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DYNAMIC CHARACTERISTICS OF A JEFFCOTT ROTOR INCLUDING FOUNDATION EFFECTS

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ABSTRACT

In this paper the dynamic characteristics of a vertical single-mass rotor are examined using noncontacting displacement probes to measure the rotor center motion and accelerometers to measure the bearing housing motion. Resonant frequencies of the rotor and foundation predicted by impact testing correlated well with those observed during actual rotor runs. Foundation resonances found to occur in the operating speed range could not be eliminated by balancing the rotor at its critical speed. The rotor was balanced a second time using a different procedure which significantly reduced the foundation motion but left the rotor highly unbalanced at its critical speed. Coupling unbalance was found to be the second source of system excitation thus indicating the need for a two-plane balancing procedure to reduce both rotor motion and foundation motion. Acceleration measurements on the bearing housing were required in addition to shaft displacement measurements to detect the coupling effects.

NOMENCLATURE

C = Rotor damping, N-s/cm

C_c = Critical damping, $2\sqrt{K \cdot M}$, N-s/cm

$F(\omega)$ = Complex force, Fx + jFy

$F^*(\omega)$ = Complex conjugate of $F(\omega)$, Fx - jFy

f = Frequency ratio, ω/ω_{cr}

$H(\omega)$ = Complex transfer function, $Hx(\omega) + jHy(\omega)$

K = Stiffness, N/cm

M = Rotor modal mass, KG

t = Time, s

U = Rotor unbalance, gm-cm

ω = Rotational frequency, 1/s

ω_{CR} = Undamped natural frequency $\sqrt{K \cdot M}$, 1/s

X = Rotor displacement, cm

$X(\omega)$ = Complex rotor displacement, Xx + jXy

ϕu = Angular position of unbalance

ξ = Damping ratio, C/C_c

INTRODUCTION

This paper presents a series of experiments performed on a simple Jeffcott test rotor mounted on a massive vertical foundation. The Jeffcott rotor is a single degree of freedom rotor having only one critical speed in the operating range. Under certain circumstances, complex multistage compressors may be reduced to simple Jeffcott models by modal techniques. Gunter studied the dynamic stability of a single-mass rotor (1). Kirk and Gunter investigated analytically the effects of foundation flexibility and damping on the synchronous response of a single-mass rotor (2).

The rotor foundation tested in this study was designed to be rigid and massive so that the rotor would behave as a simple single-mass rotor, rather than as a rotor-foundation system. The foundation was made very stiff with the intention of having all foundation resonances occur above the rotor operating speed range. Impact tests and synchronous acceleration measurements were performed on the foundation to determine how effective the design is in minimizing foundation effects in the rotor system.

Impact tests were performed on the rotor and foundation to determine the accuracy of impact testing for predicting rotor system natural frequencies. Structural frequency response testing, or modal analysis, is rapidly becoming an integral part of the development process in many industrial applications. The simplest and fastest form of structural frequency response testing is the impact test which involves striking a structure with one of various types of impact hammers while monitoring response at some specific location on the structure. A complete discussion of the theory and application of frequency response

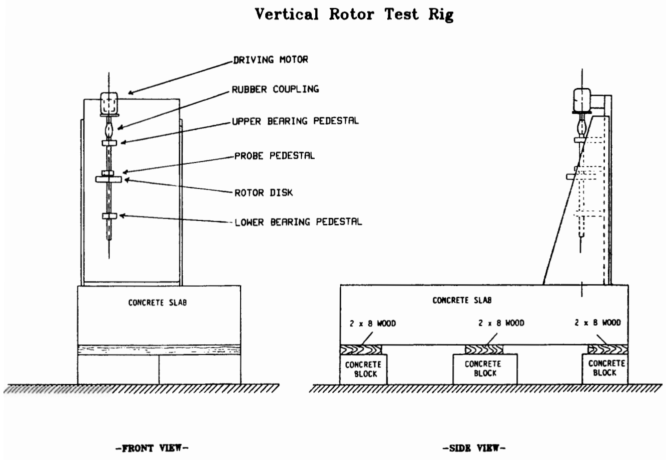

Fig. 1. Single-Mass Rotor and Foundation

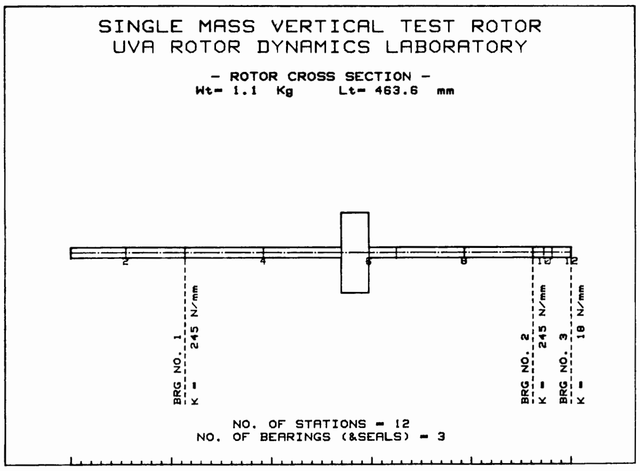

Fig. 2. Rotor Cross Section

testing is given by Halvorsen and Brown (3). Some controversy has developed concerning the practicality of impact testing applied to large rotor-foundation systems. This paper contains several experiments that were conducted to investigate the accuracy and validity of impact testing performed on the simple Jeffcott rotor mounted on a heavy foundation. The results obtained from this simple test rotor are more straightforward than one would expect from a complex steam generator. The experiments do, however, give valuable insight into the power of impact testing for determination of system natural frequencies. Impact tests were performed on the rotor and foundation separately to determine their individual frequency characteristics. Then the dynamic responses of both the rotor and the foundation were recorded during an actual run-up of the rotor from 0 to 10,000 RPM for comparison to the impact tests.

