Journal of Vibration and Acoustics

Technical Briefs

Undamped Critical Speeds of Rotor Systems Using Assumed Modes

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Introduction

For axisymmetric rotors with undamped isotropic supports, the whirl modes are circular relative to a fixed reference. These modes are then constant and planar relative to a reference which rotates at the whirl speed. The critical speeds, associated with a specified spin to whirl ratio, $1/\rho = \Omega/\omega$, are then obtained (p. 2.54; Ehrich, 1992) from the $2n$ order eigenvalue problem

$$[K_{q'}]\{\mathbf{q}'\} = \omega^2\left([M_{q'}] + \frac{1}{\rho}[\hat{G}_{q'}]\right)\{\mathbf{q}'\} \tag{1}$$

where n is the number of stations in the discrete physical model. The square arrays in Eq. (1) are symmetric and are respectively referred to as the stiffness, mass, and transformed gyroscopic matrix associated with the rotating reference displacement vector

$$\{\mathbf{q}'\}^t = \lfloor u_1' \ \beta_1' \ u_2' \ \beta_2' \ \ldots \ u_n' \ \beta_n' \rfloor \tag{2}$$

with (u_i' β_i') representing the cross-section (translation, rotation) at a typical station i as illustrated in Fig. 1. For a specified spin/whirl ratio, $1/\rho$, the associated eigenvalues are ω_r^2 (r = 1, 2, . . . , $2n$) with the critical speeds defined as

$$\Omega_r = \frac{1}{\rho}\,|\sqrt{\omega_r^2}\,| \tag{3}$$

for positive values of ω_r^2. Extraneous negative values occur for certain choices of $1/\rho$. This results when the gyroscopic stiffening is strong and prevents a full set of critical speeds (p. 2.29, Ehrich, 1992).

Crandall and Yeh (1986) and Shiau and Hwang (1989) have proposed assumed mode methods for formulating discrete models for rotordynamic systems. Their proposals both apply the assumed displacements directly to the energy functions of the system and then utilize Lagrange's equations to obtain the discrete system equations. Both methods are effective; however, they require the development of a special purpose computer code for general application to industry level problems. Most industries engaged in analyzing rotordynamic systems, have rather sophisticated finite element based codes in place which assemble the discrete rotor equations. For critical speed analyses, these equations reduce to the form of Eq. (1), and can be of fairly high order depending on the modeling assumptions. Thus, it is convenient to modify the proposals of Crandall and Yeh, and Shiau and Hwang, so that they can be applied to the FEM based model of Eq. (1) rather than directly to the energy functions. The result is an optional method for reducing the model order so as to take advantage of related computational savings.

Analytical Procedure

The assumed modes strategy is implemented by introducing the Rayleigh-Ritz transformation (p. 315, Craig, 1981)

$$\{\mathbf{q}'\} = \sum_{j=1}^{N} \{\phi\}_j a_j = [\Phi]\{\mathbf{a}\} \tag{4}$$

where the $\{\phi\}_j$'s are linearly independent assumed-mode vectors. There are many possibilities for choosing the elements ϕ_{ij} of $\{\phi\}_j$, and several possibilities are suggested below. It is assumed in this note that the shaft cross-section rotation is equal to the slope of the shape function associated with the translational constraints. The range of the index i is from 1 to n for each of the shape functions listed below.

(*i*) *Regular Polynomials*

$$\phi_{2i-1,j} = z_i^{j-1} \quad j = 1, 2, \ldots, N \tag{5}$$

The j = 1 and j = 2 terms correspond respectively to a rigid body translation and rotation about the z-origin. Thus, the choice of N = 2 provides a convenient way to convert a flexible model to a two degree-of-freedom rigid body model. This can be quite convenient and is often sufficiently accurate for many applications. A large value of N may result in ill-conditioning and should be an area of concern when utilizing regular pol-

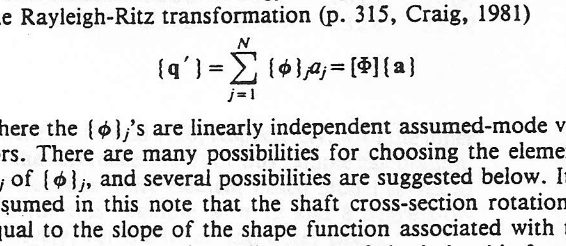

Fig. 1 Whirl reference coordinates

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Contributed by the Technical Committee on Vibration and Sound for publication in the JOURNAL OF VIBRATION AND ACOUSTICS. Manuscript received Mar. 1992; revised June 1992. Associate Technical Editor: D. Inman.

ynomial functions. Selecting the z-origin so as to minimize the distance to the most extreme station of the system is usually helpful in reducing any ill-conditioning problems. This use of regular polynomials was proposed by Shiau and Hwang (1989) with good results. A generalization of their approach would be to utilize Chebychev polynomials (p. 491, Arfken, 1966) which combine useful features of both Fourier series and orthogonality.

