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The Influence of Fluid Inertia on the Dynamic Properties of Journal Bearings

Based on a first-order perturbation solution in a modified Reynolds number an analysis is presented to determine the effect of the fluid film inertial forces on the dynamic properties of a journal bearing. The corrections to the regular amplitude and velocity coefficients are found to be small, but the accompanying acceleration coefficients which may correspond to a virtual mass of several times the mass of the journal itself, could become significant for short rotors. Numerical results are given in graphical form with dimensionless coefficients as functions of the operating eccentricity ratio.

Introduction

Consistent with the assumptions inherent in reducing the Navier-Stokes equations to Reynolds equation, the conventional laminar, thin film lubrication theory ignores the inertia forces in the fluid film [1, 2].[1] This is theoretically justified for small values of the Reynold number (of the order of 1). On the other hand, the assumption of laminar flow ceases to be valid when there is a transition to either Taylor vortex flow or to turbulent flow which, for journal bearings of typical dimensions, occurs at a Reynolds number value of approximately 1000 to 1500. Thus, there is an intermediate range, say for values of Reynolds number of the order of 10^2, where inertial effects may become noticeable without affecting the assumption of laminar flow.

Several investigators have examined the problem [1, 3, 4, 5, 6, 7, 8, 9] and found the effect to be small. The investigations, however, are usually restricted to bearings operating under steady-state conditions while dynamic conditions have only been considered for simplified bearing geometries [1, 7, 8, 9]. It is the purpose of the present paper to extend the analysis to one of the more common bearing geometries, namely the cylindrical journal bearing, and to consider the influence of the inertial terms on the dynamic properties. Based on the previous findings that show the inertial forces to be small, the analysis employs a first order perturbation expansion in Reynolds number as also done in [9].

Apart from a quantitative evaluation of the effect of the inertial forces on the dynamic bearing properties, the primary reason for undertaking the investigation is to determine the virtual mass coefficients which could be significant in certain applications, as for example short rotors. This is pointed out by Smith [10] whose analysis, however, is approximate and based on intuitive reasoning.

The Governing Equations

With the usul assumptions of thin film lubrication theory the governing differential equations for a fluid element are [1, 2]:

$$\rho\left\{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right\} = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$0 = -\frac{\partial p}{\partial y} \quad (2)$$

$$\rho\left\{\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right\} = -\frac{\partial p}{\partial z} + \mu\frac{\partial^2 w}{\partial y^2} \quad (3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (4)$$

The left-hand sides in equations (1) and (3) are the contributions from the inertia forces which disappear in classical lubrication theory. u, v and w are the fluid velocity components in the x, y and z directions, respectively, where y is the coordinate across the thickness of the film. The fluid pressure is p, the viscosity μ and the density ρ.

When the journal, with radius R, rotates with angular speed ω and x is the circumferential coordinate, the velocity boundary conditions become:

$$\begin{aligned} y &= 0 : u = v = w = 0 \\ y &= h : u = R\omega \end{aligned} \quad (5)$$

$$\begin{aligned} v &= \frac{\partial h}{\partial x}R\omega + \frac{\partial h}{\partial t} \\ w &= 0 \end{aligned} \quad (6)$$

where h is the local film thickness.

The equations are normalized by means of the dimensionless

[1] Numbers in brackets designate References at end of paper.

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quantities:

$$\theta = \frac{x}{R}\ \zeta = \frac{z}{R}\ \eta = \frac{y}{c}\ \tau = \omega t \tag{7}$$

$$\bar{u} = \frac{u}{R\omega}\ \bar{v} = \frac{v}{c\omega}\ \bar{w} = \frac{w}{R\omega} \tag{8}$$

$$\bar{p} = \frac{p}{\mu\omega(R/c)^2} \tag{9}$$

$$\bar{h} = h/c \tag{10}$$

where c is the radial journal bearing clearance. In addition, a parameter λ is required to account for the inertia forces:

$$\lambda = \frac{c}{R}R_e \tag{11}$$

where

$$\text{Reynolds number}: R_e = \frac{\rho c R\omega}{\mu} \tag{12}$$

Thus, equations (1)-(6) can be written in dimensionless form as:

$$\lambda\left\{\frac{\partial\bar{u}}{\partial\tau} + \bar{u}\frac{\partial\bar{u}}{\partial\theta} + \bar{v}\frac{\partial\bar{u}}{\partial\eta} + \bar{w}\frac{\partial\bar{u}}{\partial\zeta}\right\} = -\frac{\partial\bar{p}}{\partial\theta} + \frac{\partial^2\bar{u}}{\partial\eta^2} \tag{13}$$

$$\lambda\left\{\frac{\partial\bar{w}}{\partial\tau} + \bar{u}\frac{\partial\bar{w}}{\partial\theta} + \bar{v}\frac{\partial\bar{w}}{\partial\eta} + \bar{w}\frac{\partial\bar{w}}{\partial\zeta}\right\} = -\frac{\partial\bar{p}}{\partial\zeta} + \frac{\partial^2\bar{w}}{\partial\eta^2} \tag{14}$$

$$\frac{\partial\bar{u}}{\partial\theta} + \frac{\partial\bar{v}}{\partial\eta} + \frac{\partial\bar{w}}{\partial\zeta} = 0 \tag{15}$$

$$y = 0 : \bar{u} = \bar{v} = \bar{w} = 0 \tag{16}$$

$$y = \bar{h} : \bar{u} = 1\ \bar{w} = 0\ \bar{v} = \frac{\partial\bar{h}}{\partial\theta} + \frac{\partial\bar{h}}{\partial\tau} \tag{17}$$

The last equation, equation (2), only expresses that the pressure does not vary across the film, and is of no further interest.

From equations (13) and (14) is seen that the inertia forces are of the order (c/R) when the Reynolds number is small (of the order 1), and it is on this basis that these terms are left out in the classical lubrication theory.