Also included in the investigation is a comparison between shaft motion relative to the foundation and foundation acceleration levels. Analysts traditionally have monitored only shaft motion and have given little attention to acceleration levels in rotor foundations. There are many real-world applications such as shipboard machinery, oil platforms, and power generation plants where minimization of foundation vibrations is essential. In such applications it is necessary to monitor foundation motion as well as rotor shaft motion. Synchronous shaft displacement relative to the foundation is compared to synchronous foundation acceleration to illustrate qualitatively how various balancing procedures affect the rotor-foundation system. It will be shown that a well-balanced rotor disk does not guarantee low acceleration levels in the foundation. High foundation response obtained at the design rotor speed could not be successfully balanced by single plane balancing without re-exciting the rotor fundamental critical speed. An unbalanced coupling will be shown to be a major source of foundation excitation.

ROTOR INSTRUMENTATION AND CRITICAL SPEED ANALYSIS

Figure 1 is a schematic of front and side views of the vertically mounted single-mass rotor and foundation system used for experimentation. The rotor is housed in an aluminum framework which is bolted to a 0.76 m x .3 m x 2.1 m concrete slab. Rotor dimensions are given below.

shaft length	=	18 in. (457.2 mm)
shaft diameter	=	.375 in. (9.525 mm)
bearing span	=	12 in. (304.8 mm)
disk diameter	=	3 in. (76.2 mm)
disk width	=	1 in. (25.4 mm)
disk weight	=	2 lb. (.907 kg)

A schematic diagram of the rotor and bearing locations is shown in Figure 2. The total weight of the shaft and disk is 1.1 kg. Noncontacting Bently displacement probes monitor the rotor motion at the overhang at station 1 and near the disc center at station 6. Accelerometers are mounted in the X and Y direction on the top bearing pedestal at station 9. The X and Y directions are parallel and perpendicular, respectively, to the vertical base plate.

Figure 3 represents the predicted critical speeds and mode shapes of the vertical test rotor. A 12 station computer model was developed to calculate mode shapes by the matrix transfer method. Below 10,000 RPM the rotor will have only one critical speed which is predicted to be 3,403 RPM (56.7 Hz). The second and third critical speeds are predicted to be 11,282 RPM and 21,881 RPM, respectively, and are therefore outside the operating range of 0 to 10,000 RPM. It is also of interest to note that station 6 is a node point for the second and third modes. Hence they should not be excited by an impact.

Figure 4 represents the animated first mode shape. Below 10,000 RPM, the shaft rotor center should behave approximately as a Jeffcott rotor with a modal mass of approximately 1 kg and a natural frequency of 57 Hz. The simple Jeffcott rotor may be expressed as:

$$M\ddot{X} + C\dot{X} + KX = \omega^2 U\cos(\omega t + \phi_u) \qquad (1)$$

where the unbalance U is assumed to be acting at the disc location.

A flexible steel-neoprene coupling, 35 mm by 15 mm diameter and weighing 28 grams, connects the top end of the shaft to a series-wound motor capable of ramping speed from 0 to 10,000 RPM with constant acceleration.