(*ii*) *Constraint and Internal Modes*

$$\phi_{ij}=\psi_j(z_i) \quad j=1, 2, \ldots, N_a \tag{6a}$$

$$\phi_{ik}=\gamma_k(z_i) \quad k=N_a+1, 2, \ldots, N \tag{6b}$$

where N_a is the number of attachment stations (constraints). A *constraint mode* is a static displacement shape associated with a unit value of the jth attachment point translation and all other constraints active. For the special case when N_a = 2, the constraint modes are equivalent to rigid body modes. Otherwise, they involve deformation of the rotor and are dependent upon the flexibility characteristics of the rotor. The *internal modes*, $\gamma_k(z)$ are displacement shapes with respect to the attachment points and can be any set of independent geometrically admissible modes (Hale and Meirovitch, 1980). Glasgow and Nelson (1980) utilized natural precessional modes with fixed attachment points in a stability analysis of rotor systems; and Crandall and Yeh (1989) used a set of static deflections under an independent set of loadings. This latter strategy eliminates the need for an eigenvector extraction and substitutes a static analysis which generally requires less computational effort. If N_a is quite large, a small number of internal modes is usually adequate for a model of reasonable accuracy.

(*iii*) *Free-Free Modes*

$$\phi_{2i-1,1}=1 \tag{7a}$$

$$\phi_{2i-1,2}=z_i-L/2 \tag{7b}$$

$$\phi_{2i-1,j+2}=(\cos\beta_jL-\cosh\beta_jL)(\sin\beta_jz_i+\sinh\beta_jz_i)$$
$$-(\sin\beta_jL-\sinh\beta_jL)(\cos\beta_jz_i+\cosh\beta_jz_i)$$
$$(j=1, 2, \ldots, N-2) \tag{7c}$$

The first two functions represent rigid body modes and the remaining shape functions correspond to the flexible free-free modes of a uniform beam (p. 438, Thomson, 1988).

With a choice of assumed-mode vectors, the transformation of Eq. (4) is established. The introduction of this transformation into the eigenvalue problem of Eq. (1) yields the following reduced order eigenvalue problem in terms of the modal displacements a_j (j = 1, 2, . . . , N).

$$[K_a]\{\mathbf{a}\}=\omega^2\left([M_a]+\frac{1}{\rho}[\hat{G}_a]\right)\{\mathbf{a}\} \tag{8}$$

where $[K_a]$ = $[\Phi]^t[K_q'][\Phi]$, etc. This eigenvalue problem provides the system critical speeds Ω_r using Eq. (3), and the associated physical coordinate modal vectors by back-substitution in the transformation of Eq. (4).

Example

A dual-shaft system is used to illustrate the ability of the method to accurately predict critical speeds for general and complicated configurations. This system was previously analyzed by Glasgow and Nelson (1980) using component mode synthesis. A schematic of the system is shown in Fig. 2. Details of the rotor configuration, rotating assembly mass and stiffness distribution, and bearing properties are contained in the referenced paper. The system consists of two flexible rotors interconnected by an intershaft bearing. The operating speed of the high speed (outer) rotor is considered here to be 1.5 times that of the low speed rotor. Spin/whirl ratios for critical speed calculations are then 1.00 and 1.50, respectively, with the inner shaft chosen as the reference shaft for this sample calculation. A 20 degree-of-freedom finite element based model serves as the base system for this example with 12 and 8 degrees-of-freedom, respectively, for the inner and outer shafts.

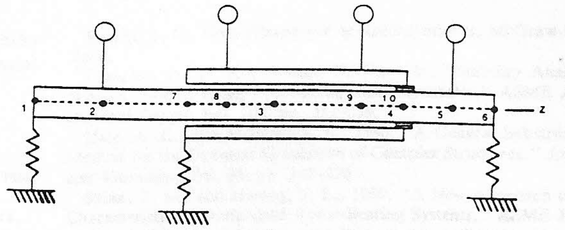

Fig. 2 Dual-shaft rotor configuration

Table 1 Reduced model critical speeds (rad/s): ρ_1 = 1.00, ρ_2 = 1.50 for various types of assumed modes

a. Regular Polynomials
b. Constraint and Internal Modes
c. Free-Free Modes

	N_1/N_2					
Mode	3/2	4/3	5/3	6/4	7/4	8/4
1a	902	900	869	869	867	867
2a	1957	1700	1695	1617	1617	1609
3a	2734	2349	2345	2324	2323	2313
1b	876	874	868	867	866	866
2b	1954	1627	1626	1610	1609	1608
3b	2722	2336	2336	2315	2308	2304
1c	899	897	874	874	867	867
2c	1956	1665	1663	1620	1617	1607
3c	2719	2366	2365	2329	2324	2312

The first three critical speeds are listed in Table 1 below for three different sets of shape functions: regular polynomials, constraint and internal modes, and free-free modes. The number of assumed modes for the inner and outer shafts are denoted by N_1 and N_2, respectively. The degrees-of-freedom of the reduced order model is then N = N_1 + N_2. The internal modes in this example were chosen as the simply supported vibration modes of a uniform beam. The first three critical speeds of this dual-shaft system, from the 20 degree-of-freedom finite element model, were calculated to be 864, 1601, and 2287 rad/s.

The introduction of the assumed-mode approximation to the 20 degree-of-freedom finite element model constitutes the introduction of constraints. Thus, the critical speeds associated with any of the reduced order models are higher than those of the base model. For any of the assumed modes chosen, it is seen that a reasonable number of modes provides quite accurate critical speed predictions in comparison with the base system.

Conclusions

A procedure is presented above to reduce the degrees-of-freedom of a discrete rotordynamics model by utilizing an assumed modes Rayleigh-Ritz approximation. Many possibilities exist for the assumed modes and any reasonable choice will yield a reduced order model with adequate accuracy for most applications. The above reduction procedure provides an option which can be implemented with relative ease and may prove beneficial for many applications where computational efficiency is particularly important.

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