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The Influence of Internal Friction on the Stability of High Speed Rotors

1 Introduction

With the increase in performance requirements of high speed rotating machinery in various fields such as process equipment, auxiliary power machinery, and space applications, the engineer is faced with the problem of designing a unit capable of smooth operation under various conditions of speed and load. As an example, turborotors in the auxiliary power systems of space applications must be designed to perform satisfactorily under adverse load conditions ranging from the high acceleration forces encountered at takeoff to the zero gravity load condition encountered in orbit. In many of these applications the design operating range may be well above the rotor first critical speed. Under these circumstances the problem of insuring that a turbomachine will perform with a stable, low level amplitude of vibration is extremely difficult.

Under certain conditions of high speed and light loading, a situation can arise in which the rotor system is capable of orbiting or precessing in its bearings at a rate less than the total rotor angular speed. This nonsynchronous precessive motion, which has often been referred to in the literature as whirling or whipping, can lead to destruction of the rotor if the whirl threshold speed is exceeded. Nonsynchronous rotor precession is a self-induced vibration and has sometimes been described as "sustained transient motion." In general, a self-excited or self-induced vibration is defined as a phenomenon in which the excitation forces inducing the vibration are controlled by the motion. This is in contrast to a forced vibration in which the external excitation is a function of time only.

With a self-excited whirl instability, unbalance is of minor importance. At the onset of whirl, the rotor behavior is unlike a critical speed resonance where the amplitude of motion builds up to a maximum value and then decreases. At the inception of nonsynchronous whirling, the rotor motion will continually build up with speed since the self-excitation increases the energy transfer into the system with increased speed. If the rotor speed is increased appreciably above the whirl threshold speed, the large orbiting obtained will usually result in rotor or bearing failure.

This paper deals with the influence of internal rotor friction on stability. The equations of motion for the single-mass flexible rotor are developed and analyzed to determine the stability threshold. An analog computer was used to simulate the rotor motion, and comparison is made with experimental data.

The investigation shows that large gains in stability can be achieved by incorporating bearing support flexibility and damping into the system.

2 Background

At the turn of the century, the design philosophy applied to rotating equipment was to construct rotors sufficiently stiff to insure operation below the first natural critical speed. In 1919, H. H. Jeffcott [1],[1] a noted English dynamicist, was asked to investigate the effect of rotor unbalance on the whirl amplitudes and forces transmitted to the bearings. Jeffcott analyzed the motion of a single-mass flexible rotor on fixed bearing supports (see Fig. 1). Jeffcott's splendid analysis of a relatively simple model revealed to rotating machinery designers the possibility of operation above the first critical speed providing good rotor balance is acquired.

The 1920's saw a trend reversing the rotor design concepts of the previous decade. Turbine and particularly compressor and pump manufacturers were beginning to construct higher speed, lighter weight rotors designed for operation well above the first critical speed. As more manufacturers adopted this "flexible" rotor design, several encountered severe difficulties when operating above the first critical speed. These problems were at first attributed to the lack of proper rotor balance. At this time, a major United States manufacturer encountered a series of failures of pumps designed to operate above the first critical speed. Dr. B. L. Newkirk set up a series of experiments with several units to observe the rotor dynamic behavior. It was observed that at speeds above the first critical speed these units would enter into a violent whirling in which the rotor center line precessed at a rate equal to the first critical speed. If the unit rotational speed were

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[1] Numbers in brackets designate References at end of paper.

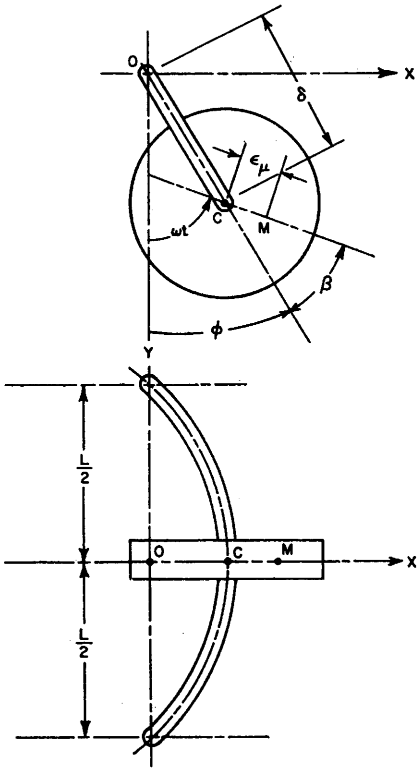

Fig. 1 Single-mass flexible rotor

increased above its initial whirl speed, the whirl amplitude would increase, leading to eventual rotor failure. To further investigate all aspects and contributing factors to this problem, an experimental test rotor was constructed to simulate a typical pump unit. Extensive testing of this unit revealed the following important facts concerning this phenomenon [2]:

1 The onset speed of whirling or the whirl amplitude was unaffected by refinement in rotor balance.

2 A well-balanced rotor sometimes required an external disturbance to initiate whirl.

3 Whirling always occurred above the first critical speed.

4 The whirl threshold speed could vary widely between machines of similar construction.

5 The precession (or whirl) speed was constant regardless of the unit rotational speed.

6 Whirling was encountered only with built-up rotors.

7 Increasing the foundation flexibility would increase the whirl threshold speed.

8 Introducing damping into the foundation would increase the whirl threshold speed.

9 Increasing the axial thrust bearing load would increase the whirl threshold speed.

It became clear to Dr. Newkirk that the rotor dynamic behavior could not be attributed to a critical speed resonance since the high vibrations encountered always occurred above the first critical speed and refinement of balance had no effect upon diminishing the whirl amplitudes. There was nothing in the literature at that time to indicate that any mode of motion, other than synchronous whirl, was possible. During the course of the investigation, a theory of the cause of the vibration was postulated by A. L. Kimball [3]. Kimball suggested that forces normal to the plane of the deflected rotor could be produced by the hysteresis of the metal undergoing alternate stress reversal cycles. Newkirk concluded that these out-of-phase forces could also be developed

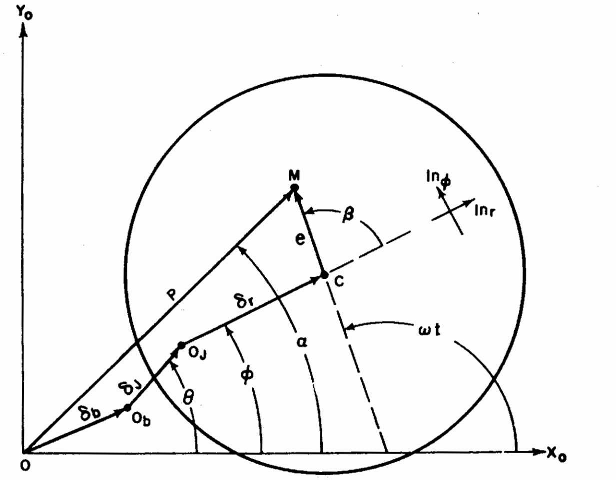

Fig. 2 Vectorial representation of a cross section of a deflected rotor

by a disk shrunk on the shaft. Upon re-examination of Jeffcott's model and by introducing this additional force normal to the deflected rotor, he was able to demonstrate that the rotor was indeed unstable above the first critical speed, and thus was partially able to explain some of his experimental findings. Since Newkirk made no attempt to extend Jeffcott's model by considering a flexible foundation with damping, he was unable to explain theoretically several of the key points of his experimental investigations, particularly as to why increased bearing or foundation flexibility and damping will improve rotor whirl stability.

This phenomenon became known as whirling above the first critical speed, or shaft whirling, and its nature is completely different from the vibrations encountered when operating in the vicinity of a critical speed.

3 Discussion

In general, a self-excited or self-induced vibration is defined as a phenomenon in which the excitation forces inducing the vibration are controlled by the motion. This is in contrast to a forced vibration, in which the external excitation is a function of time only. As an example, the excitation forces usually considered with rotating machinery are alternating and impulse forces such as caused by unbalance and shock. These force systems are expressed as explicit functions of time and are unaltered by the mode of vibration of the system.

The exciting force for the case of shaft whirling, as described earlier, is provided by the frictional forces developed between two mating surfaces when undergoing deformation. This frictional force will henceforth be referred to as rotating or rotary damping and (for constant δ_r) can be expressed in the form

$$F = -C_2\delta_r(\dot{\phi} - \omega) \tag{1}$$

This force is developed only if the whirl speed $\dot{\phi}$ is different from the rotational speed ω (see Fig. 2). When the motion of the system is such that $\dot{\phi} > \omega$ (which occurs below the first critical speed), the whirl motion is damped out and the system is stable. When the precession rate $\dot{\phi}$ is smaller than the rotational speed ω, the rotary damping force becomes a source of excitation; that is, energy is added to the system causing the whirl amplitude to increase.