At moderate values of the Reynolds number, however, the influence of the inertia terms may become noticeable. To investigate the effect, a first-order perturbation solution is carried out where the nonlinear equations are linearized by setting [9]:

$$\bar{p} = \bar{p}^{(0)} + \lambda\bar{p}^{(1)} + o(\lambda^2) \tag{18}$$

$$\bar{u} = \bar{u}^{(0)} + \lambda\bar{u}^{(1)} + o(\lambda^2) \tag{19}$$

and analogously for $\bar{v}$ and $\bar{w}$. Substituting these expressions into equations (13)-(17) and collecting terms of like order in λ results in the following zero-order equations:

$$\frac{\partial\bar{p}^{(0)}}{\partial\theta} = \frac{\partial^2\bar{u}^{(0)}}{\partial\eta^2} \tag{20}$$

$$\frac{\partial\bar{p}^{(0)}}{\partial\zeta} = \frac{\partial^2\bar{w}^{(0)}}{\partial\eta^2} \tag{21}$$

$$\frac{\partial\bar{u}^{(0)}}{\partial\theta} + \frac{\partial\bar{v}^{(0)}}{\partial\eta} + \frac{\partial\bar{w}^{(0)}}{\partial\zeta} = 0 \tag{22}$$

with the boundary conditions:

$$\eta = 0 : \bar{u}^{(0)} = \bar{v}^{(0)} = \bar{w}^{(0)} = 0$$
$$\eta = \bar{h} : \bar{u}^{(0)} = 1;\ \bar{w}^{(0)} = 0;\ \bar{v}^{(0)} = \frac{\partial\bar{h}}{\partial\theta} + \frac{\partial\bar{h}}{\partial\tau} \tag{23}$$

Integrating equations (20) and (21) twice yields the well-known velocity profiles of lubrication theory:

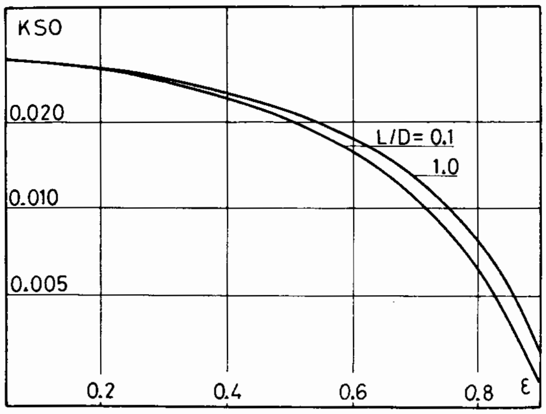

Fig. 1 Correction to the inverse Sommerfeld number.

$$\bar{u}^{(0)} = -\frac{1}{2}\frac{\partial\bar{p}^{(0)}}{\partial\theta}\eta(\bar{h} - \eta) + \frac{\eta}{\bar{h}}$$
$$\bar{w}^{(0)} = -\frac{1}{2}\frac{\partial\bar{p}^{(0)}}{\partial\zeta}\eta(\bar{h} - \eta) \tag{24}$$

Inserting $\bar{u}^{(0)}$ and $\bar{w}^{(0)}$ into the continuity equation, equation (22), $\bar{v}^{(0)}$ can be determined by integration. Using the boundary condition at $\eta = \bar{h}$, equation (23) results in Reynolds equation:

$$\frac{\partial}{\partial\theta}\left\{\bar{h}^3\frac{\partial\bar{p}^{(0)}}{\partial\theta}\right\} + \frac{\partial}{\partial\zeta}\left\{\bar{h}^3\frac{\partial\bar{p}^{(0)}}{\partial\zeta}\right\} = 6\frac{\partial\bar{h}}{\partial\theta} + 12\frac{\partial\bar{h}}{\partial\tau} \tag{25}$$

while $\bar{v}^{(0)}$ becomes:

$$\bar{v}^{(0)} = \frac{\eta^2(2\bar{h} - \eta)}{\bar{h}^3}\frac{\partial\bar{h}}{\partial\theta} - \frac{\eta^2(\bar{h} - \eta)}{2\bar{h}} \times \left\{\frac{\partial\bar{h}}{\partial\theta}\frac{\partial\bar{p}^{(0)}}{\partial\theta} + \frac{\partial\bar{h}}{\partial\zeta}\frac{\partial\bar{p}^{(0)}}{\partial\zeta}\right\} + \frac{\eta^2(3\bar{h} - 2\eta)}{\bar{h}^3}\frac{\partial\bar{h}}{\partial\tau} \tag{26}$$

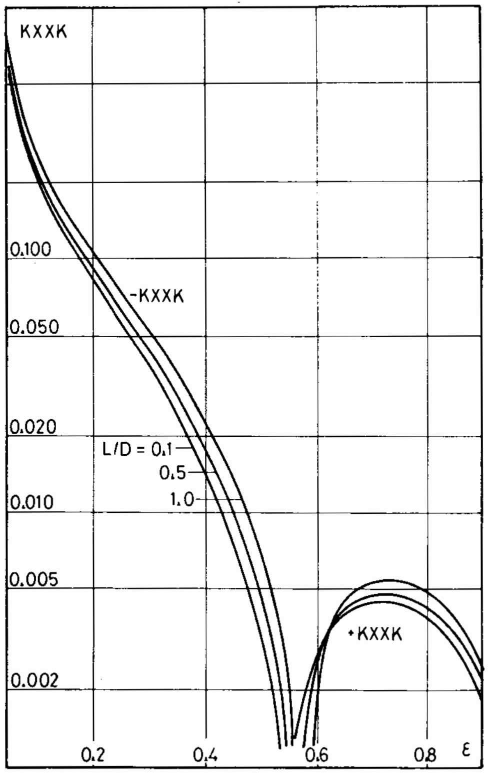

Fig. 2 Correction to the direct amplitude coefficient K_{xx}

The first order perturbation equations, akin to equations (20)–(22), are:

$$\frac{\partial^2 \bar{u}^{(1)}}{\partial \eta^2} = \frac{\partial \bar{p}^{(1)}}{\partial \theta} + \bar{u}^{(0)} \frac{\partial \bar{u}^{(0)}}{\partial \theta} + \bar{v}^{(0)} \frac{\partial \bar{u}^{(0)}}{\partial \eta} + \bar{w}^{(0)} \frac{\partial \bar{u}^{(0)}}{\partial \zeta} + \frac{\partial \bar{u}^{(0)}}{\partial \tau} \quad (27)$$

