AN INTRODUCTION TO VIBRATION ANALYSIS USING *MATLAB*

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Introduction and Background

The development of the computer from the late 1950's has resulted in more powerful computers of smaller size and of such low cost that extensive computational power is available to engineers at an affordable cost. The recent extensive advances in the power of microprocessors has resulted in a generation of new desktop and portable computers that rival mainframe, and even super-computers, of only a decade ago.

With these high-powered personal computers have also come the development of powerful engineering software programs for use in computer-aided design and manufacturing (***CAD/CAM***) and extensive finite element packages for the analysis of various statics, dynamics, and stress problems of interest to the engineer. Although there is an extensive array of pre-programmed software available to the engineer and scientist for various problems and tasks, this does not preclude his need or desire to perform his own specialized programming tasks.

Computer Languages

Computer languages can be described in terms of levels. The lowest level is machine language, which is closely tied to the computer hardware and is a binary language, since the instructions are written in binary sequences of zeros and ones. An assembly language is used to generate a binary code and also is machine specific. The instructions may resemble written statements instead of binary code, but the language does not have a rich array of commands or statements at its disposal. Thus, writing programs in assembly language can be very tedious and is generally not done by the engineer for normal problem solution. Programs written in assembly language are done in order to achieve the maximum speed of execution. Instrumentation that contains microprocessors are programmed in assembly language in order to run at maximum speed. These programs are called real time programs. For example, a controller for a magnetic bearing needs to run in real time to operate without failure.

Engineers usually write programs in terms of a higher level language such as ***FORTRAN*** (FORmula TRANslation). The initial version of ***FORTRAN*** was developed in the late 1950's for solving engineering and scientific problems. ***FORTRAN*** is a third-generation, high-level language, as compared to machine and assembly language programs. ***FORTRAN*** must first be compiled in order to execute on a computer. ***FORTRAN*** has been constantly updated, with new versions such as ***FORTRAN 77*** and the current standard, ***FORTRAN 90.*** ***FORTRAN 90*** has strong numerical computational capabilities and many of the features and structures found in ***C***. Although ***FORTRAN*** has been the past choice of the scientific community, it lacks many features such as built-in graphical capabilities, ***GUI*** (graphical user interface), and advanced matrix and file manipulation capabilities. In order to enhance the capabilities of ***FORTRAN***, various extensive library subroutines have been developed, such as the IMSL library. The main problem with ***FORTRAN*** is that it is a compiled language, rather than an interactive language, and hence the program must be compiled first, and errors in syntax corrected, before the program may be run. This process can be extremely time consuming and some bugs are difficult to trace and correct.

In order to teach computer programming with greater ease, ***BASIC*** (Beginner's All-Purpose Symbolic Instruction Code) was developed in the mid-1960's as an educational tool. Many PC computers have been supplied with a ***BASIC*** computer language, such as ***GW BASIC*** and ***MS BASIC***. These ***BASIC*** programs are hardly adequate for scientific programming because of their lack of structure and inability to operate with large matrices in a rapid fashion. However, a major advantage of ***BASIC*** is that the language is interpretive in nature. That is, it is computed line by line, and hence one could identify the location of an error in logic and syntax with greater ease. This made debugging extremely easy. However, the normal ***BASIC*** languages lack speed and robustness for consideration as a serious engineering platform. Simultaneous with the development of ***BASIC***, various Computer Science Departments were pushing the teaching of ***ALGOL*** and ***PASCAL*** as teaching languages. These languages had little appeal to the practicing engineer as they were simply lacking in any desirable mathematical or graphical features.

Matrix Theory in Vibration Analysis

Matrix theory had been well established by the 1950's as evidenced by texts such as Hildebrand, 1960. However, from an engineering standpoint, matrix theory had little practical application since it was impossible to compute even small order matrices without the aid of a computer. The idea of inversion of large matrices and eigenvalue extraction was not in the normal toolbox of practicing engineers without the aid of mainframe computers.