Fig. 2 represents a typical cross section of the idealized rotor taken at the nth mass station. From the examination of this schematic representation of a rotor section, it is possible to precisely define whirling of a system and also write the governing equations of motion of the mass section. The position vector $\mathbf{P}_n$ of the nth mass center is given by

$$\mathbf{P}_n = \boldsymbol{\delta}_b + \boldsymbol{\delta}_j + \boldsymbol{\delta}_r^{(n)} + \mathbf{e}^{(n)} \tag{2}$$

where

$\boldsymbol{\delta}_b$ = absolute displacement vector of the bearing

$\boldsymbol{\delta}_j$ = displacement vector of the journal relative to the bearing

$\boldsymbol{\delta}_r^{(n)}$ = displacement vector of the deflected rotor center line relative to the journal at the nth station

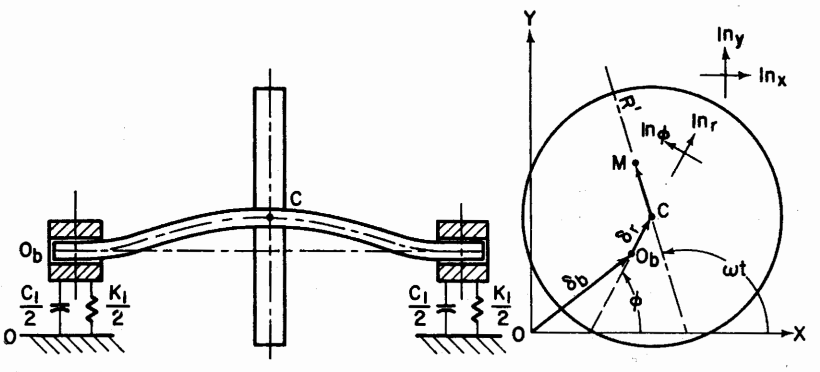

Fig. 3 Schematic diagram of a flexible rotor on an elastic foundation

$\mathbf{e}^{(n)}$ = position vector of the rotor mass center relative to the rotor volume center at the nth station.

Examination of equation (2) and Fig. 2 shows that seven degrees of freedom are required to completely specify the location of the nth mass station. In the subsequent analysis, the following assumptions will be employed:

1 $\boldsymbol{\delta}_j = 0$, which implies no displacement of the journal center relative to the bearing (no fluid film bearing).
2 $e = 0$, no rotor unbalance.
3 $g = 0$, no body forces acting on rotor.
4 $n = 1$, single-mass rotor.

These assumptions reduce the rotor problem to a single-mass dynamical system possessing four degrees of freedom. This configuration is shown in Fig. 3.

4 Equations of Motion

A. Deflection Analysis

The displacement of the journal center is given by the position vector (see Fig. 3)

$$^0\mathbf{P}^{0_b} = \boldsymbol{\delta}_b = X_1\mathbf{n}_x + Y_1\mathbf{n}_y \quad (3)$$

where X_1 and Y_1 are the Cartesian coordinates of point 0_b relative to point 0, and the displacement of the rotor volume center from the journal center is given by

$$^0\mathbf{P}^c = \boldsymbol{\delta}_r = X_2\mathbf{n}_x + Y_2\mathbf{n}_y \quad (4)$$

where X_2 and Y_2 are the Cartesian coordinates of point C relative to point 0_b, and $\mathbf{n}_x$, $\mathbf{n}_y$ represent a set of fixed unit vectors.

The total displacement of the rotor center is given by

$$^0\mathbf{P}^c = X\mathbf{n}_x + Y\mathbf{n}_y \quad (5)$$

where

$$X = X_1 + X_2 \quad (6a)$$

$$Y = Y_1 + Y_2 \quad (6b)$$

In general, the mass center of the rotor will not correspond to the rotor volume center. Only for the case of perfect balance will point M correspond to point C. In this case, $^0P^c = {}^0P^m$ and the equations of motion of the rotor will be given by

$$M\ddot{X} - F_x = 0; \quad M\ddot{Y} - F_Y = 0 \quad (7)$$

B. Bearing Forces

For the case of a symmetric bearing support, the elastic and damping characteristics are uniform in all transverse directions. The bearing or foundation at 0_b are assumed to be of the form

$$F_x = -C_1\dot{X}_1 - K_1X_1 \quad (8a)$$

$$F_y = -C_1\dot{Y}_1 - K_1Y_1 \quad (8b)$$

(linear elastic and damping forces).

C. Shaft Characteristics

The forces acting on the shaft are the elastic restoring forces and the damping forces. Of importance in the calculation of rotor stability is the inclusion of rotary damping on the shaft. This will be defined as the damping which resists a change of strain of the flexible members.