$$\frac{\partial^2 \bar{w}^{(1)}}{\partial \eta^2} = \frac{\partial \bar{p}^{(1)}}{\partial \zeta} + \bar{u}^{(0)} \frac{\partial \bar{w}^{(0)}}{\partial \theta} + \bar{v}^{(0)} \frac{\partial \bar{w}^{(0)}}{\partial \eta} + \bar{w}^{(0)} \frac{\partial \bar{w}^{(0)}}{\partial \zeta} + \frac{\partial \bar{w}^{(0)}}{\partial \tau} \quad (28)$$

$$\frac{\partial \bar{u}^{(1)}}{\partial \theta} + \frac{\partial \bar{v}^{(1)}}{\partial \eta} + \frac{\partial \bar{w}^{(1)}}{\partial \zeta} = \quad (29)$$

with the boundary conditions:

$$\eta = 0 \text{ and } \eta = \bar{h} : \bar{u}^{(1)} = \bar{v}^{(1)} = \bar{w}^{(1)} = 0 \quad (30)$$

As $\bar{p}^{(1)}$ is independent of η, (equation (2)), equations (27) and (28) may be integrated twice, making use of equations (24), (26) and (30), to calculate $\bar{u}^{(1)}$ and $\bar{w}^{(1)}$. The boundary conditions, equation (30), together with equation (29) can be applied to derive the first-order equivalent of Reynolds equation. After rather extensive algebraic operations the final equation becomes:

$$\frac{\partial}{\partial \theta}\{\bar{h}^3 \frac{\partial \bar{p}^{(1)}}{\partial \theta}\} + \frac{\partial}{\partial \zeta}\{\bar{h}^3 \frac{\partial \bar{p}^{(1)}}{\partial \zeta}\} = \frac{\partial}{\partial \theta}\{- \frac{3\bar{h}^7}{560} \frac{\partial}{\partial \theta}\{(\frac{\partial \bar{p}^{(0)}}{\partial \theta})^2 + (\frac{\partial \bar{p}^{(0)}}{\partial \zeta})^2\} - \frac{3\bar{h}^6}{140} \frac{\partial \bar{h}}{\partial \theta} (\frac{\partial \bar{p}^{(0)}}{\partial \theta})^2 + \frac{\bar{h}^5}{20} \frac{\partial^2 \bar{p}^{(0)}}{\partial \theta^2} + \frac{13\bar{h}^4}{140} \frac{\partial \bar{h}}{\partial \theta} \frac{\partial \bar{p}^{(0)}}{\partial \theta}$$

$$- \frac{\bar{h}^2}{10} \frac{\partial \bar{h}}{\partial \theta} - \frac{3\bar{h}^6}{140} \frac{\partial \bar{h}}{\partial \zeta} \frac{\partial \bar{p}^{(0)}}{\partial \theta} \frac{\partial \bar{p}^{(0)}}{\partial \zeta} + \frac{13\bar{h}^4}{70} \frac{\partial \bar{p}^{(0)}}{\partial \theta} \frac{\partial \bar{h}}{\partial \tau} + \frac{\bar{h}^5}{10} \frac{\partial^2 \bar{p}^{(0)}}{\partial \theta \partial \tau}\}$$

$$+ \frac{\partial}{\partial \zeta}\{- \frac{3\bar{h}^7}{560} \frac{\partial}{\partial \zeta}\{(\frac{\partial \bar{p}^{(0)}}{\partial \theta})^2 + (\frac{\partial \bar{p}^{(0)}}{\partial \zeta})^2\} - \frac{3\bar{h}^6}{140} \frac{\partial \bar{h}}{\partial \theta} \frac{\partial \bar{p}^{(0)}}{\partial \theta} \frac{\partial \bar{p}^{(0)}}{\partial \zeta}$$

$$+ \frac{\bar{h}^5}{20} \frac{\partial^2 \bar{p}^{(0)}}{\partial \theta \partial \zeta} + \frac{13\bar{h}^4}{140} \frac{\partial \bar{h}}{\partial \theta} \frac{\partial \bar{p}^{(0)}}{\partial \zeta} - \frac{3\bar{h}^6}{140} \frac{\partial \bar{h}}{\partial \zeta} (\frac{\partial \bar{p}^{(0)}}{\partial \zeta})^2 + \frac{13\bar{h}^4}{70} \frac{\partial \bar{p}^{(0)}}{\partial \zeta} \frac{\partial \bar{h}}{\partial \tau} + \frac{\bar{h}^5}{10} \frac{\partial^2 \bar{p}^{(0)}}{\partial \zeta \partial \tau}\} \quad (31)$$

The Dynamic Coefficients

The filmthickness is a function of the journal center position. Introducing an x-y-coordinate system with origin in the bearing center and the x-axis in the direction of the applied static load, the journal center has the coordinates ($\bar{x}$, $\bar{y}_0$) under static conditions. The coordinates are normalized with respect to the radial clearance. Superimposing a dynamic motion with amplitudes $\Delta\bar{x}$