In the late 1970's the Hewlett Packard Corp. introduced a remarkable BASIC language which was to be later referred to as Rocky Mountain Basic or simply ***HP BASIC***. This language could multiply and invert matrices. This lead to the possibility of using the numerical and transfer matrix methods of Myklestad, Prohl, and Lund in the direct matrix formulation of rotor dynamics problems. The author presented a paper at the Vibration Institute in 1981 on *Rotor Dynamics on The Minicomputer*. The HP 9845 computer with the HP Basic language used at that time cost over $35,000. Although newer and more powerful and cheaper HP workstations were developed, the ***HP Basic*** was abandoned in favor of ***C*** (***HP BASIC*** , however, is available as ***HT BASIC***.). The abandonment of ***HP BASIC*** with its extensive matrix and graphical commands was felt as a considerable loss to the vibration analysis community.

The ***MATLAB*** (or Matrix Laboratory) language was developed originally for ***UNIX*** in ***C*** and was later ported to the PC computers. The current Windows NT computers with 128Meg of memory are equivalent to mainframe computers of the previous decade (only better since they are more user friendly and in direct control of the engineer). This has given the engineer a new set of tools that may be applied to control theory, finite elements, graphics, as well as vibration analysis. Currently matrix theory is of great interest to the engineer and scientist as it now is easy to apply. The analogy is that one does not need to be a car mechanic to drive a car (a little bit of training, however, does help to keep one from crashing!). One does not need to be an expert in matrix theory to use it. There are currently a number of excellent texts on vibrations and dynamics that make use of matrix theory as listed in the references such as Bathe, Diamarogonous, Genta, and Kramer. Of particular interest is the text by Kwon & Bang on *The Finite Element Method using MATLAB*.

In this paper some ***MATLAB*** examples are presented to illustrate the application of MATLAB to vibration analysis. The examples represent about 1 % of the commands and functions available. The commands presented are sufficient to enable one to solve a large class of problems from forced response, transient analysis and eigenvalue analysis. One may also develop a multi-plane balancing program using the such matrix functions as singular value decomposition.

2. *MATLAB* Language and Syntax

The syntax of ***MATLAB*** is very easy to master if one has familiarity with any of the higher languages, such as ***FORTRAN***, ***BASIC***, or ***C***. For engineers who are used to writing in ***FORTRAN***, a ***MATLAB*** file can be constructed to resemble ***FORTRAN***. However, its power resides in its extensive matrix capabilities. The language is constructed on the ***C*** platform and, hence, users who are familiar with ***C*** or ***C++*** will recognize some of the more advanced constructs of the language. For simple problems, program commands may be generated at the ***MATLAB*** prompt. Since the program is interpretive, an answer is given immediately. However, the real power of ***MATLAB*** resides in the development of m files. The m file is simply an ASCII, or script, file representing a set of commands with an extension of .m, instead of .txt for the normal text file. All ***MATLAB*** script, or function files have the extension .m.

Common Functions

Table 2.1 represents some common functions used in ***MATLAB***. The variable x may represent a real or an imaginary number. In addition to the standard functions familiar to those used in ***FORTRAN*** and ***BASIC***, there are certain interesting functions in ***MATLAB*** that can operate on complex numbers. These are ABS, ANGLE, REAL, and IMAG. These functions return the absolute value of the complex number, the angle, and real and imaginary components. The ability to deal with complex numbers makes the language particularly suitable for vibration analysis and control theory. When entering a complex number with a real component of five and an imaginary component of ten, one could use the following:

```
A = 5.0 + 10.0 i
      or
A = 5.0 + 10.0*j
```

Note that either i or j could be used to indicate the imaginary value. Lower case must be used. Complex numbers may be multiplied or divided, similar to real numbers.

Table 2.2 represents more advanced functions and operations. The stated functions, as shown in Table 2.2, will provide the significant tools required for vibration analysis of a large class of problems. Since most of the listed functions are m files, the functions may be viewed in detail and even modified to suit a particular application.