Consider a rotating reference from R' which is revolving with an angular velocity of ω. The rotor forces will be expressed in this system, since damping in the shaft is brought about by a change in configuration of the rotating shaft. The forces acting at C are given by

$$\mathbf{F}_c = -[C_2{}^{R'}\mathbf{V}^{c/0_b} + K_2\boldsymbol{\delta}_r] \quad (9)$$

where

$^{R'}\mathbf{V}^{c/0_b}$ = velocity of point C relative to 0_b in reference frame R'

or

$$^{R'}\mathbf{V}^{c/0_b} = {}^{R}\mathbf{V}^{c/0_b} - {}^{R}\boldsymbol{\omega}^{R'} \times \boldsymbol{\delta}_r = \dot{\phi}\delta_r\,\mathbf{n}_\phi + \dot{\delta}_r\mathbf{n}_r - \omega\delta_r\mathbf{n}_\phi \quad (10)$$

Hence

$$\mathbf{F}_c = -[(C_2\dot{\delta}_r + K_2\delta_r)\mathbf{n}_r + C_2\delta_r(\dot{\phi} - \omega)\mathbf{n}_\phi] \quad (11)$$

The equations of transformation between the $\mathbf{n}_r$, $\mathbf{n}_\phi$ and the fixed Cartesian unit vector set $\mathbf{n}_x$ and $\mathbf{n}_y$ are given by

$$\begin{pmatrix}\mathbf{n}_r\\ \mathbf{n}_\phi\end{pmatrix} = \begin{pmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}\mathbf{n}_x\\ \mathbf{n}_y\end{pmatrix} \quad (12)$$

By taking the dot product of equation (11) with $\mathbf{n}_x$ and $\mathbf{n}_y$, the horizontal and vertical components are obtained as

$$F_x = \mathbf{F}_c\cdot\mathbf{n}_x$$
$$= -[(C_2\dot{\delta}_r + K_2\delta_r)\cos\phi - C_2\delta_r(\dot{\phi} - \omega)\sin\phi] \quad (13a)$$

$$F_y = \mathbf{F}_c\cdot\mathbf{n}_y$$
$$= -[(C_2\dot{\delta}_r + K_2\delta_r)\sin\phi + C_2\delta_r(\dot{\phi} - \omega)\cos\phi] \quad (13b)$$

since

$$X_2 = \delta_r\cos\phi; \qquad Y_2 = \delta_r\sin\phi$$

and (14)

$$\dot{X}_2 = \dot{\delta}_r\cos\phi - \delta_r\dot{\phi}\sin\phi; \qquad \dot{Y}_2 = \dot{\delta}_r\sin\phi + \delta_r\dot{\phi}\cos\phi$$

Hence

$$F_x = -[C_2(\dot{X}_2 + \omega Y_2) + K_2X_2] \quad (15a)$$

$$F_y = -[C_2(\dot{Y}_2 - \omega X_2) + K_2Y_2] \quad (15b)$$

Combining equations (8a, b) and (15a, b) subject to the condition that

$$X = X_1 + X_2$$
$$Y = Y_1 + Y_2$$

$$F_x = -\frac{C_2K_1}{K_1 + K_2}(\dot{X}_2 + \omega Y_2) - \frac{K_2C_1\dot{X}_1}{K_1 + K_2} - \frac{K_1K_2}{K_1 + K_2}X \quad (16a)$$

$$F_y = -\frac{C_2K_1}{K_1 + K_2}(\dot{Y}_2 - \omega X_2) - \frac{K_2C_1\dot{Y}_1}{K_1 + K_2} - \frac{K_1K_2}{K_1 + K_2}Y \quad (16b)$$

If it is assumed that the damping forces are much smaller than the elastic forces, then equations (16a, b) become

$$F_x = -C_1\left(\frac{K_2}{K_1 + K_2}\right)^2\dot{X} - C_2\left(\frac{K_1}{K_1 + K_2}\right)^2 \times (\dot{X} + \omega Y) - \frac{K_1K_2}{K_1 + K_2}X \quad (17a)$$

$$F_y = -C_1\left(\frac{K_2}{K_1 + K_2}\right)^2\dot{Y} - C_2\left(\frac{K_1}{K_1 + K_2}\right)^2 \times (\dot{Y} - \omega X) - \frac{K_1K_2}{K_1 + K_2}Y \quad (17b)$$

The equations of motion of the system become

$$\ddot{X} + (n_1 + n_2)\dot{X} + \omega n_2Y + \omega_{cr}^2X = 0 \quad (18a)$$

$$\ddot{Y} + (n_1 + n_2)\dot{Y} - \omega n_2X + \omega_{cr}^2Y = 0 \quad (18b)$$

where

$n_1 = \dfrac{C_1}{M}\left(\dfrac{K_2}{K_1 + K_2}\right)^2$ = stationary damping coefficient of the bearing supports

$$n_2 = \frac{C_2}{M}\left(\frac{K_1}{K_1 + K_2}\right)^2 = \text{rotating damping coefficient of the shaft}$$

$$\omega_{cr}^2 = \frac{K_1 K_2}{M(K_1 + K_2)} = \text{critical speed squared for case of light damping}$$