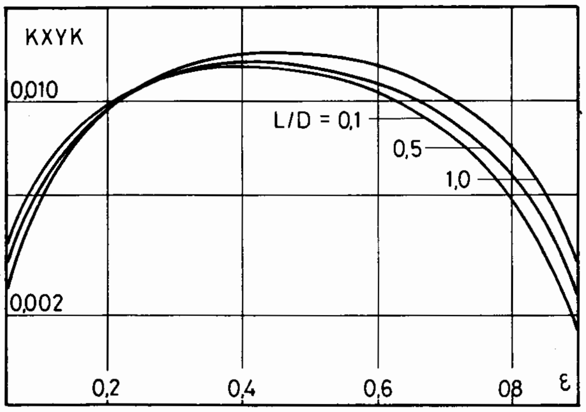

Fig. 4 Correction to the cross-coupling amplitude coefficient K_{xy}

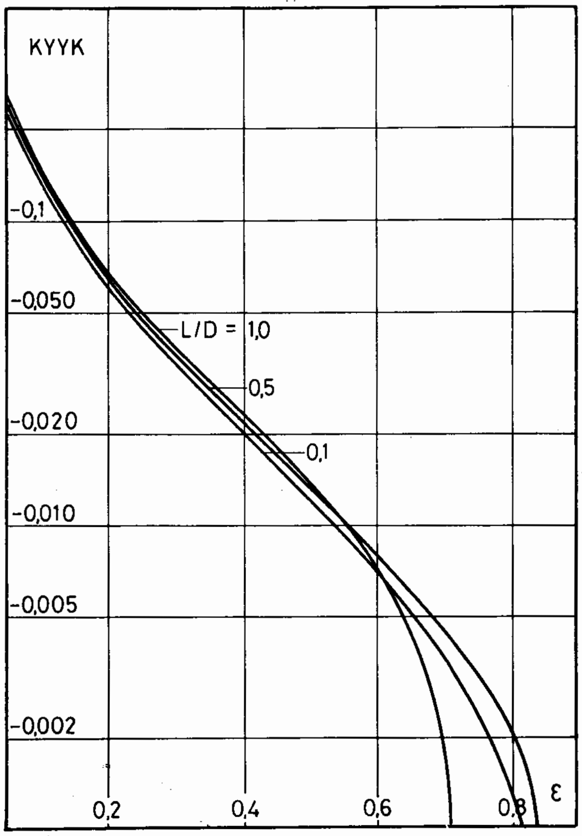

Fig. 3 Correction to the direct amplitude coefficient K_{yy}

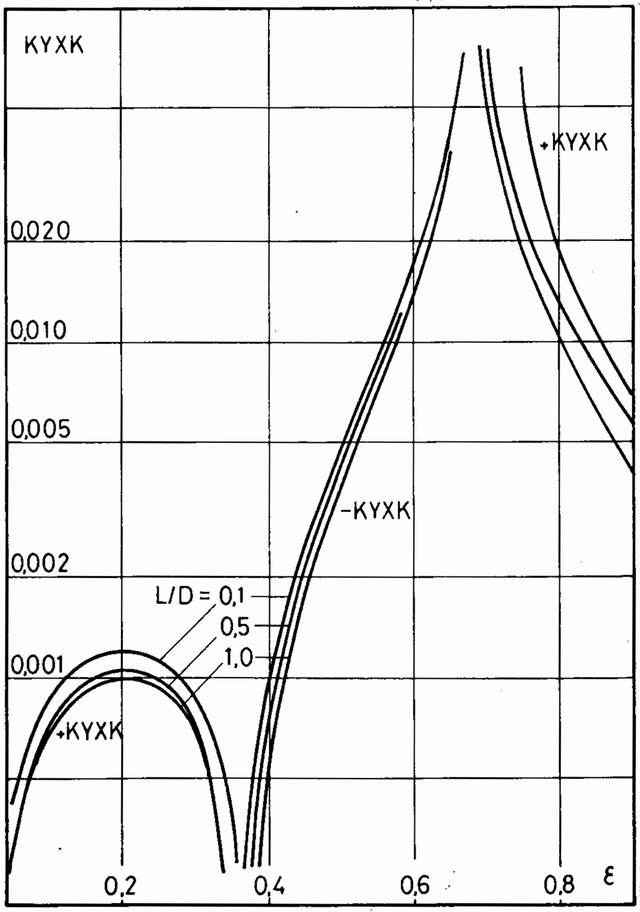

Fig. 5 Correction to the cross-coupling amplitude coefficient K_{yx}

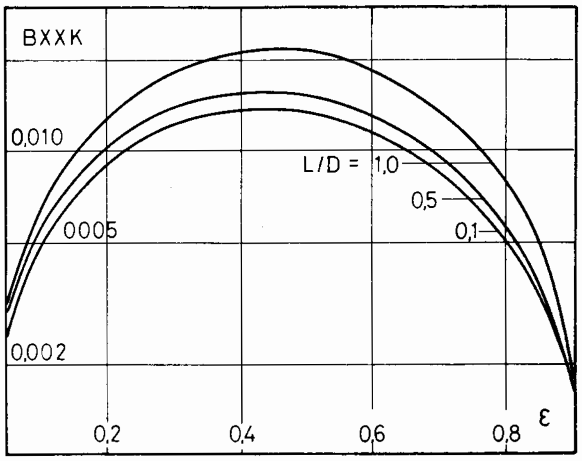

Fig. 6 Correction to the direct velocity coefficient B_{xx}

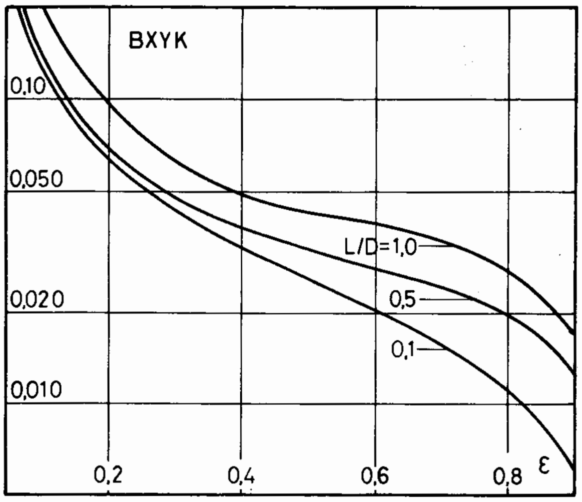

Fig. 8 Correction to the cross-coupling velocity coefficient B_{xy}

and $\Delta\bar{y}$ around this equilibrium position, the dimensionless local filmthickness can be written as:

$$\bar{h} = \bar{h}_0 + \Delta\bar{x}\cos\theta + \Delta\bar{y}\sin\theta \tag{32}$$

where:

$$\bar{h}_0 = 1 + \bar{x}_0\cos\theta + \bar{y}_0\sin\theta \tag{33}$$

and where the coordinate angle θ is measured from the negative x-axis. Assuming the amplitudes $\Delta\bar{x}$ and $\Delta\bar{y}$ to be small the pressure may be expanded in a first order perturbation such that equation (18) becomes:

$$\bar{p} = \bar{p}_0^{(0)} + \bar{p}_x^{(0)}\Delta\bar{x} + \bar{p}_y^{(0)}\Delta\bar{y} + \bar{p}_{\dot{x}}^{(0)}\Delta\dot{\bar{x}} + \bar{p}_{\dot{y}}^{(0)}\Delta\dot{\bar{y}} + \lambda\{\bar{p}_0^{(1)} + \bar{p}_x^{(1)}\Delta\bar{x} + \bar{p}_y^{(1)}\Delta\bar{y} + \bar{p}_{\dot{x}}^{(1)}\Delta\dot{\bar{x}} + \bar{p}_{\dot{y}}^{(1)}\Delta\dot{\bar{y}} + \bar{p}_{\ddot{x}}^{(1)}\Delta\ddot{\bar{x}} + \bar{p}_{\ddot{y}}^{(1)}\Delta\ddot{\bar{y}}\} \tag{34}$$

Similarly, a first order expansion of equation (32) yields:

$$\bar{h}^n = \bar{h}_0^n + n\bar{h}_0^{n-1}(\Delta\bar{x}\cos\theta + \Delta\bar{y}\sin\theta) \tag{35}$$

Substitution of these equations into equations (25) and (31) and collecting terms according to the perturbation variables, results in 12 equations to determine the 12 quantities: $\bar{p}_0^{(0)}$, $\bar{p}_x^{(0)}$, etc.

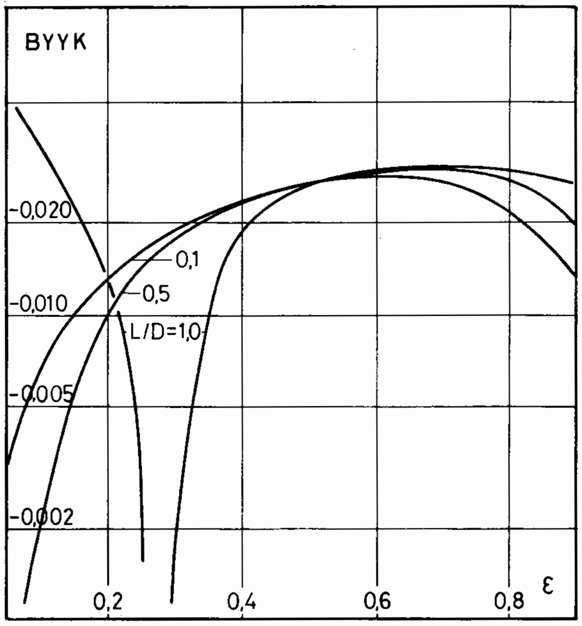

Fig. 7 Correction to the direct velocity coefficient B_{yy}

With $\bar{p}_k^{(j)}$ representing the general quantity these equations are of the form:

$$\frac{\partial}{\partial\theta}\{\bar{h}^3\frac{\partial\bar{p}_k^{(j)}}{\partial\theta}\} + \frac{\partial}{\partial\zeta}\{\bar{h}^3\frac{\partial\bar{p}_k^{(j)}}{\partial\zeta}\} = RHS_k^{(j)} \tag{36}$$

where the right-hand side, $RHS_k^{(j)}$, depends on $\bar{p}^{(0)}$ or $\bar{p}^{(1)}$ and their first-order derivatives (second order derivatives can be eliminated by means of the governing equations for $\bar{p}^{(0)}$ or $\bar{p}^{(1)}$). As the expressions are quite lengtly they are omitted.

Allowing for film rupture, the adopted boundary condition are:

$$\begin{aligned} &\zeta = \pm(L/D): \bar{p} = 0 \\ &\theta = \theta_1(\zeta),\ \theta = \theta_2(\zeta): \bar{p} = \frac{\partial\bar{p}}{\partial\bar{n}} = 0 \end{aligned} \tag{37}$$

where $\bar{n}$ indicates the direction of the normal to the boundary curve. To implement the last condition, consider a point (θ_0, ζ_0) on the free boundary under static conditions. Owing to a pressure perturbation $\Delta\bar{p}$ this point moves the increment $(\Delta\theta, \Delta\zeta)$ to a point $(\theta = \theta_0 + \Delta\theta,\ \zeta = \zeta_0 + \Delta\zeta)$. Requiring the pressure to be zero on the new boundary curve, a first order expansion yields:

$$\bar{p}(\theta,\zeta) = 0 = \bar{p}(\theta_0,\zeta_0) + (\frac{\partial\bar{p}}{\partial\theta})_0\Delta\theta + (\frac{\partial\bar{p}}{\partial\zeta})_0\Delta\zeta \tag{38}$$

With $\bar{p} = \bar{p}_0 + \Delta\bar{p}$ and retaining only first order terms, this equation reduces to:

$$\bar{p}(\theta,\zeta) = 0 = \bar{p}_0(\theta_0,\zeta_0) + \Delta\bar{p}(\theta_0,\zeta_0) + (\frac{\partial\bar{p}_0}{\partial\theta})_0\Delta\theta + (\frac{\partial\bar{p}_0}{\partial\zeta})_0\Delta\zeta \tag{39}$$

Because $\bar{p}_0(\theta_0, \zeta_0) = (\partial\bar{p}_0/\partial\theta)_0 = (\partial\bar{p}_0/\partial\zeta)_0 = 0$, the boundary condition for the pressure perturbation becomes: $\Delta\bar{p}(\theta_0, \zeta_0) = 0$.