Special Variables and Commands

ans - default name used for results
pi - default value = 3.1416
eps - smallest number when
whos - list of variables and sizes of arrays
clear - clears variables from work space
print - dwinc - send figure to color printer
help 'topics' - gives help on listed topics
diary (*filename*) - saves <u>all</u> terminal input and output to *filename*

Simple Matrix Commands

The ability to generate and manipulate matrices is the essence of ***MATLAB*** and is what makes the language so powerful and attractive to use for engineering analysis. A few simple matrix commands will be given to illustrate the ability to deal with matrices. The symbol >> represents the interactive ***MATLAB*** command window.

```
>> A = -3:3                                          (specify array from -3 to 3)
     A =  -3  -2  -1  0  1  2  3
>> A = zeros (3)                                     (specify 3×3 matrix of zeros)
     A = 0  0  0
         0  0  0
         0  0  0
>> A = [  1  -2  1  ;  -2  3  -2  ;  1  -2  4  ]    (specify a 3×3 matrix)
     A = 1  -2  1
        -2   3 -2
         1  -2  4
>> det (a)                                           (determinant of a)
     ans = -3
>> b = inv(a)                                        (compute inverse matrix a)
     b = -2.6607   -2.0000   -0.3333
        -2.0000   -1.0000         0
        -0.3233         0    0.3333
>> c = a*b                                           (verify c = identity matrix)
     c = 1.0000         0         0
              0    1.0000         0
              0         0    1.000
>> RT = [  1  4  1  ]                                (generate row vector)
>> R = RT'                                           (transpose of RT to generate a column vector)
     R =  1
          4
          1
>> X = A \ R                                         (solution of R = A * X)
     X = -11
         - 6
           0
>> X = inv(A) * R                                    (solution of R = A * X by inversion of A matrix)
     X = -11
         - 6
           0
```

Table 2.1 - Common Functions	
angle(z)	Angle of complex vector z, radians
abs(x)	Absolute value or magnitude of complex number
acos(x)	Inverse cosine
acosh(x)	Inverse hyperbolic cosine
angle(x)	Four-quadrant angle of complex number
asin(x)	Inverse sine
asinh(x)	Inverse hyperbolic sine
atan(x)	Inverse tangent
atan2(x,y)	Four-quadrant inverse tangent
atanh(x)	Inverse hyperbolic tangent
ceil(x)	Round toward plus infinity
conj(x)	Complex conjugate
cos(x)	Cosine
cosh(x)	Hyperbolic cosine
det(A)	Determinant of square matrix A, scaler
exp(x)	Exponential: e^x
fix(x)	Round toward zero
floor(x)	Round toward minus infinity
gcd(x,y)	Greatest common divisor of integers *x* and *y*
imag(x)	Complex imaginary part
lcm(x,y)	Least common multiple of integers *x* and *y*
log(x)	Natural logarithm
log10(x)	Common logarithm
max(x)	Maximum value of vector x
real(x)	Complex real part
rem(x,y)	Remainder after division Rem(x,y) gives the remainder of x/y
round(x)	Round toward nearest integer
sign(x)	Signum function: return sign of argument, e.g., sign(1.2)=1,sign(-23.4)= -1,sign(0)=0
sin(x)	Sine
sinh(x)	Hyperbolic sine
sqrt(X)	Square root
tan(x)	Tangent
tanh(x)	Hyperbolic tangent

Table 2.2 - Advanced Functions and Operations

Plot(x,y) - plots arrays x and y in a graph and scales both x and y coordinates. X and y must be the same dimension

y = det(A) - calculates the determinate of a matrix and returns scales value

b = Inv(A) - computes inverse of a matrix

xt = X′ - computes transpose of x

X = A/R - computes solution of n simultaneous equations with {R} = [A] {X} where [A] may be real or imaginary

[V, D] = eig (A,B) - solves the general eigenvalue problem of A * V = B * V * D where A and B are square matrices
V = eigenvectors and D is a diagonal matrix of the eigenvalues
or