Note that

$$\frac{K_1 K_2}{K_1 + K_2} = K_T \text{ effective system spring rate.}$$

5 Stability Analysis

Note that the rotary damping coefficient n_2 couples the two equations:

$$\ddot{X} + a_1\dot{X} + a_2 Y + a_3 X = 0 \qquad (19a)$$

$$\ddot{Y} + a_1\dot{Y} - a_2 X + a_3 Y = 0 \qquad (19b)$$

Now by eliminating the cross-coupling term in equations (19), one obtains a fourth-order equation in X:

$$\ddot{\ddot{X}} + 2a_1\dddot{X} + (2a_3 + a_1^2)\ddot{X} + 2a_1a_3\dot{X} + (a_2^2 + a_3^2)X = 0 \qquad (20)$$

Assume a solution of the form $X = Ae^{st}$. Then equation (20) becomes

$$S^4 + A_3S^3 + A_2S^2 + A_1S + A_0 = 0 \qquad (21)$$

where

$$A_3 = 2a_1 = 2(n_1 + n_2)$$
$$A_2 = 2a_3 + a_1^2 = 2\omega_{cr}^2(n_1 + n_2)^2$$
$$A_1 = 2a_1a_3 = 2\omega_{cr}^2(n_1 + n_2)$$
$$A_0 = a_2^2 + a_3^2 = (\omega n_2)^2 + \omega_{cr}^4$$

Rather than attempt at this point to solve for the roots of the frequency equation (21), we will instead examine the stability of the system by Routh's criterion.

Consider the general equation:

$$\sum_{n=0}^{N} A_n S^n = 0 \qquad (22)$$

For $N = 4$, Routh's criterion of stability is

$$A_1A_2A_3 > A_1^2 + A_0A_3^2 \qquad (23)$$

$$\left.\begin{aligned} &2(n_1 + N_2)\omega_{cr}^2[2\omega_{cr}^2 + (n_1 + n_2)^2]2(n_1 + n_2) > \\ &4(n_1 + n_2)^2\omega_{cr}^4 + 4[(\omega n_2)^2 + \omega_{cr}^4](n_1 + n_2)^2 \end{aligned}\right\} \qquad (24)$$

This reduces to

$$\omega_{cr}^2(n_1 + n_2)^2 > (\omega n_2)^2 \qquad (25)$$

or

$$\omega < \omega_{cr}\left[1 + \left(\frac{C_1}{C_2}\right)\left(\frac{K_2}{K_1}\right)^2\right] \quad \text{for stability} \qquad (26)$$

This is the stability criterion of a flexible rotor subjected to internal damping, C_2. The stability condition states that the rotor may become unstable or whirl at some speed above the first critical speed. The exact onset speed of whirling may vary, depending upon the ratio of the damping terms C_1 and C_2 and the spring rates K_1 and K_2. Thus, if the bearing damping term is zero or small in comparison to the rotary damping term, the threshold of instability is $\omega = \omega_{cr}$, the rotor critical speed. Conversely, if the rotary damping is small, or if the rotor stiffness K_2 is much higher than the bearing stiffness (rotor behaves as a "rigid body"), then the threshold of instability is $\omega \gg \omega_{cr}$, as shown by Fig. 4.

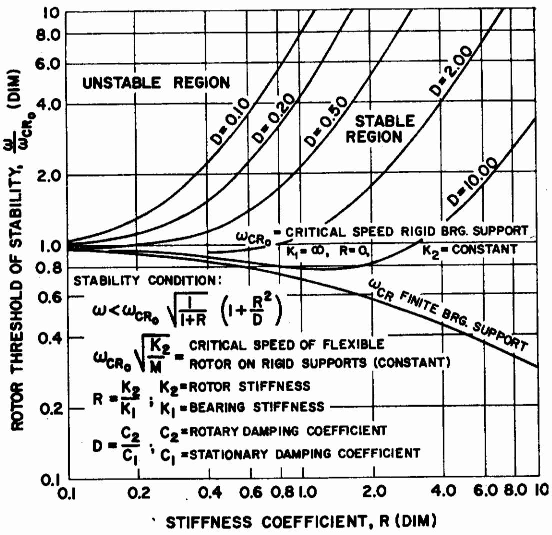

Fig. 4 Stability threshold of a flexible rotor with internal friction on a symmetric elastic bearing support