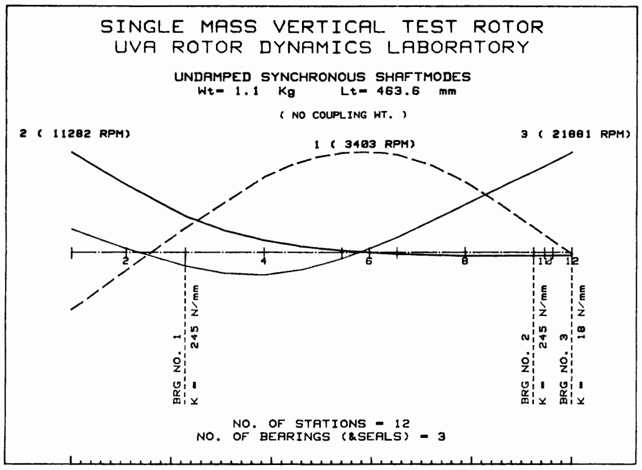

Fig. 3. Undamped Rotor Modeshapes

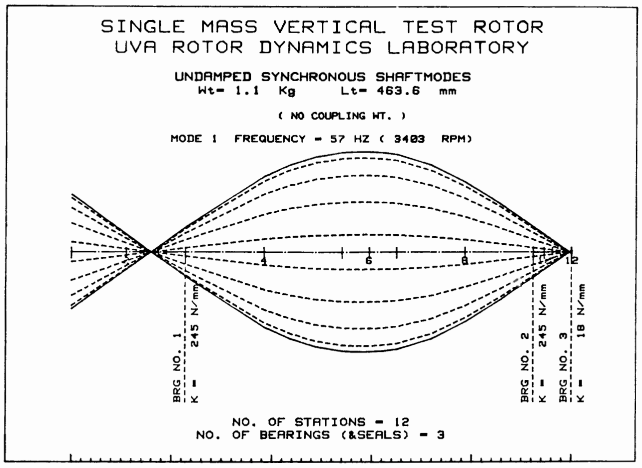

Fig. 4. Animated First Modeshape

Transfer function analyses were performed with the aid of a Hewlett Packard Model 5240A FFT Signal Analyzer and a PCB Piezotronics Impact Testing Kit. Further details concerning the impact hammer are given in Section III.

A Hewlett Packard Model 9845B Desktop Computer was used in conjunction with a Bently Nevada Synchronous Tracking Filter to facilitate data acquisition during rotor runs. The software used was a rotor system diagnostic package capable of sampling and plotting digitized data.

IMPACT TESTING FOR DETERMINATION OF NATURAL FREQUENCIES

The elegance of impact testing lies in the fact that an impact excites a wide range of frequencies at once, as opposed to slower methods which involve sweeping an excitation through a range of frequencies. One can simply strike a structure once and perform an FFT analysis of the response to determine natural frequencies. Applications in the field of rotor dynamics include determination of rotor critical speeds, measurement of foundation flexibility and analysis of damping characteristics.

A popular method of processing impact data is the transfer function. This commonly involves excitation of a structure with an impact while monitoring response with an accelerometer, velocimeter, or displacement probe. When a transfer function is performed, the output signal is divided by the input. A transfer function, therefore, is the ratio of the reponse to the excitation over a selected frequency range. System damping may be estimated by various procedures using transfer function and phase data.

The general complex transfer function is defined as the ratio of the cross power spectrum to the auto power spectrum and is expressed as

$$H(\omega) = \frac{X(\omega)\cdot F^*(\omega)}{F(\omega)\cdot F^*(\omega)} = \frac{\text{Cross Power Spectrum}}{\text{Auto Power Spectrum}} \tag{2}$$

For a multi-degree of freedom system the complex responses are the product of the complex influence coefficient matrix and the complex forcing function. This can be expressed in matrix form as

$$\begin{Bmatrix} X_1(\omega) \\ X_2(\omega) \\ X_3(\omega) \\ \cdot \\ \cdot \\ X_n(\omega) \end{Bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) & \cdots & H_{1n} \\ H_{21}(\omega) & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ H_{n1}(\omega) & \cdots & \cdots & H_{nn}(\omega) \end{bmatrix} \begin{Bmatrix} F_1(\omega) \\ F_2(\omega) \\ \cdot \\ \cdot \\ \cdot \\ F_n(w) \end{Bmatrix} \tag{3}$$

For the ideal single degree of freedom rotor the transfer function is given by

$$H(\omega) = \frac{1}{K[1-f^2+2j\xi f]} = \frac{\text{OUTPUT (DISPLACEMENT)}}{\text{INPUT (FORCE)}} \tag{4}$$

The excitation used in this study was an impact which has relatively "flat" magnitude over the range of frequencies involved. An impact excitation, therefore, is both a rapid input and one in which the magnitude is essentially constant with frequency. A PCB Piezotronics Impact Hammer was used for the impact testing. The hammer consists of a weighted head to which a force transducer is attached. To the force transducer is fixed a nylon striker which can be changed to either rubber or steel to allow for variable impact characteristics. The usable range of the force transducer is from 0 to 5,000 lb_f, with a calibration factor of .98 mv/lb_f. An amplitude vs. frequency plot of an impact performed with this particular hammer showed that the excitation is relatively constant over the range of frequencies in question.