[V, D] = eig (A) - solves the eigenvalue problem of
A * V = V * D

[U,S,V] = svd(A) - singular value decomposition of n×m A matrix
S = diagonal matrix of singular values
May be used for least squared error solutions and more accurate eigenvalue compositions over QR

Y1 = Interp1 (x,y,x1, 'cubic') - Generates an interpolated set of points y1 for given x1 vector using cubic spline interpolation

a = polyfit (x,y,n) - Generates the coefficients for an nth order polynomial such that $f(x) = a_o X^n + a_1 X^{n-2} + \ldots a_{n-1} X + a_n$

Y = polyval (a,x) - computes values of y for vector x corresponding to polynomial with coefficients a

Y = quad8('F',A,B) - Numerical evaluation of integral F from A to B using 8-panel Newton-Cotes 8-panel method

Y = FFT(x,n) - Performs FFT analysis over vector x with n points

[T,Y] = ODE23('F',Time,Yo) -Integrates the equations Y′ = F(t,y) with initial condition Yo

3 *MATLAB* Applications For Vibration Analysis

In this section some simple examples will be presented on the use of MATLAB for vibration analysis. The examples presented here are given in order to illustrate some of the power and ease of use of the ***MATLAB*** command structure. One of the problems facing anyone attempting to use a new programming language is the overwhelming number of commands available and the syntax of the language. However, to perform many of the basic computations involved in vibration analysis, one may be very effective with the use of only 1% of the commands available to the programmer.

3.1 Fourier Approximation of a Square Wave

In Example 3.1, a square wave is approximated by a Fourier series of 3 and 5 terms. It is a simple procedure to extend the analysis to 10 or 20 terms by means of a recursion loop. The equation for the square wave is given by:

$$x(t) = \frac{4}{\pi} \times (\cos(\omega * t) - \frac{\cos(3\omega * t)}{4} + \frac{\cos(5\omega * t)}{5} - \frac{\cos(7 * \omega * t)}{7} + ...)$$