6 Rotor Precession Speed

Let $Z = X + iY$. Then equations (19a, b) may be written as

$$\ddot{Z} + a_1\dot{Z} + (a_3 - ia_2)Z = 0 \qquad (26a)$$

Assume a solution of the form

$$Z = ae^{\alpha t}$$

This results in the following frequency equation:

$$\alpha^2 + a_1\alpha + (a_3 - ia_2) = 0 \qquad (27)$$

In general, the value of α is complex:

$$\alpha = P + iS$$

Substitution of the above into (27) and separating real and imaginary parts results in

$$P^2 - S^2 + a_1P + a_3 = 0 \qquad (28a)$$

$$2PS + a_1S - a_2 = 0 \qquad (28b)$$

Eliminating S from (28a) and (28b), one obtains

$$4P^4 + 8a_1P^3 + (5a_1^2 + 4a_3)P^2 + (4a_1a_3 + a_1^3)P + a_1^2a_3 - a_2^2 = 0 \qquad (29)$$

From the assumed form of the solution, it can be seen that if $P > 0$, the system is unstable (displacements will grow exponentially with time); and if $P < 0$, the system is stable (motion is damped out). At the threshold of stability, we have the condition that $P = 0$, which implies that

$$a_1^2a_3 - a_2^2 = 0; \quad (m + n)^2\omega_{cr}^2 - (n\omega)^2 = 0$$

or

$$\omega = \omega_{cr}\left(1 + \frac{n_1}{n_2}\right) \qquad (30)$$

defines the threshold of stability. It was seen from Routh's criterion that, if $\omega > \omega_{cr}(1 + m/n)$, the system is unstable.

By eliminating P from (28a) and (28b), we obtain

$$S^4 + \left(\frac{a_1^2}{4} - a_3\right)S^2 - \frac{a_2^2}{4} = 0 \qquad (31)$$

Solving for S^2:

$$S^2 = \frac{-\left(\frac{a_1^2}{4}\right) - a_3 \pm \sqrt{\left(\frac{a_1^2}{4} - a_3\right)^2 + a_2^2}}{2}$$

or

$$S^2 = \frac{\omega_{cr}^2 - \left(\frac{n_1+n_2}{2}\right)^2 \pm \sqrt{\left(\left\{\frac{n_1+n_2}{2}\right\}^2 - \omega_{cr}^2\right)^2 + (\omega n_2)^2}}{2} \tag{32}$$

From equation (13) it was found that at the threshold of instability, $\omega = \omega_{cr}(1 + m/n)$. Introducing this condition into equation (32), we obtain

$$S_T^2 = \frac{\omega_{cr}^2 - \left(\frac{n_1+n_2}{2}\right)^2 \pm \sqrt{\left(\frac{n_1+n_2}{2}\right)^4 + \frac{1}{2}\omega_{cr}^2(n_1+n_2)^2 + \omega_{cr}^4}}{2}$$

$$= \frac{\omega_{cr}^2 - \left(\frac{n_1+n_2}{2}\right)^2 \pm \left[\omega_{cr}^2 + \frac{n_1+n_2}{2}\right]^2}{2} \tag{33}$$

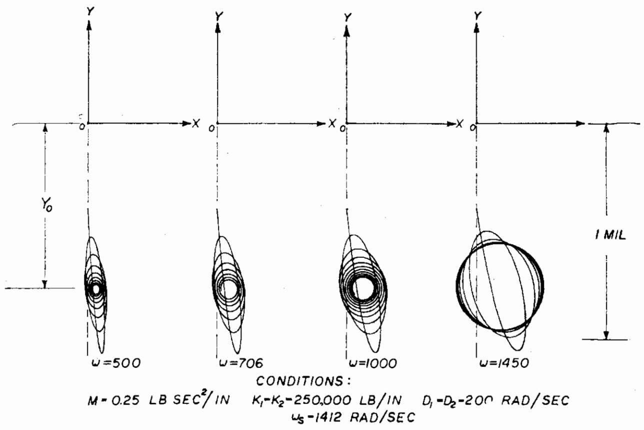

Fig. 5 Whirl orbits of a balanced horizontal rotor below the threshold of stability, $\omega < \omega_s$