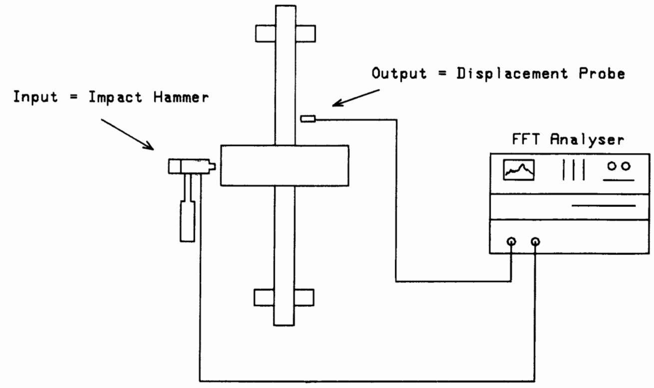

Fig. 5. Schematic of Single-Mass Rotor Transfer Function

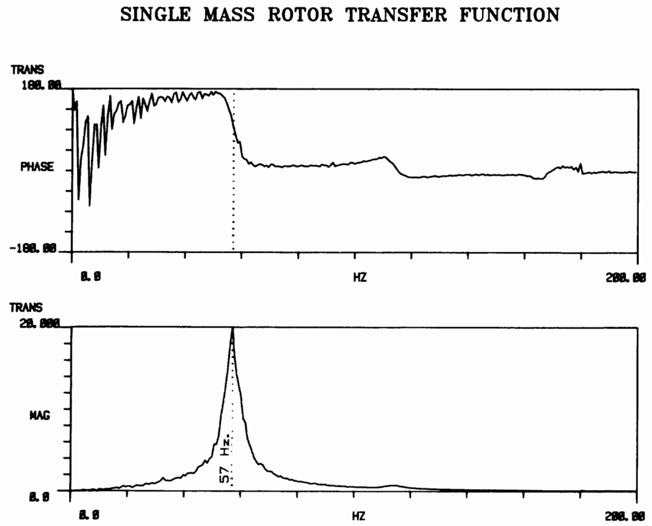

Fig. 6. Single-Mass Rotor Transfer Function (Input = Impact to Shaft, Output = Shaft Displacement)

DETERMINATION OF JEFFCOTT ROTOR CRITICAL SPEEDS

The static (nonrotating) natural frequencies of the vertical test rotor were experimentally determined with the use of an FFT analyzer by performing a transfer function. The input excitation is an impact to the rotor, and the response is shaft movement monitored by displacement probes mounted on the base plate. A schematic of such a test is shown in Figure 5. Results of an impact test performed on the single-mass test rotor are shown in Figure 6. A frequency range of 0 to 200 Hz was selected because this roughly coincides with the rotor's operating range of 0 to 10,000 RPM (0 to 166 Hz). The static natural frequency is found from Figure 6 to be 57 Hz (3,420 RPM). The existence of a natural frequency is further confirmed by the presence of a -180 degree phase change at 57 Hz. There also appears to be an indication of a resonance near 115-117 Hz (6,900 RPM) which could be from the foundation. Static damping of the rotor-bearing system may also be determined with the FFT analyzer. The rotor first mode, in this case, has approximately 2-3% modal damping.

In an effort to validate the impact test, the synchronous response of the single-mass rotor with center plane unbalance was observed during a run-up from 0 to 10,000 RPM. Figure 7 shows a plot of the relative horizontal shaft motion in mils vs rotor speed. The dynamic critical speed occurs at 56 Hz (3,350 RPM), which is in close agreement with the impact test prediction of 57 Hz. Some difference between static and dynamic resonant frequency is expected, since bearing characteristics and rotor gyroscopics are functions of speed. These differences would be much more pronounced in a rigid machine supported in fluid film bearings where a higher percentage of the total energy is absorbed in the bearings. It appears, however, that the static test has accurately located the first critical speed of this simple flexible rotor.

The synchronous rotor response shown in Figure 7 corresponds closely to the theoretical Jeffcott rotor with shaft bow. Note that the relative horizontal displacement probe does not indicate the presence of any structural resonance frequencies in the system.

Figure 8 represents the analysis of an extended Jeffcott rotor on a foundation with a mass ratio of 40 times the rotor weight. The foundation resonance

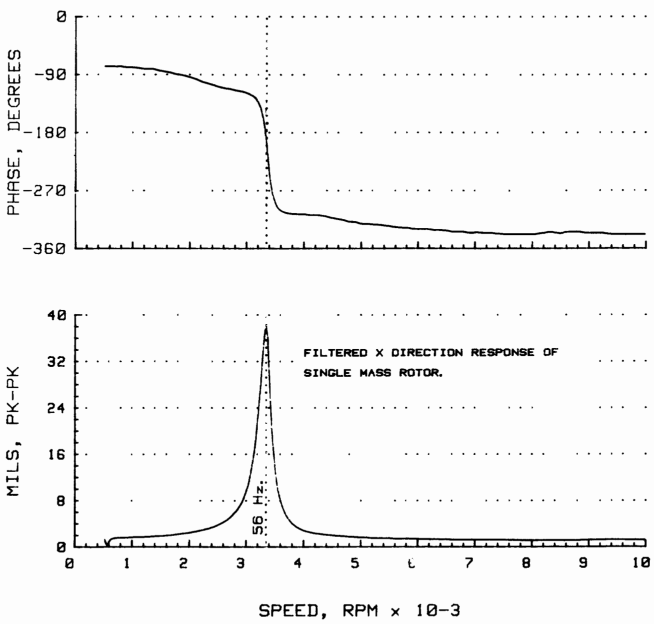

Fig. 7. Synchronous Shaft Motion of Single-Mass Rotor

frequency is around 8,200 RPM and the foundation critical damping was adjusted to 2.5% (A_1 = 20). Only a small perturbation in the phase angle is observed in the relative shaft to foundation response. This phase and amplitude response is similar to the experimental response curves obtained from the static impact tests. This indicates that relative displacement readings may not necessarily be sensitive to the existence of foundation or housing modes in a turbomachine.

