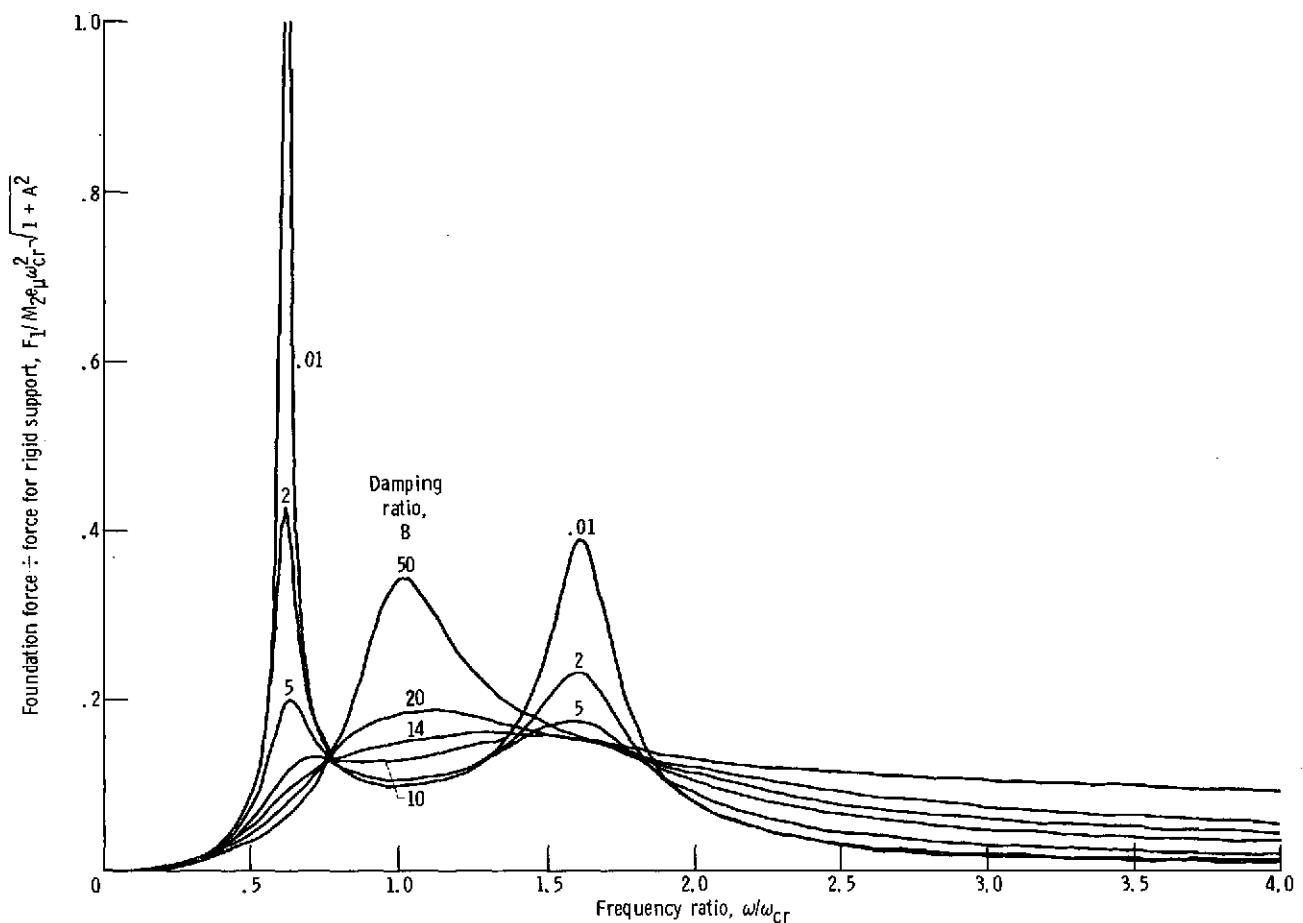


Figure 9. - Dimensionless force transmitted to foundation as function of speed ratio for various values of support damping (from ref. 3).
Stiffness ratio, 1; mass ratio, 1; amplification factor, 10.