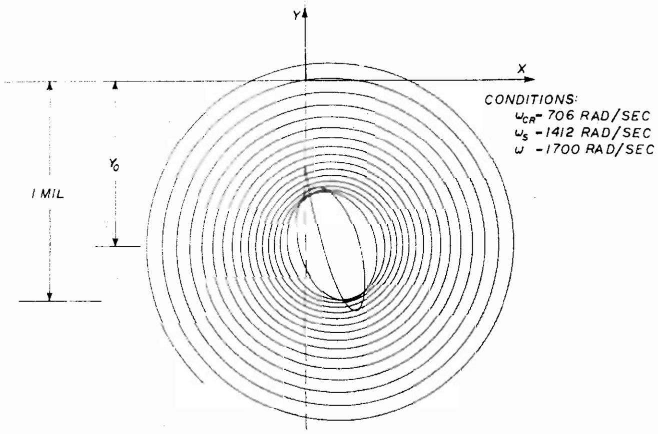

Fig. 6 Whirl orbit of a balanced horizontal rotor above the threshold of stability, $\omega > \omega_s$

or

$$S_T^2 + = \omega_{cr}^2; \qquad S_T^2 - = -\frac{1}{2}(n_1+n_2)^2$$

Consider only the positive root or that

$$S_T = \omega_{cr} \tag{34}$$

The foregoing statement implies that, at the threshold of instability, the volume center of the rotor will precess in a forward direction at a rate equal to the first system critical speed.

Example Problem

Let

$M = 0.25 \dfrac{\text{lb-sec}^2}{\text{in.}}$ (96.6-lb rotor)

$K_1 = K_2 = 250{,}000$ lb/in.

$D_1 = D_2 = 200$ rad/sec

$\omega_{cr_0} = \sqrt{\dfrac{K_2}{M}} = 1000$ rad/sec = rotor natural frequency (on rigid supports)

$\omega_{cr} = \sqrt{\dfrac{K_T}{M}} = 706$ rad/sec = system natural frequency

$\omega_s = \omega_{cr}\left(1 + \dfrac{n_1}{n_2}\right) = 1412$ rad/sec = threshold speed

The whirl precession rate is given by equation (32):

$$\omega_p = \sqrt{\frac{\omega_{cr}^2 - \left(\frac{n_1+n_2}{2}\right)^2 + \sqrt{\left\{\left(\frac{n_1+n_2}{2}\right)^2 - \omega_{cr}^2\right\}^2 + (\omega n_2)^2}}{2}}$$

$$\omega_p = \frac{\omega_{cr}}{2}\sqrt{1 - \left(\frac{D}{4\omega_{cr}}\right)^2 + \sqrt{\left\{1 - \left(\frac{D}{4\omega_{cr}}\right)^2\right\}^2 + \left(\frac{\omega}{\omega_{cr}}\right)^2\left(\frac{D}{4\omega_{cr}}\right)^2}}$$

note that

$$\left(\frac{D}{4\omega_{cr}}\right)^2 = \left(\frac{200}{4 \times 706}\right)^2 = 0.005 \ll 1$$

Hence, expanding in terms of $\left(\dfrac{D}{4\omega_{cr}}\right)^2$,

$$\omega_p = \omega_{cr}\left[1 + \frac{1}{8}\left(\frac{\omega}{\omega_{cr}}\right)^2\left(\frac{D}{4\omega_{cr}}\right)^2\right]$$

or

$$\omega_p = \omega_{cr}$$

Thus it can be seen that the precession rate of the rotor is in all practical cases a constant, regardless of the total rotor rotational speed.

7 Analog Computer Solution of the Rotor Motion

Equation (18) was programmed on the analog computer, and the rotor motion was investigated over a wide range of support stiffness and damping values. Also included in the analog program was the influence of rotor unbalance and nonlinear shaft stiffness. For example, Fig. 5 represents the whirl orbits of a balanced horizontal rotor for a speed range of 500 to 1450 rad/sec. Notice that as the rotor speed approaches the threshold of stability, the time required for the transient motion to die out increases. At the threshold of stability, a sustained transient motion is developed.

Fig. 6 represents the rotor motion above the threshold of stability. In the linear system, the unstable rotor motion is unbounded.

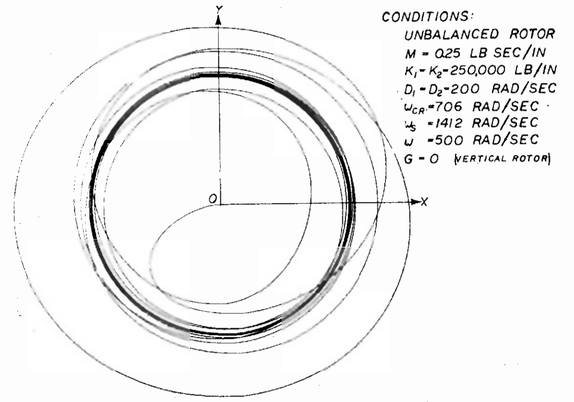

Fig. 7 Whirl orbit of an unbalanced rotor with internal friction damping below the threshold of stability, $\omega < \omega_s$