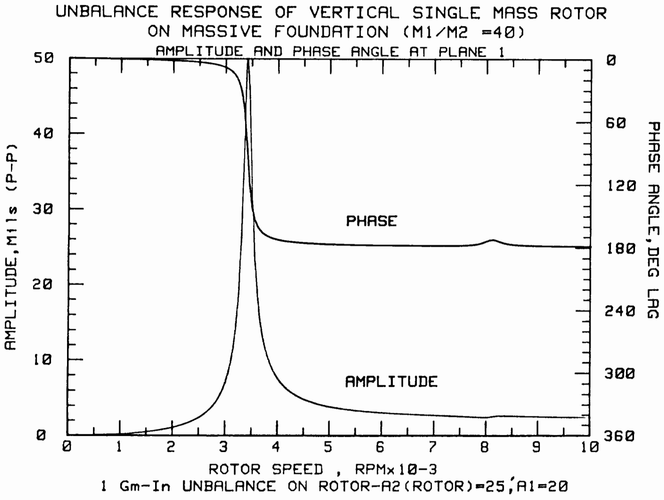

Fig. 8. Computer Model of Single-Mass Rotor Mounted on Heavy Foundation

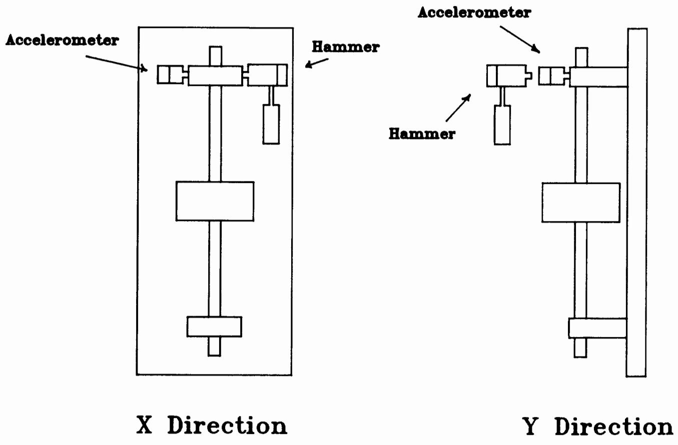

Fig. 9. Schematic of X and Y Direction Foundation Transfer Functions

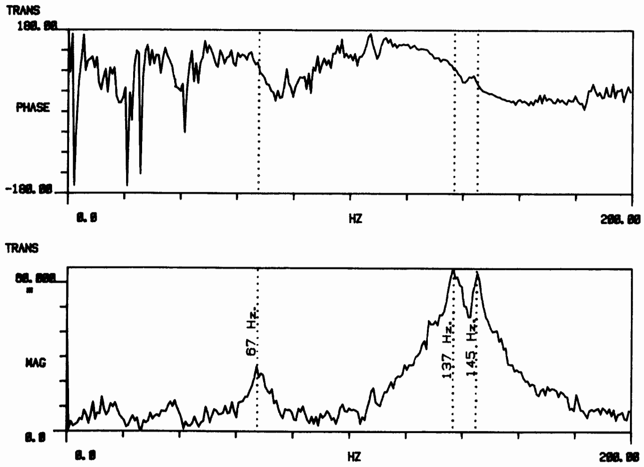

Fig. 10. X Direction Transfer Function on Foundation (Input = Impact Force; Output = Acceleration)

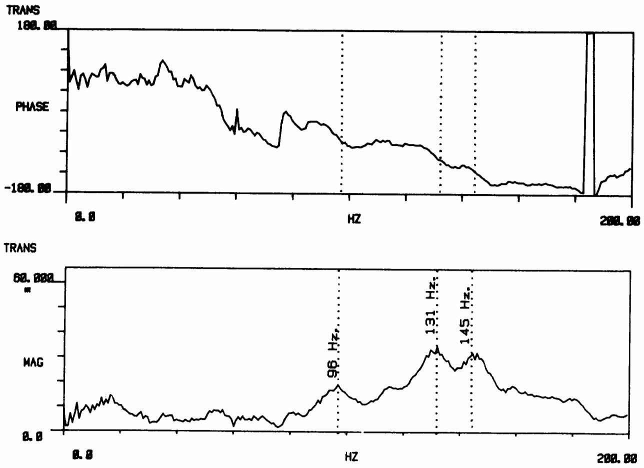

Fig. 11. Y Direction Transfer Function on Foundation (Input = Impact Force; Output = Acceleration)

DETERMINATION OF ROTOR FOUNDATION RESONANCES

Another useful application of impact testing is the determination of rotor foundation resonances. From the impact test performed on the rotor center, and from observation of the relative rotor displacement, there was a slight indication of foundation resonance frequencies in the operating speed range. Foundation characteristics can have adverse effects on rotor performance and on surrounding equipment when foundation resonances occur in the operating speed range.

Figure 9 shows a schematic of the instrumentation used to perform the X and Y direction impact tests on the foundation of the single-mass rotor. The first test was performed in the X direction (parallel to the vertical base plate) and the response was measured with an accelerometer attached directly to the bearing pedestal. One might expect considerably different characteristics in the X and Y directions because of foundation asymmetry.

Results of the X direction impact test are shown in Figure 10. A very prominent double peak and the associated phase change occur at 137 Hz and 145 Hz. Also appearing in Figure 10 is a less prominent resonance at 67 Hz. In both cases the phase changes are approximately 180 degrees, indicating that these are indeed natural frequencies. The rotor fundamental critical speed was not detected during this impact test.