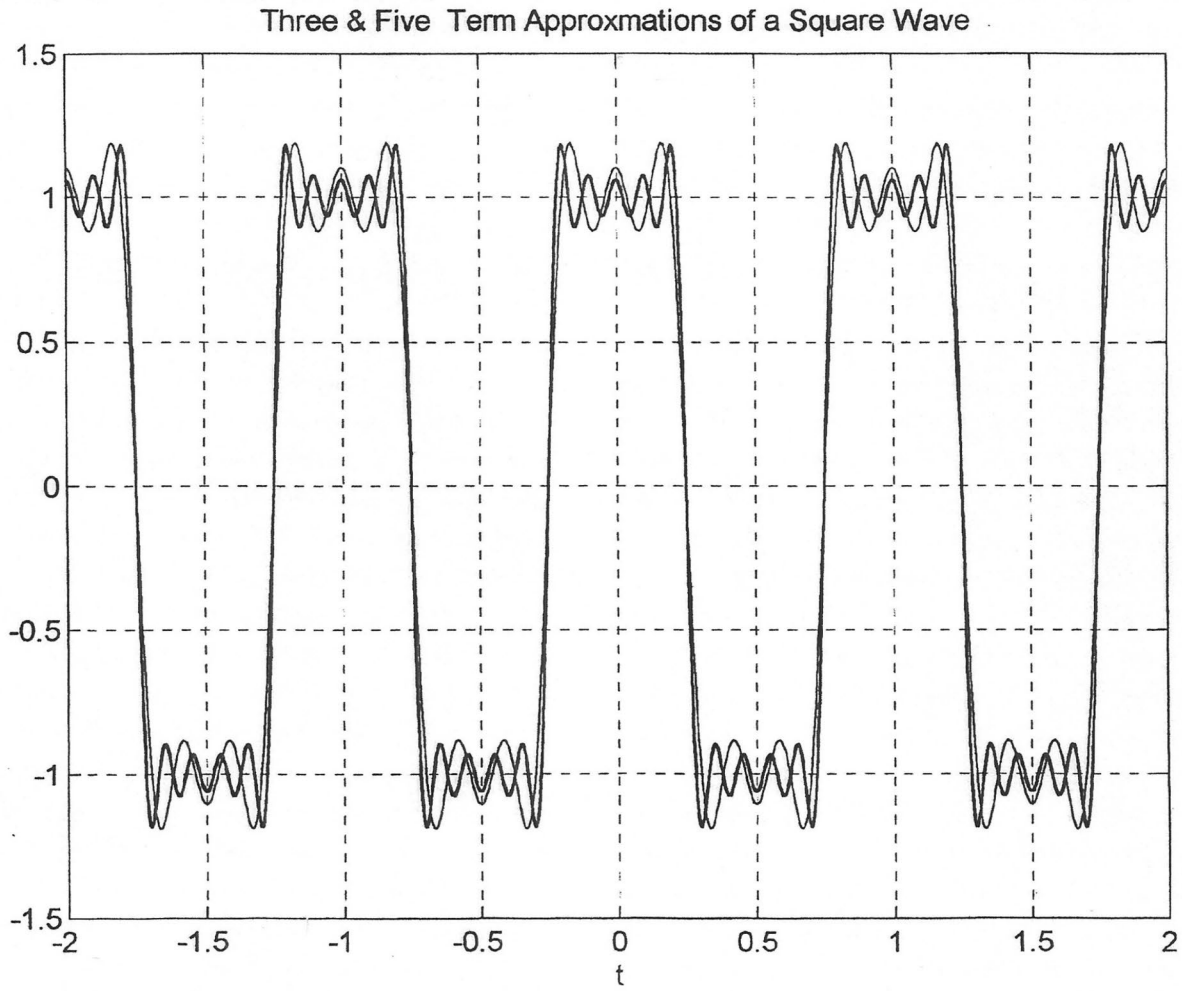

Figure 3.1 Three and Five Term Approximation of a Square Wave

Table 3.1 Fourier Series Script File

```
% sqrt3t.m
% Plot Fourier approximation to a square wave
t=-2:   0.005   :   2;          % generate t vector with 801 elements
omega=2*pi;                     % pi is 3.1416 built in function
x1= cos (omega*t) ;             % calculate x1 vector with 801 elements
x2=-cos(3*omega*t) /3;
x3= cos (5*omega*t) /5;
x =4*(x1  +x2  +x3) /pi;
plot(t,x, '-' ),grid            % plot and scale the t vector vs x
title('Three & Five Term Approximations of a Square Wave')
xlabel ('t')
hold on                         % hold figure for a second plot

x4=-cos(7*omega*t) /7;
x5=cos(9*omega*t) /9;
x7=4*(x1+x2+x3+x4+x5) /pi;
plot(t,x7,'linewidth',1)
```

Table 3.1 represents the script file for the Fourier series representation of the square wave. This simple file illustrates the generation of a vector t from -2 to 2 with steps of 0.005, having 801 elements. Therefore, the computations for x1, x2, and x3 represent vectors with the same dimension as t. These elements also have 801 elements. The vectors of x1, x2, and x3 are automatically generated by the calculations involving the vector t. Thus, the programmer does not need to specify the dimension statement.

The graph is generated simply by the plot command of ***plot (t,x), grid*** to generate a plot with a grid. The scaling and labeling is automatically done. By using the command ***hold on***, the graph is kept open and a second plot is generated. A title may be added with the use of the ***title*** command. Labels along the x and y axes are generated by ***xlable*** and ***ylabel*** statements, respectively. If one wishes to change the scaling in the x and y directions, one would use the command ***axis*** (***[xmin, xmax, ymin, ymax]***). For example, if one wished to plot the x axis between -1 and +1 and the y axis from -2 to +2, the corresponding statement would be ***axis*** (***[-1, 1, -2, 2]***). The capabilities of ***MATLAB*** are such that one can use it for generating graphs, instead of using a standard plotting program. There are many advantages to using ***MATLAB*** for graphics and plotting of functions, as it is possible to plot in various colors, change line thickness, interpolate or curve fit experimental data, or numerically differentiate or integrate the area under the curves.