With the boundary conditions established, equation (37) can be written in finite difference form and solved numerically [1, 14]. By integrating the film pressures the reaction forces are obtained as:

$$\left.\begin{matrix} F_x \\ F_y \end{matrix}\right\} = -\int_{-(L/2)}^{(L/2)}\int_{\theta_1}^{\theta_2} p\left\{\begin{matrix}\cos\theta \\ \sin\theta\end{matrix}\right\} Rd\theta\, dz$$

or in dimensionless form:

$$\left.\begin{matrix} \bar{F}_x = \dfrac{F_x}{\mu NDL(R/c)^2} \\ \bar{F}_y = \dfrac{F_y}{\mu NDL(R/c)^2} \end{matrix}\right\} = -\frac{\pi/2}{L/D}\int_{-(L/D)}^{(L/D)}\int_{\theta_1}^{\theta_2}\bar{p}\left\{\begin{matrix}\cos\theta \\ \sin\theta\end{matrix}\right\} d\theta\, d\zeta \tag{40}$$

where N is the rotational speed in rps ($N = \omega/2\pi$). Substituting for $\bar p$ from equation (34) the perturbation expansion of the reaction forces becomes:

$$\bar F_x = \bar F_{xo}^{(0)} + \bar K_{xx}^{(0)}\Delta\bar x + \bar K_{xy}^{(0)}\Delta\bar y + \bar B_{xx}^{(0)}\Delta\dot{\bar x} + \bar B_{xy}^{(0)}\Delta\dot{\bar y} + \lambda\{\bar F_{xo}^{(1)} + \bar K_{xx}^{(1)}\Delta\bar x + \bar K_{xy}^{(1)}\Delta\bar y + \bar B_{xx}^{(1)}\Delta\dot{\bar x} + \bar B_{xy}^{(1)}\Delta\dot{\bar y} + \bar C_{xx}\Delta\ddot{\bar x} + \bar C_{xy}\Delta\ddot{\bar y}\} \quad (41)$$

and analogously for $\bar F_y$ where, as an example:

$$\left.\begin{matrix}\bar K_{xx}^{(0)}\\ \bar K_{yx}^{(0)}\end{matrix}\right\} = -\frac{\pi/2}{L/D}\int_{-(L/D)}^{(L/D)}\int_{\theta_1}^{\theta_2}\bar p_x^{(0)}\left\{\begin{matrix}\cos\theta\\ \sin\theta\end{matrix}\right\}d\theta\,d\xi \quad (42)$$

and similarly for the remaining coefficients. In dimensional form the reaction forces are:

$$F_x = (1 + \lambda\Delta\bar F_{xo})F_{xo} + (1 + \lambda\Delta\bar K_{xx})K_{xx}\Delta x + (1 + \lambda\Delta\bar K_{xy})K_{xy}\Delta y + (1 + \lambda\Delta\bar B_{xx})B_{xx}\frac{d\Delta x}{dt} + (1 + \lambda\Delta\bar B_{xy})B_{xy}\frac{d\Delta y}{dt} + C_{xx}\frac{d^2\Delta x}{dt^2} + C_{xy}\frac{d^2\Delta y}{dt^2} \quad (43)$$

and analogously for F_y. The relationship between the dimensional and the dimensionless coefficients are:

$$K_{xx} = \frac{1}{c}\mu NDL(R/C)^2\bar K_{xx} = S\frac{W}{c}\bar K_{xx} \quad \{N/m\} \quad (44)$$

$$B_{xx} = S\frac{W}{\omega c}\bar B_{xx} \quad \{Ns/m\} \quad (45)$$

$$C_{xx} = \rho\pi R^2L(R/c)\frac{1}{\pi^2}\bar C_{xx} \quad \{Ns^2/m\} \quad (46)$$

and similarly for the other coefficients. S is the Sommerfeld number and W is the static load such that:

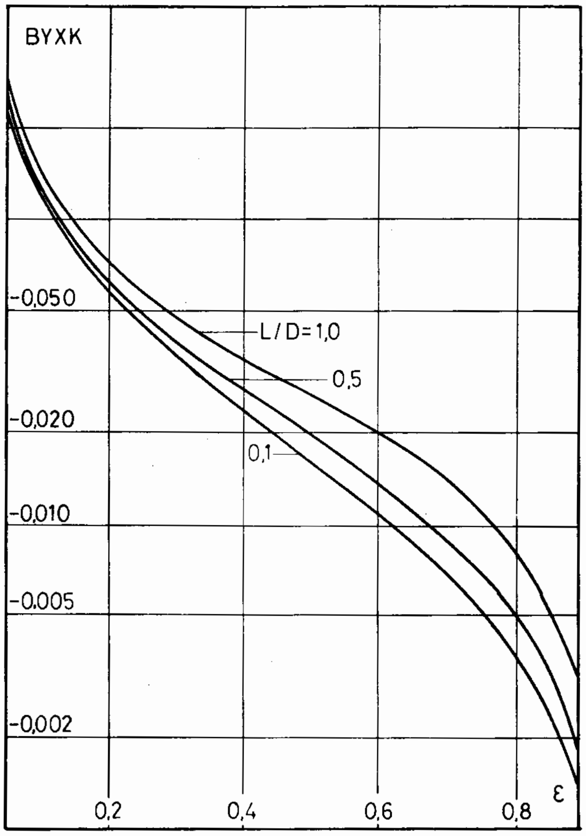

Fig. 9 Correction to the cross-coupling velocity coefficient B_{yx}

$$W = (1 + \lambda\Delta\bar F_{xo})F_{xo} \quad (47)$$

or, in dimensionless form:

$$\bar F_{xo}^{(0)} + \lambda\bar F_{xo}^{(1)} = (1 + \lambda\Delta\bar F_{xo})\bar F_{xo}^{(0)} = \frac{W}{\mu NDL(R/c)^2} = \frac{1}{S} \quad (48)$$

Static equilibrium requires that $\bar F_{yo}^{(0)} + \lambda\bar F_{yo}^{(1)} = 0$ which means that the equilibrium coordinates $(\bar x_0, \bar y_0)$ depend on λ. As the effect is small it has been ignored in order to simplify the calculation of the coefficients.