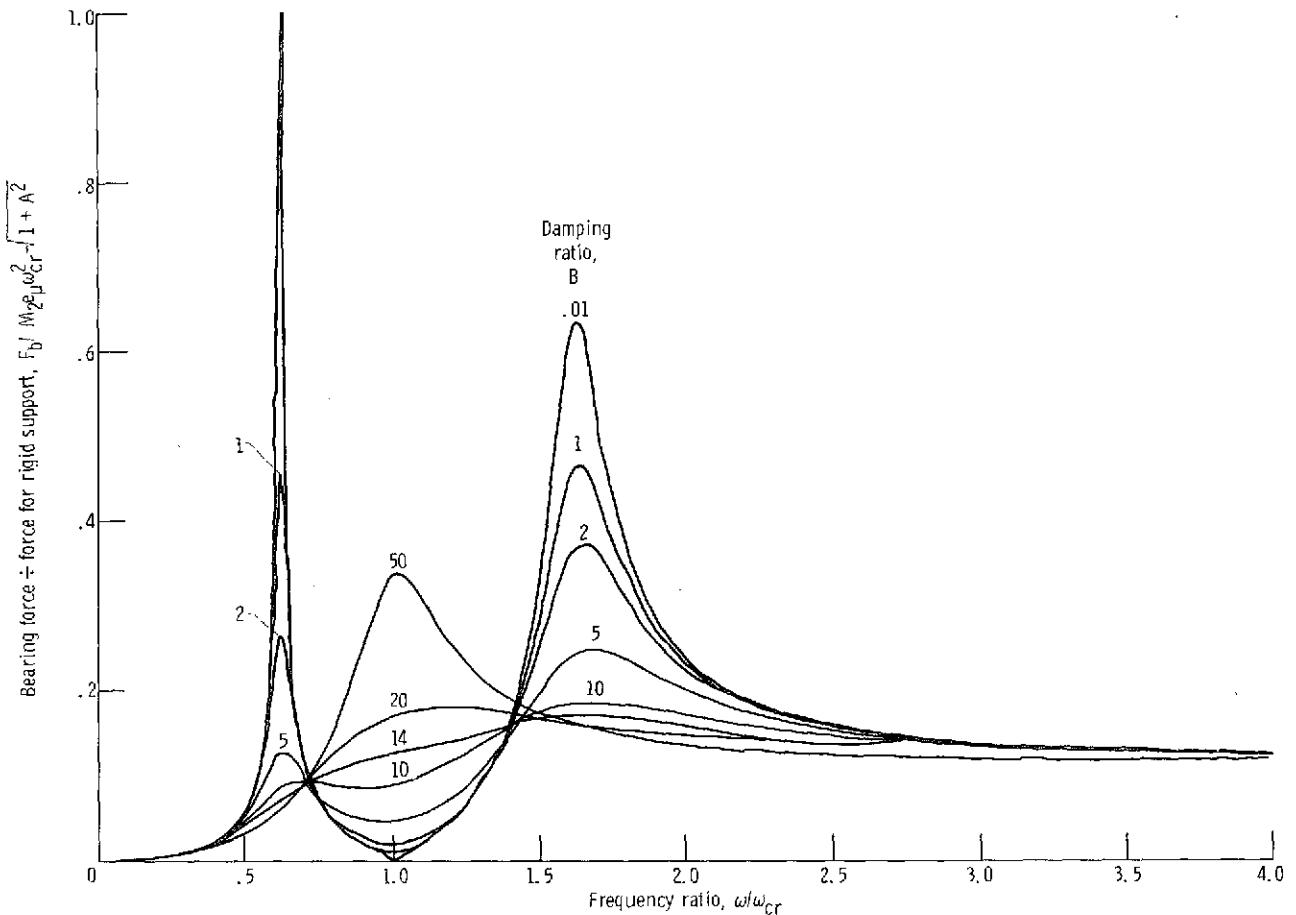


Figure 10. - Dimensionless force transmitted to bearings as function of speed ratio for various values of support damping (from ref. 3).
 Stiffness ratio, 1; mass ratio, 1; amplification factor, 10.