A value at any point on the curve may be obtained by the command

[x,y] = ginput(n) ;

where:

n = no. of desired points

[x,y] = x, y vectors of coordinates of points

3.2 Forced Response of Single-Mass Systems

In this example, the forced response of a single degree-of-freedom system is presented. The equations of motion for the system is given by

$$M\ddot{X} + C\dot{X} + KX = F_o \cos \omega t$$

Figure 3.2 represents the dimensionless amplitude and dimensionless forces transmitted for a range of $\xi = 0.005$ to 0.5.

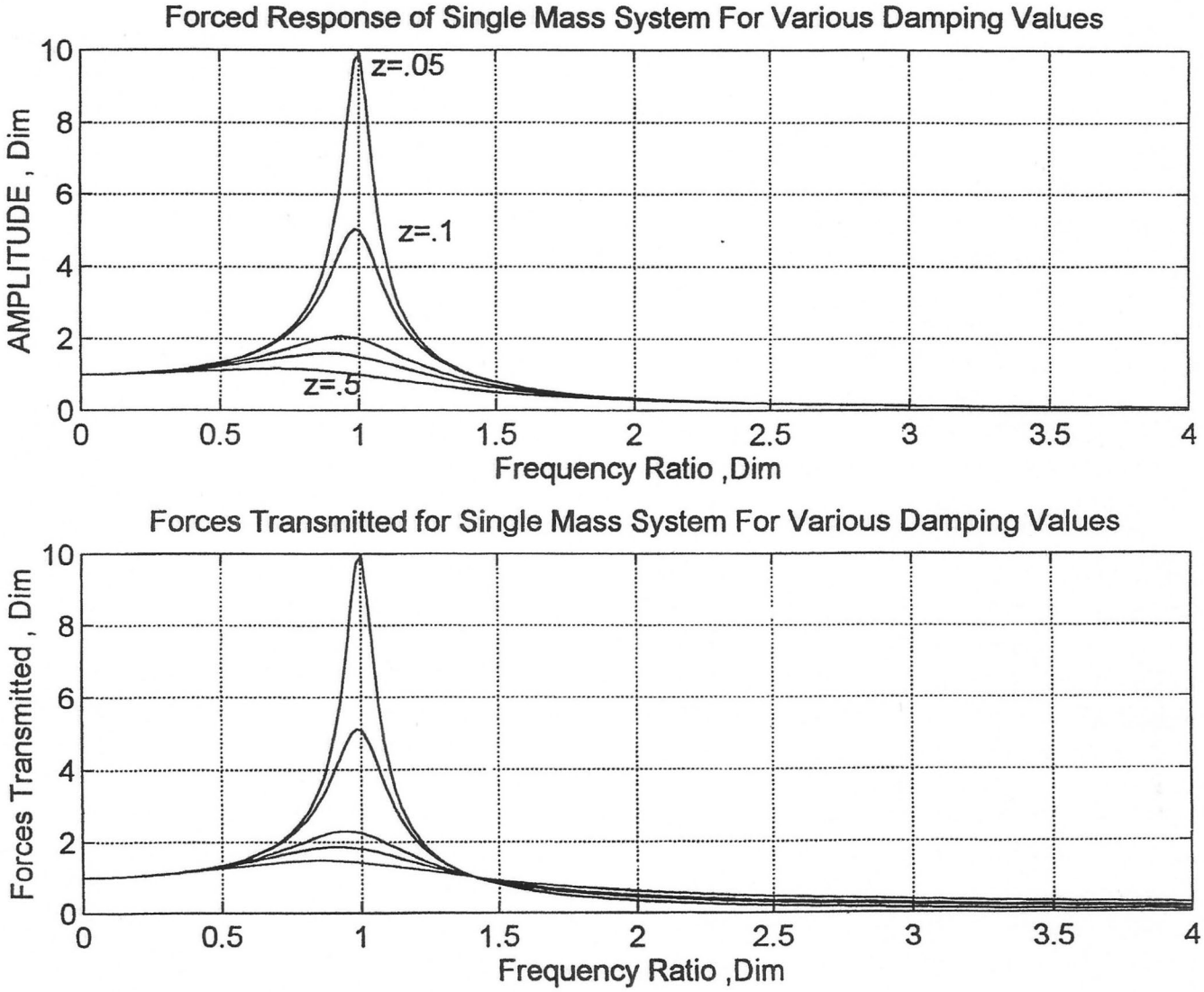

Figure 3.2 Forced Response and Transmitted Forces for a Single Degree-of-Freedom System

Table 3.2 Single Degree-of-Freedom Unbalance Response Script File

```
% Unbalance Response of Single Degree of Freedom System
% Constant Forcing Function
% E.J. Gunter Dept of Mech & Aerospace Engineering
% A = amplitude vector for the various damping cases
     % TRD = dynamic transmissibility = Ft/Fo

     zeta=[0.05  0.1  0.25   0.333   0.5];
     %freq=0:0.05:2;
     freq=linspace(0,4,200);

     for k=1:5;
           z=zeta(k);
        for l=1:200 % frequency sweep
             f=freq(l) ;
             f2=f*f;
             n1=(1.0 -f2)^2;
             n2=(2*z*f)^2;
             d =sqrt(n1 +n2);
             A(k,l)= 1.0/d  ; %  amplitude
             n=sqrt( 1 + n2);
             TRD(k,l)=n/d;
        end;
     end;
     subplot(2,1,1)
      plot(freq,A)
      grid
      xlabel(' Frequency Ratio ,Dim');
      ylabel('AMPLITUDE , Dim');
      title('Forced Response of Single Mass System For Various Damping
      text(.8,.75, 'z=.5');
      text(1.15,5,'z=.1');
      text(1.05,9.6,'z=.05')

pause
subplot(2,1,2)
plot(freq,TRD);
hold on
set(gca,'DefaultTextFont','Times','DefaultFontSize',14)
grid
xlabel(' Frequency Ratio ,Dim');