Figure 11 shows the results of the Y direction impact test. In this case the bearing pedestal response is lower in magnitude and contains a peak near 131 Hz that was not seen in the X direction. The same peak seen in the X direction at 145 Hz is present, but lower in magnitude than the peak at 131 Hz. As in the X direction, there is also a smaller, lower-frequency resonance that appears at 96 Hz. Again, the rotor fundamental critical speed was not detected in the impact test.

To verify the impact tests, plots of X and Y direction synchronous foundation acceleration vs RPM were made during a rotor run-up from 0 to 10,000 RPM. In this test the foundation is being dynamically excited by rotor unbalance vibration, instead of an impact. Response is again measured with X and Y direction accelerometers. Phase is measured relative to shaft rotation. Figure 12 shows X direction foundation response vs. rotor RPM. Appearing on this plot are actually several natural frequencies, one associated with the rotor critical at 56 Hz (3,375 RPM), and larger ones from the foundation at 66 Hz, 137 Hz and 144 Hz (3,975, 8,250 and 8,650 RPM). The maximum rotor vibration occurs at the first critical speed at 56 Hz, and is observed on the foundation even though the foundation has no resonance there. At the higher rotor speeds of 3,975 RPM (66 Hz), 8,250 RPM (137 Hz) and 8,650 RPM (144 Hz), the response is larger as foundation resonances are excited by relatively low rotor motion. These results are in very good agreement with impact test results which predicted X direction foundation resonances at 67 Hz, 137 Hz and 145 Hz.

Figure 13 shows a similar plot for the Y direction where the first peak is again caused by high rotor vibration corresponding to the rotor first critical speed. The higher frequency peaks are foundation resonances. This time there is a much greater foundation response to the rotor critical at 56 Hz than was seen in the X direction. Referring to Figure 10 and Figure 11, we see that this is not predicted by the transfer functions which show that near 56 Hz there is almost the same flexibility in the X and Y directions. The foundation responses near 100 Hz, 133 Hz and 144 Hz are consistent with the impact tests which predicted Y direction foundation resonances at 96 Hz, 131 Hz and 145 Hz. The same predominant Y direction peak at 133 Hz that was visible in the transfer function has appeared again during the run-up.

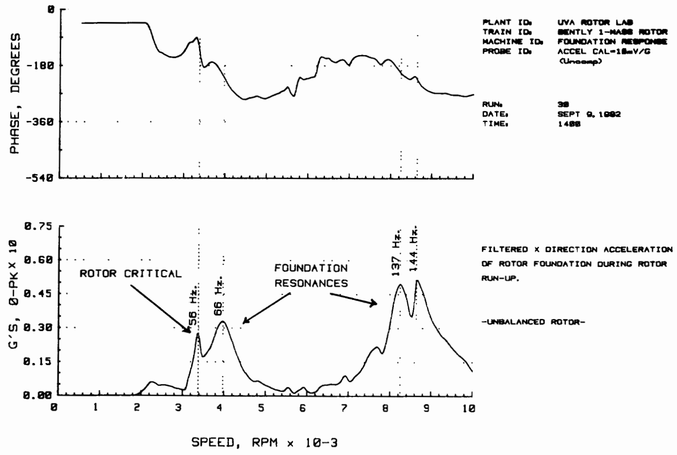

Fig. 12. X Direction Synchronous Acceleration Levels in Foundation During Rotor Run

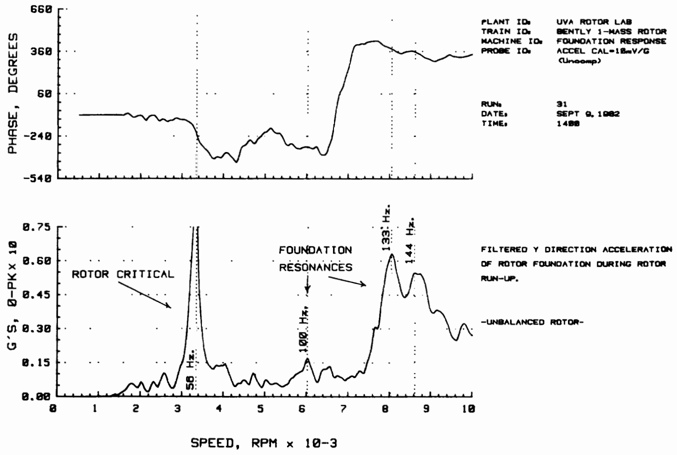

Fig. 13. Y Direction Synchronous Acceleration Levels in Foundation During Rotor Run

COMPARISON OF RELATIVE ROTOR DISPLACEMENT RESPONSE TO FOUNDATION ACCELERATION

Comparison of shaft relative displacement with foundation acceleration during the same run reveals significant differences in the frequencies excited as shown in Figure 14. Hereafter all data will be given for the X direction only, because this will sufficiently show the effects of various balancing techniques. Rotor amplitude measured with displacement probes is shown in the lower half of each figure. Notice the absence of any significant peaks in the rotor response plots other than the fundamental rotor critical speed. The shaft-foundation relative displacement measurements do not detect the presence of foundation resonance frequencies. The reason for this is that the displacement probes mounted on the foundation are calibrated in a range appropriate for shaft displacement which is much higher than foundation displacement. It is apparent from Figure 14 that the accelerometer mounted on the bearing housing detects the rotor critical speed as well as the various foundation resonances. Again, it is clear that high rotor amplitude is exciting the foundation at the critical speed and at higher speeds the foundation resonances are excited by relatively low rotor motion.