Fig. 7 represents the case identical to Fig. 5, of ω = 500 rad/sec with unbalance added. When the transient motion dies out, the orbit reaches the steady-state motion caused by the rotor unbalance. Fig. 8 represents the unbalance rotor motion slightly above the threshold speed. The equations of motion of the system in complex vector form are

$$\dot{Z} + (n_1 + n_2)\dot{Z} + [\omega_{cr}^2(1 + \delta Z\bar{Z}) - i\omega n_2]Z = \omega^2 e^{i\omega t} + iG/M \quad (35)$$

It was found that only several percent change in radial nonlinear stiffness parameter δ was necessary to produce a limit cycle at the threshold of stability. In the linear case, the total rotor orbit forms a slowly divergent spiral. The introduction of the nonlinear component (δ = 0.01 and 0.04) causes a finite orbit to develop. When the rotor speed is increased above the threshold speed, the orbit grows but remains bounded. In actual rotor systems, some small nonlinearity is usually always present to produce finite limit cycles.

8 Discussion and Conclusions

From the stability criterion developed in this analysis, it is now possible to theoretically explain the experimental findings of Dr. Newkirk in 1924. Dr. Newkirk investigated the whirl behavior of shafts with shrink fit disks and from experimental observations arrived at several important conclusions concerning this type of behavior. By examination of the graph of the plotted stability criterion (Fig. 4), these conclusions can be verified.

Examination of the governing equations of motion shows that rotor unbalance doesn't appear in the equations, and hence the stability criterion should not be affected by the degree of balance. The stability criterion shows that whirling must take place above the first critical speed. The value ω_{cr_0} represents the critical speed of the unit with rigid bearing supports. The whirl speed is dependent upon two important parameters, the flexibility ratio R and the damping ratio D. If $R = 0$ (rigid support), the threshold region increases with increasing D. Unless one has a built-up rotor or shaft with shrink fits, it is impossible to obtain rotary damping and D will be zero. Hence this motion will only occur with built-up rotors. It has been observed that the whirl threshold speed would vary considerably between units of similar design and even for the same unit under different test conditions. This can be readily accounted for by the fact that the shrink fits of the built-up sections control the value of the rotary damping coefficient. Some variations of fit are to be expected even between units of identical construction. It was observed that increasing the thrust bearing load increases stability. This can be accounted for by the fact that the stationary damping coefficient C_1 is increased, reducing the damping parameter D.

Of utmost interest in this analysis is the effect of foundation or bearing housing flexibility on rotor stability. In the original experimental investigation, it was found that decreasing the foundation flexibility and introducing damping into the foundation had an extremely beneficial effect on the whirl threshold. From the stability plot, Fig. 4, it is seen that a reduction in the foundation flexibility will increase R and reduce the system critical speed. If the damping parameter D is less than 2, the effect of increased foundation flexibility will increase the whirl threshold speed. In some instances, it was observed that bearing damping was necessary (along with reduced support flexibility) in order to improve the whirl region. If the D-value is too high, that is, larger than 3, a reduction in support flexibility will not only reduce the system critical speed but will also reduce the whirl threshold speed. Introduction of damping into the support is necessary to reduce the damping parameter D and hence raises the whirl speed for the case of isotropic support stiffness.

Reference [4] shows that rotor stability may be appreciably increased without the addition of damping by means of anisotropic support stiffness. Reference [4] also shows that rotor instability may be induced by other factors such as fluid film bearings.

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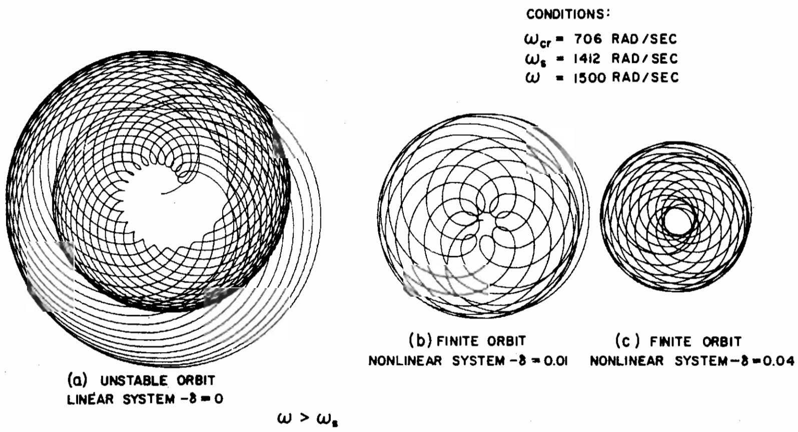

Fig. 8 Effect of nonlinearity on rotor motion above the threshold of stability, $\omega > \omega_s$