The coefficients K_{xx}, B_{xx}, K_{xy}, B_{xy}, etc., are the 8 regular coefficients which are derived from Reynolds equation without inertia forces [12]. When inertia effects are included the coefficients are modified as shown in equation (43) where:

$$\Delta\bar K_{xx} = \bar K_{xx}^{(1)}/\bar K_{xx}^{(0)} \quad (49)$$

$$\Delta\bar B_{xx} = \bar B_{xx}^{(1)}/\bar B_{xx}^{(0)} \quad (50)$$

and similarly for the remaining corrections. These corrections are small as shown later.

The Acceleration Coefficients

In addition to the usual amplitude and velocity coefficients, equation (43) shows that, when inertia forces are considered, the dynamic fluid film reaction forces also depend on the journal accelerations [10]. The 4 acceleration coefficients, C_{xx}, C_{xy}, C_{yx} and C_{yy}, are computed from equation (46) where:

$$\left.\begin{matrix}\bar C_{xx}\\ \bar C_{yx}\end{matrix}\right\} = -\frac{\pi/2}{L/D}\int_{-(L/D)}^{(L/D)}\int_{\theta_1}^{\theta_2}\bar p_x^{(1)}\left\{\begin{matrix}\cos\theta\\ \sin\theta\end{matrix}\right\}d\theta\,d\xi \quad (51)$$

and similarly for $\bar C_{xy}$ and $\bar C_{yy}$. The perturbated pressures $\bar p_x^{(1)}$ and $\bar p_y^{(1)}$ are determined from equation (36) in the following form:

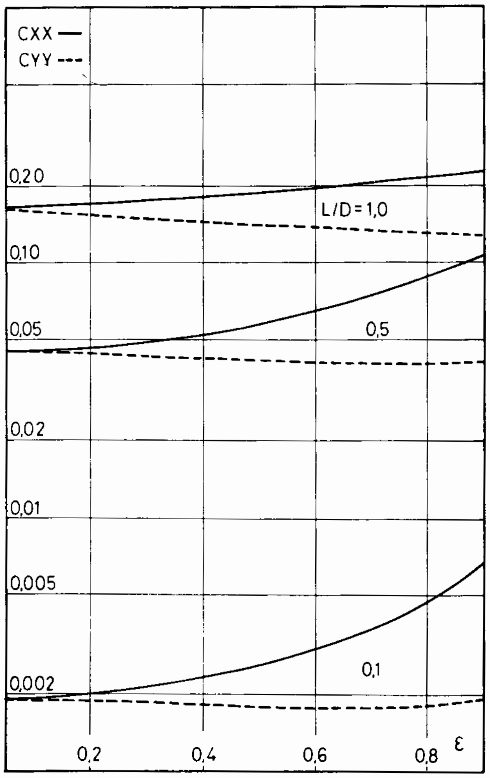

Fig. 10 The dimensionless direct acceleration coefficients C_{xx} and C_{yy}

$$\left\{\frac{\partial}{\partial\theta}\left(\bar h_0^3 \frac{\partial}{\partial\theta}\right) + \frac{\partial}{\partial\zeta}\left(\bar h_0^3\frac{\partial}{\partial\zeta}\right)\right\}\left\{\begin{matrix}\bar p_{\dot x}^{(1)}\\ \bar p_{\dot y}^{1}\end{matrix}\right\} = \frac{6}{5}\bar h_0^2\left\{\begin{matrix}\cos\theta\\ \sin\theta\end{matrix}\right\} + \frac{\bar h_0^4}{5}\frac{\partial \bar h_0}{\partial\theta}\left\{\begin{matrix}\dfrac{\partial \bar p_{\dot x}^{(0)}}{\partial\theta}\\ \dfrac{\partial\bar p_{\dot y}^{(0)}}{\partial\theta}\end{matrix}\right\} \quad (52)$$

where $\bar p_x^{(0)}$ and $p_y^{(0)}$ are found from:

$$\left\{\frac{\partial}{\partial\theta}\left(\bar h_0^3 \frac{\partial}{\partial\theta}\right) + \frac{\partial}{\partial\zeta}\left(\bar h_0^3\frac{\partial}{\partial\zeta}\right)\right\}\left\{\begin{matrix}\bar p_{\dot x}^{(0)}\\ \bar p_{\dot y}^{(0)}\end{matrix}\right\} = \left\{\begin{matrix}12\cos\theta\\ 12\sin\theta\end{matrix}\right\} \quad (53)$$

In the special case of concentric journal ($\bar x_o = \bar y_o = 0 \rightarrow \bar h = 1$) the solution becomes:

$$\left.\begin{matrix}\bar p_{\dot x}^{(1)}\\ \bar p_{\dot y}^{(1)}\end{matrix}\right\} = -\frac{6}{5}\left(1 - \frac{\cosh\zeta}{\cosh(L/D)}\right)\left\{\begin{matrix}\cos\theta\\ \sin\theta\end{matrix}\right\} \quad (54)$$

whereby ($\bar x_o = \bar y_o = 0$):

$$C_{xx} = C_{yy} = \frac{3}{5}\left(1 - \frac{\tanh(L/D)}{L/D}\right)\rho\pi R^2 L(R/c)$$
$$C_{xy} = C_{yx} = 0 \quad (55)$$

For small values of the length-to-diameter ratio this may also be written:

$$\bar x_o = \bar y_o = 0,\ L/D \lesssim 0.5 : C_{xx} = C_{yy} = \frac{1}{5}(L/D)^2\rho\pi R^2 L(R/c) \quad (56)$$

Assuming the lubricant to be oil while the journal material is steel, and setting $L/D = 0.5$ and $R/c = 10^3$, it is found that C_{xx} and C_{yy} equal 6 times the journal mass. This effect could be