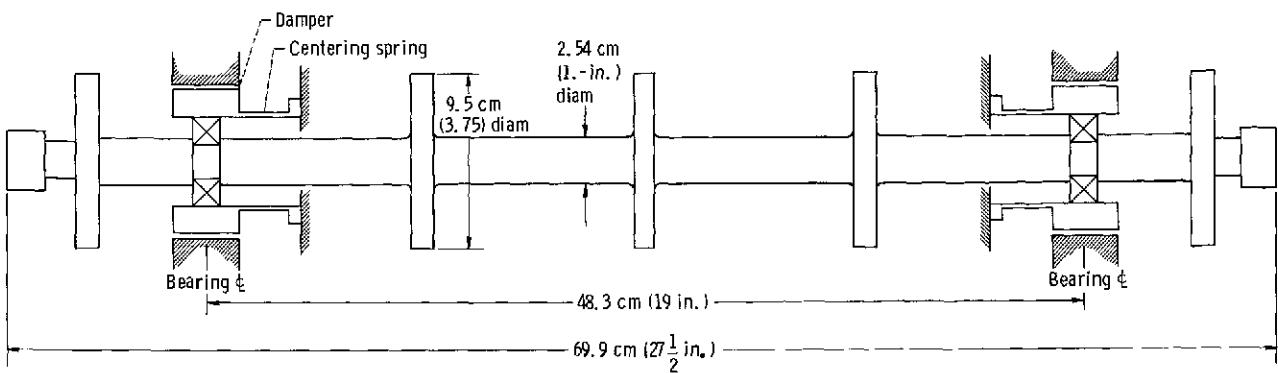


Figure 11. - Five mass flexible rotor.