Figure 15 shows results of single-plane balancing the single-mass rotor at its critical speed (4,5). The rotor center plane response is a combination of disc unbalance response, shaft bow, and mechanical runout. The shaft response was minimized by balancing the compensated amplitude (low speed motion subtracted) at the rotor critical speed. An exceptionally well balanced rotor was obtained as can be seen from the lower trace of Figure 15. Although both rotor motion and foundation motion have been reduced at the critical speed, there remains a substantial amount of foundation motion at higher frequencies.

It is apparent that for this particular rotor, balancing at the rotor critical speed is not effective in eliminating the source of excitation for the higher frequency foundation resonances. Since the balanced rotor causes only a minimal excitation to the foundation at the higher frequencies, there must be an additional source of excitation for the high frequency foundation acceleration observed in Figure 15. This might be a reasonable balance if one were concerned primarily with minimizing critical speed response and could tolerate the high-speed foundation motion. However, if high-speed foundation response is also to be minimized, the additional source of unbalance must be identified and eliminated.

A different type of single-plane rotor balance based on foundation acceleration measurements is shown in Figure 16, where the rotor has now been balanced to reduce the high-speed foundation motion at 8,000 RPM. However, this procedure now leaves the rotor highly unbalanced at the critical speed. It is apparent from the experimental data that a single-plane balance at the rotor mid-span can eliminate either rotor motion at

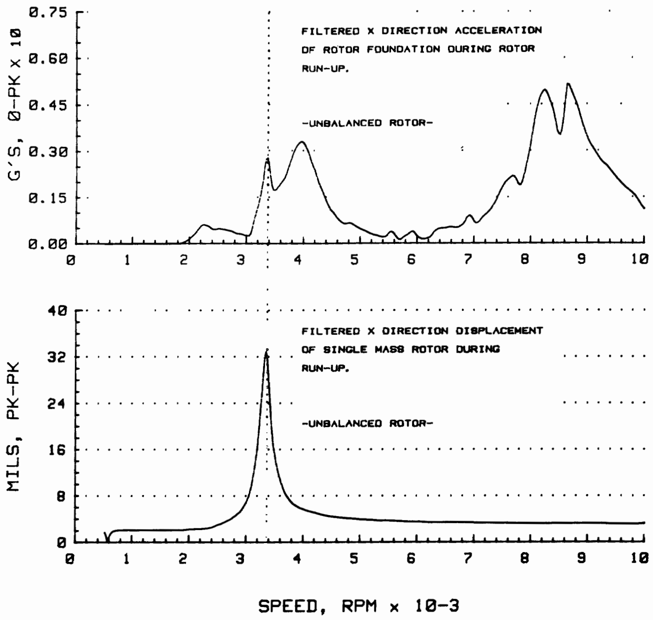

Fig. 14. Comparison of X Direction Rotor Motion and X Direction Foundation Acceleration During Run-Up (Rotor Unbalanced)

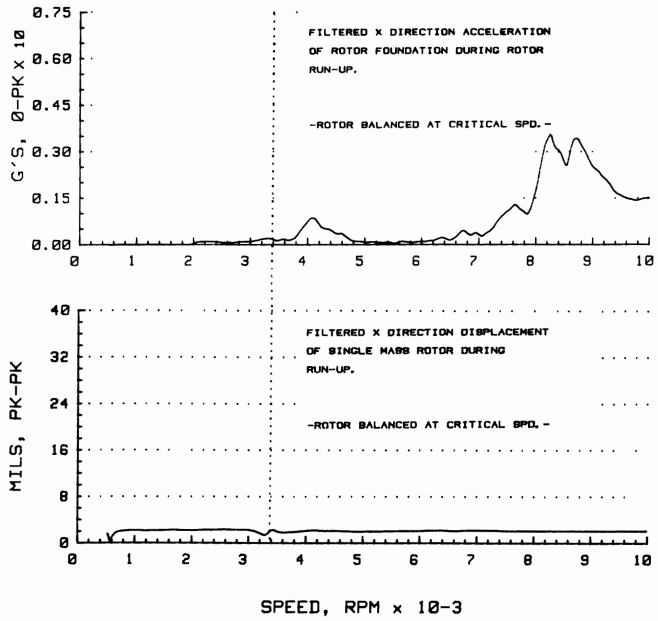

Fig. 15. Comparison of X Direction Rotor Motion and X Direction Foundation Acceleration During Run-Up (Rotor Balanced at Critical Speed)