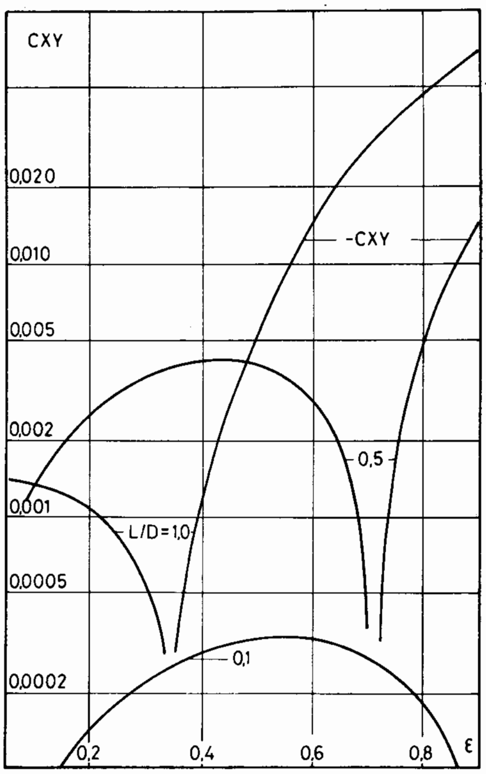

Fig. 11 **The dimensionless cross-coupling acceleration coefficient C_{xy}**

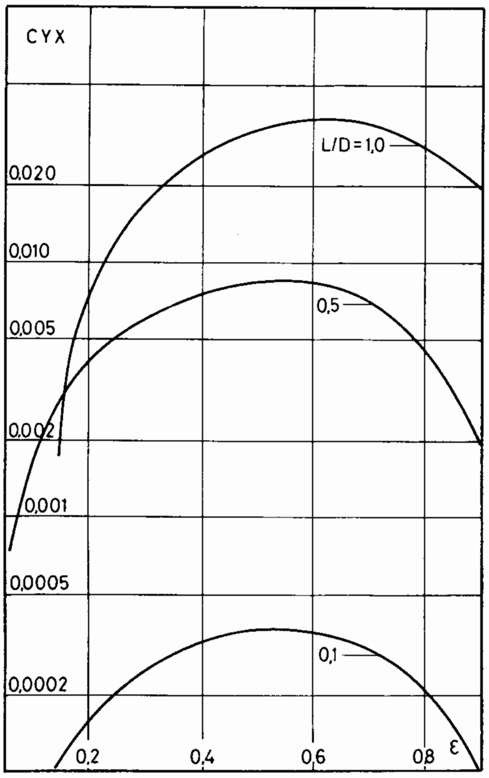

Fig. 12 **The dimensionless cross-coupling acceleration coefficient C_{yx}**

appreciable for small, compact rotors but should be of no concern in larger machines.

If the complete 360-deg film is considered to contribute, the results in equations (55) and (56) should be multiplied by 2 such that the factor 3/5 is replaced by 6/5. This latter value may be compared with the results in [11] where the factor equals 1, derived on the basis of an ideal, inviscid fluid. The same result is obtained in [10] from considerations of continuity of accelerated flow.

Numerical Results

Calculations are performed for a plain cylindrical journal bearing with film rupture. Three values of the length-to-dia ratio are considered: $L/D = 0.1$, 0.5 and 1.0, and the results are obtained as functions of the static equilibrium eccentricity ratio:

$$\epsilon = \sqrt{\bar x_o^2 + \bar y_o^2}$$

As the actual values of the Sommerfeld number and the amplitude and velocity coefficients can be found elsewhere, Figs. 1 to 9 only show the corrections to these quantities. Referring to equation (43) the figures give the corrections as:

$$\text{in Fig. 1} : KSO = \Delta\bar F_{xo}$$

$$\text{in Fig. 2} : KXXK = \Delta\bar K_{xx}$$

and analogously for the remaining figures. Hence, to arrive at the true Sommerfeld number and the true coefficients the values obtained on the basis of the conventional Reynolds equation should be multiplied by ($1 + \lambda \times$ correction). As the maximum allowable value of λ is of the order of 1 to stay within the laminar regime, the figures show that the corrections at most amount to a few per cent. When Figs. 2, 3, 8 and 9 give large corrections as ϵ tends to zero and fig. 5 shows a similar singularity around $\epsilon \simeq 0.7$ the reason is simply that the base coefficients themselves become zero.

While Figs. 1 to 9 must be multiplied by λ before they are applied, Figs. 10 to 12 give the actual acceleration coefficients in the dimensionless form:

$$CXX = \frac{C_{xx}}{\rho\pi R^2 L(R/c)}$$

and similarly for CYY, CXY and CYX. Noticing that $\rho\pi R^2 L$ is the mass of that volume of oil which could be contained in the bearing cavity when the journal is removed, and with R/c being typically of the order of 10^3 it is seen that the acceleration coefficients represent an added mass of several times that of the journal itself. For large, heavy rotors the effect is of no importance, but it could become quite significant for small, short rotors. It should be of particular concern in experiments set up to test for the threshold of instability (oil whip) where the journal frequently is a sizeable part of the total rotor. If no allowance is made for the virtual mass effect under such conditions serious errors could occur where the experimentally observed critical journal mass value would be considerably less than that predicted from a theoretical calculation which does not include the acceleration coefficients.

Conclusions

The analysis and the results confirm that the contribution from the inertial film forces to the load carrying capacity and the dynamic reaction forces of journal bearings is quite limited. The corrections to the regular amplitude and velocity coefficients amount at most to a few percent which, in general, is less than the tolerance errors and uncertainties from other sources. In practice, therefore, these corrections can be ignored.

In addition to the amplitude and velocity coefficients, there are also associated acceleration coefficients as already shown by Smith [10]. They act as a virtual mass and, for a bearing of typical dimensions and design, the effect may be equivalent to an increase of the journal mass by a factor of as much as 5 to 10. While insignificant in larger machines the effect could be pronounced for small, compact rotors.

Special attention should be given to experimental apparatus for testing dynamic response or stability of journal bearings where the journal often is a significant part of the total rotor for reasons of convenience and simplicity. In such cases, disregarding the acceleration coefficients could lead to serious discrepancies in correlating experimental and theoretical results.

Whereas the analysis should account satisfactorily for the inertia forces in the film flow, the effect that these forces may have on the extend and the instantaneous location of the active film domaine has not been explored. Indications are that for sufficiently fast motions of the journal, the film domaine will lag behind which probably influences the dynamic characteristics of the bearing far more than the effects examined in the preceeding analysis. It is hoped that the presented analysis may serve as a first step in a more comprehensive study of this problem.

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