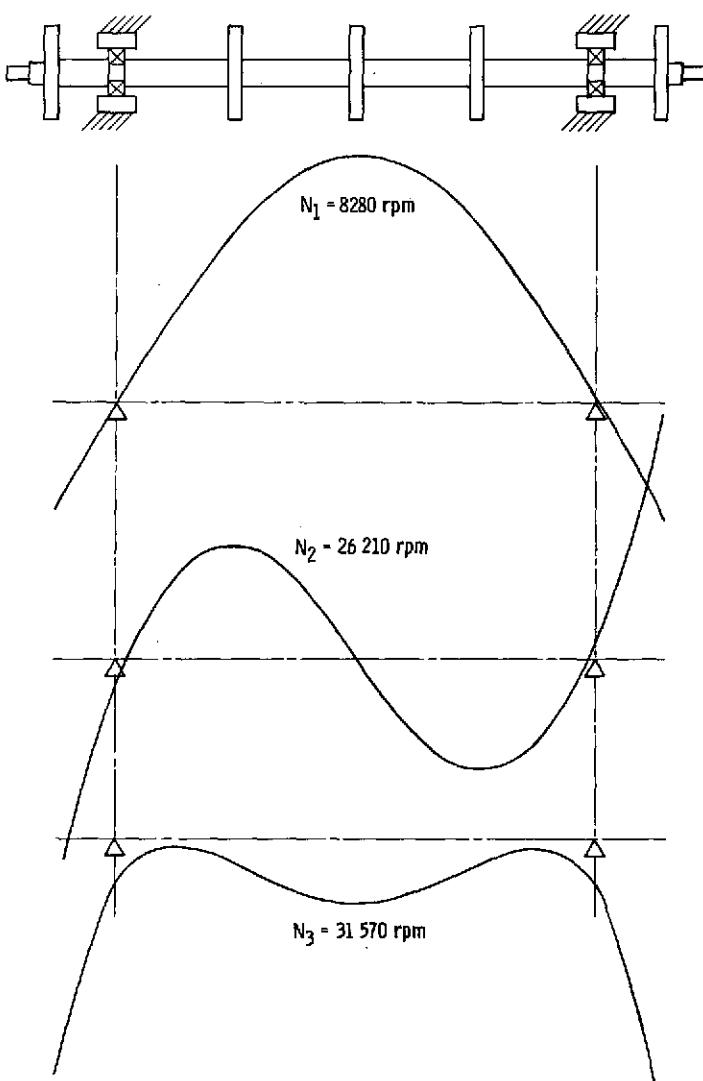


Figure 12. - Undamped critical speeds and mode shapes of five-mass rotor on rigid supports. Bearing stiffness, 65.5 meganewtons per meter (375 000 lb/in.).

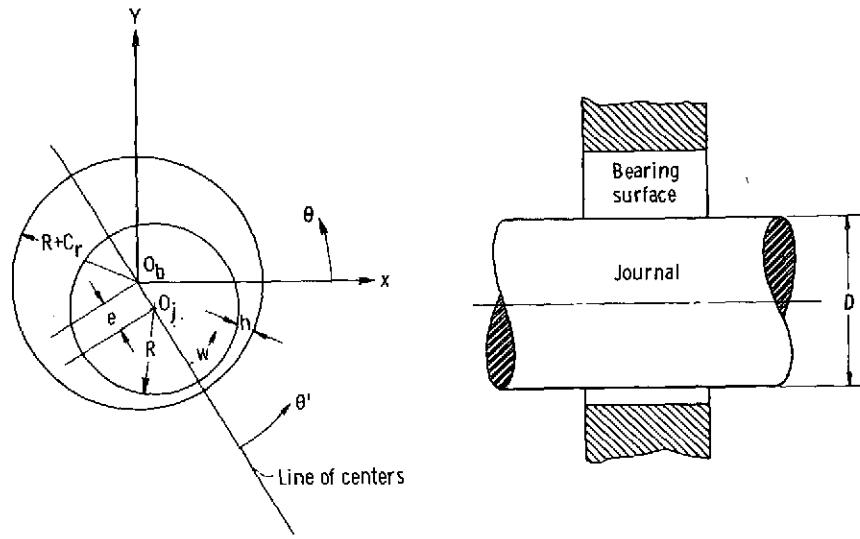


Figure 13. - Squeeze-film damper bearing in fixed and rotating coordinate systems.