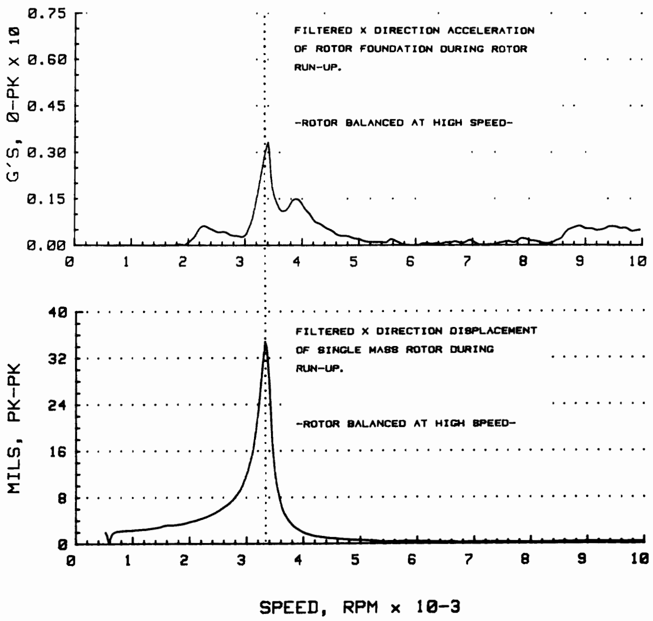

Fig. 16. Comparison of X Direction Rotor Motion and X Direction Foundation Acceleration During Run-Up (Rotor Balanced at High Speed)

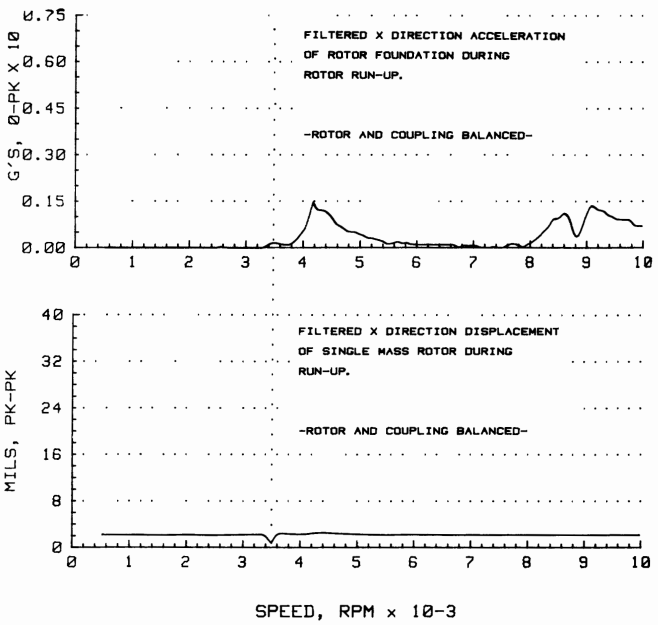

Fig. 17. Comparison of X Direction Rotor Motion and X Direction Foundation Acceleration During Run-Up (Rotor and Coupling Balanced)

the critical speed, or foundation motion at high speed, but not both. Again, the presence of other unbalanced components in the system is indicated. To completely balance this rotor will require a two-plane or multi-plane balancing procedure which will eliminate all system unbalances resulting in reduced vibration levels at all speeds.

In this rotor system the only other source of unbalance is the coupling. This was proven by detaching the rotor and coupling from the drive motor and running the motor alone while monitoring foundation acceleration. Very little foundation response was observed, indicating that the motor was not a source of unbalance excitation. The only remaining source of unbalance excitation is the coupling.

Figure 17 shows the results of balancing both the rotor and the coupling. This was done by first balancing the rotor at the critical speed using displacement probe measurements and then balancing the coupling at high speed using foundation accelerometer measurements. The accelerometer measurements were found to be the most sensitive for balancing the system at high-speed.

Details on the balancing procedures and a mathematical analysis of the rotor and foundation will be presented in a later paper.

SUMMARY AND CONCLUSIONS

In this paper the experimental characteristics of a vertical Jeffcott rotor mounted on a heavy foundation were examined. Noncontacting position probes were placed on the supporting base plate to monitor the relative shaft center motion, and accelerometers were placed on the bearing pedestals to measure the foundation acceleration. Relative shaft displacement monitoring probes were found to be insensitive to the higher frequency foundation motion. The accelerometers mounted on the bearing pedestal detected the foundation resonances as well as the fundamental rotor critical speed.

The original intent of the foundation design was to minimize foundation motion. Experimental tests indicated, however, that foundation resonance frequencies were in the operating speed range and that these modes could be excited by rotor unbalance, shaft bow, or coupling unbalance.

It was shown through experimentation that the popular impact testing method is a valuable tool for predicting rotor system resonant frequencies. An impact hammer was used in conjunction with an FFT analyzer to predict accurately the location of the first critical speed of a single-mass rotor. The impact test prediction was 57 Hz and the actual critical speed was found to be 56 Hz (3,350 RPM). The rotor foundation was then analyzed in two directions using similar procedures, and again the impact tests gave accurate predictions of resonant frequencies. All major resonant peaks which appeared in the impact spectra were also seen during an actual run in which the foundation was excited by the rotor.

Comparisons made between rotor motion and foundation acceleration during the same run for an unbalanced rotor and coupling showed high rotor motion at the rotor critical speed, and high foundation acceleration at both the rotor critical, and at high speed where foundation resonances occurred. Two types of single-plane balancing procedures were performed on the rotor and results showed that neither was effective in eliminating both low-speed motion at the rotor critical and high-speed foundation response. In this case, a two-plane balancing procedure was required to eliminate both coupling unbalance and rotor unbalance. This resulted in a well-behaved system at all speeds.

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