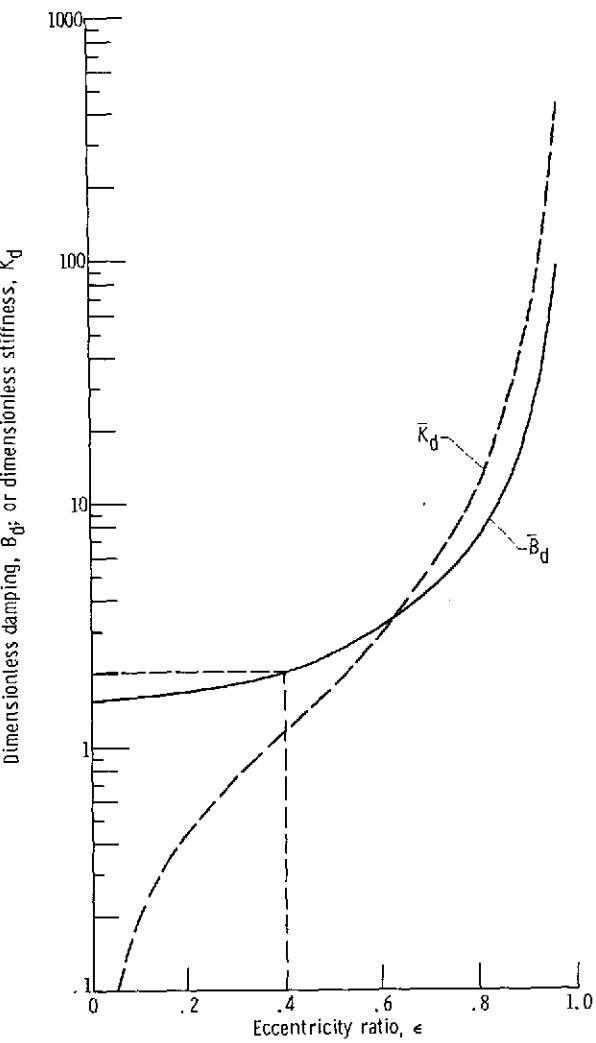


Figure 14. - Dimensionless damping (eq. (20)) and stiffness (eq. (19)) as functions of eccentricity ratio.

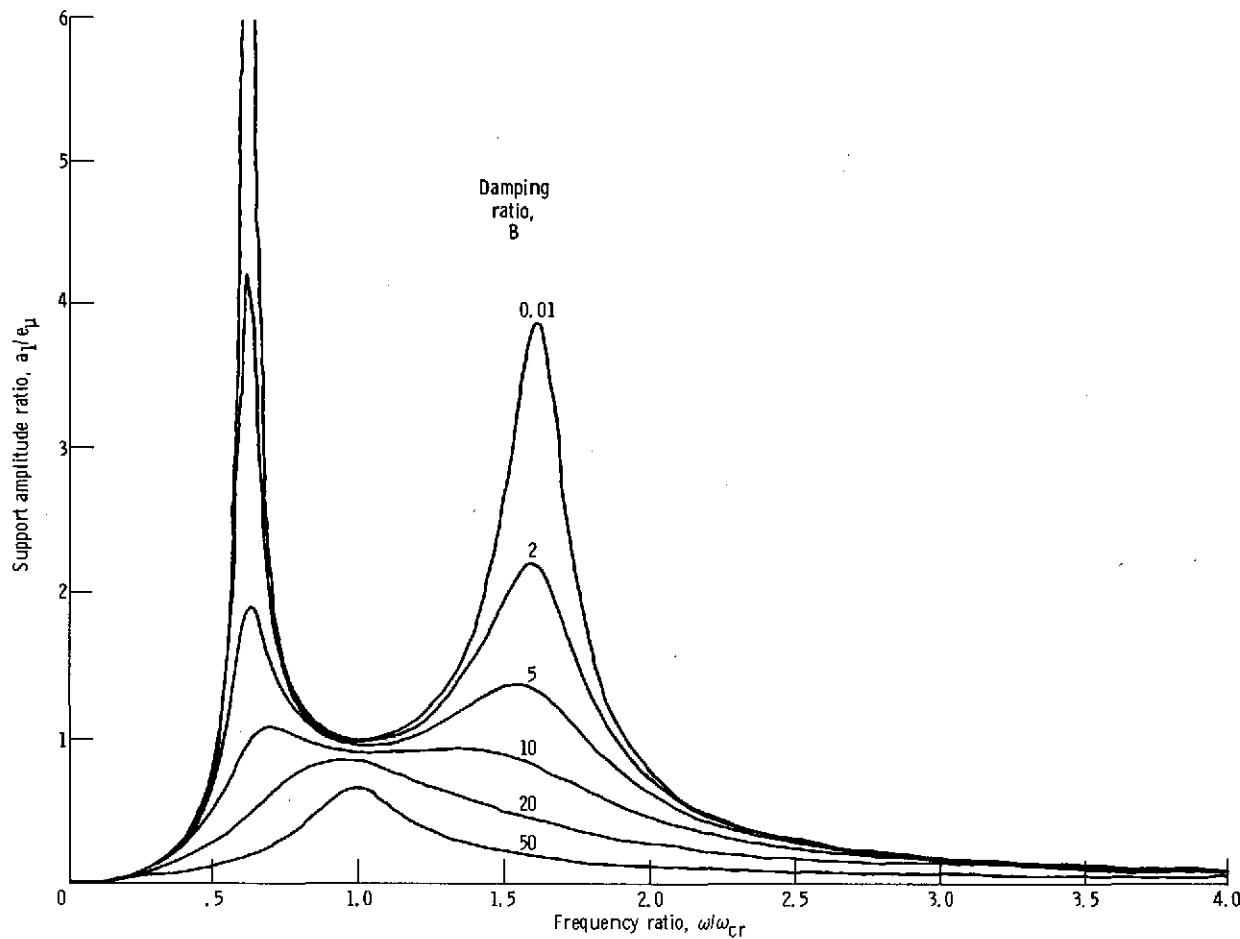


Figure 15. - Support amplitude as function of speed for various values of support damping (from ref. 3). Stiffness ratio, 1; mass ratio, 1; amplification factor, 10.

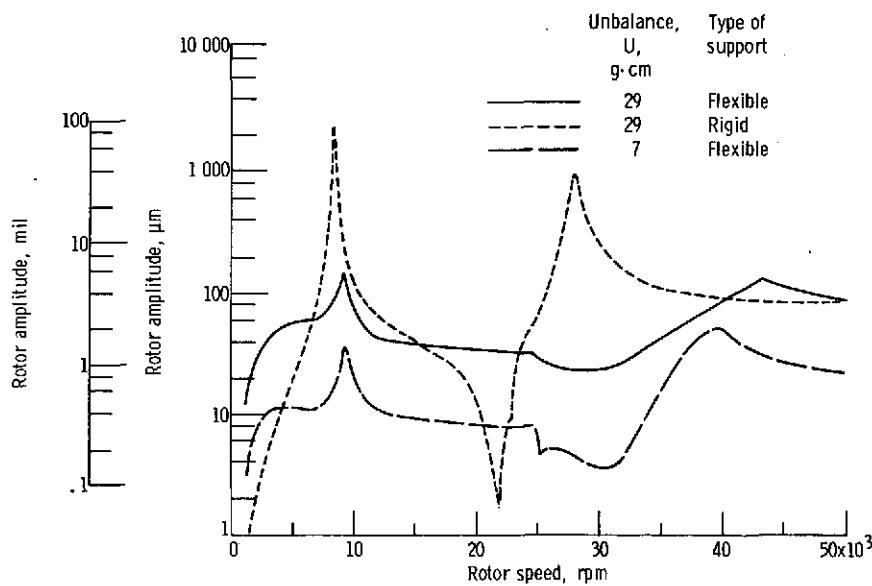


Figure 16. - Amplitude at rotor midspan for rigidly and flexibly supported five-mass rotor.

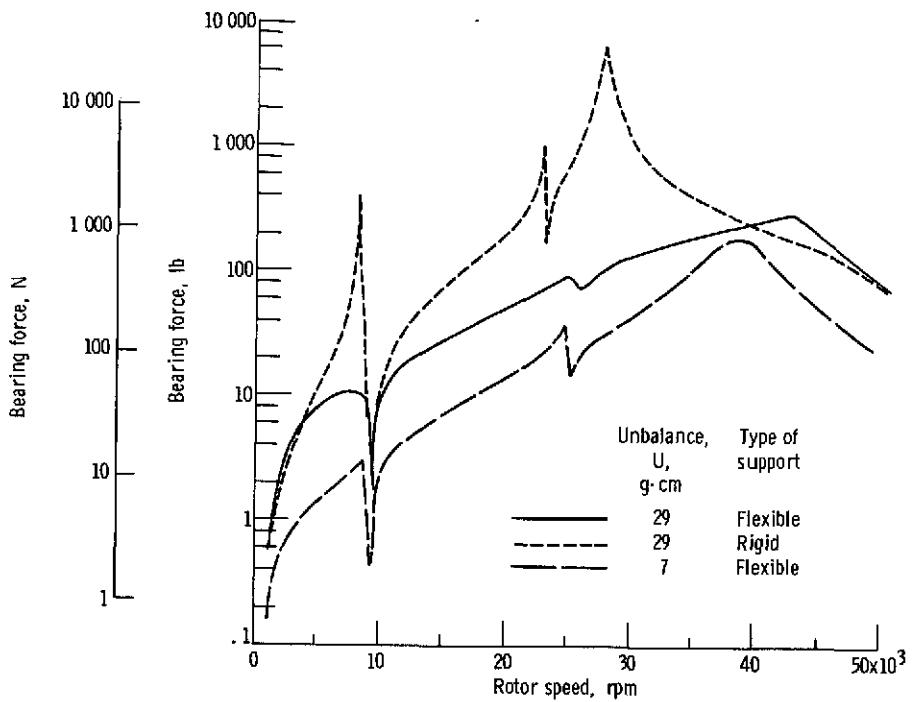


Figure 17. - Bearing forces for rigidly and flexibly supported five-mass rotor.

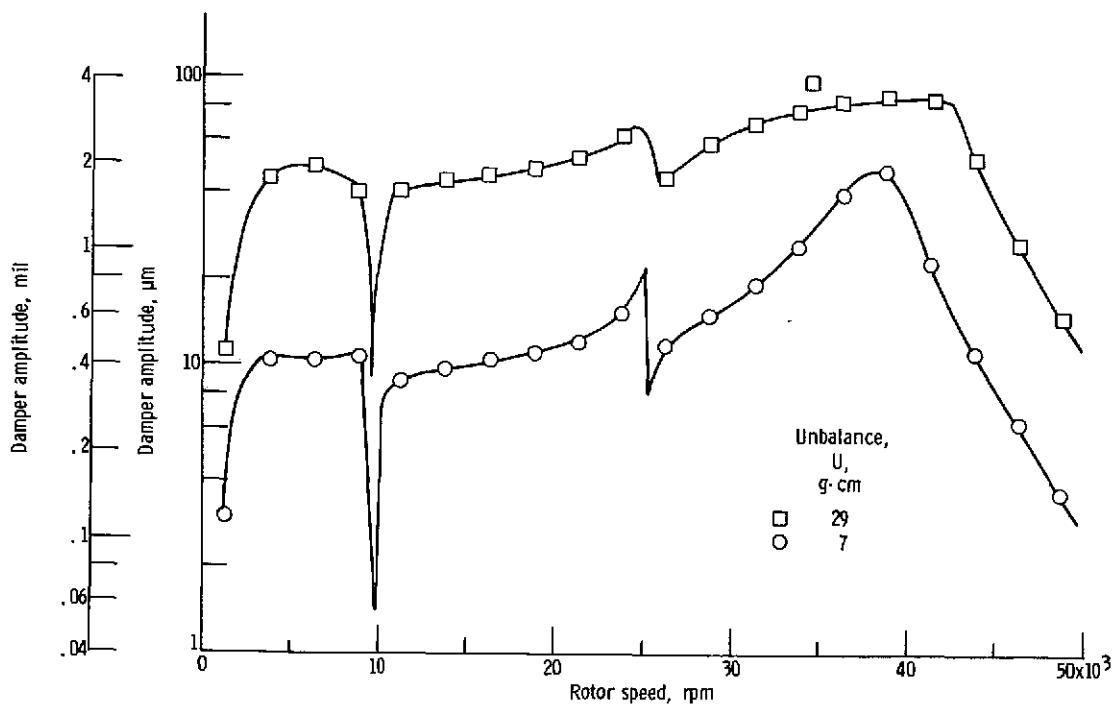


Figure 18. - Damper amplitude for flexibly supported rotor.