ylabel('Forces Transmitted , Dim');
title('Forces Transmitted for Single Mass System For Various Damping
```

In Table 3.2, the calculation for the response was computed by using two nested **do** loops for k = 1:5 for the five values of damping, and l = 1:200 for the 200 values of the frequency. With ***MATLAB***, one or more of the **do** or **for** loops may be condensed, but for those familiar with ***FORTRAN***, one can write the program to be similar to ***FORTRAN***. This procedure is recommended until one becomes more familiar with the compact ***MATLAB*** matrix statements.

In Figure 3.2, two plots are generated using the **subplot** command. **Subplot(2,1,1)** represents the first plot of two. The first plot is in the first column and first row. **Subplot(2,1,2)** represents the second plot in the second row. By using the **subplot** command, one may have as many plots on a page as desired. It should be noted that with the **set** command, one may change the font type and size.

Table 3.2 Single Degree-of-Freedom Unbalance Response Script File

```
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% Constant Forcing Function
% E.J. Gunter Dept of Mech & Aerospace Engineering
% A = amplitude vector for the various damping cases
     % TRD = dynamic transmissibility = Ft/Fo

     zeta=[0.05  0.1  0.25   0.333   0.5];
     %freq=0:0.05:2;
     freq=linspace(0,4,200);

     for k=1:5;
           z=zeta(k);
        for l=1:200 % frequency sweep
             f=freq(l) ;
             f2=f*f;
             n1=(1.0 -f2)^2;
             n2=(2*z*f)^2;
             d =sqrt(n1 +n2);
             A(k,l)= 1.0/d  ; %  amplitude
             n=sqrt( 1 + n2);
             TRD(k,l)=n/d;
        end;
     end;
     subplot(2,1,1)
      plot(freq,A)
      grid
      xlabel(' Frequency Ratio ,Dim');
      ylabel('AMPLITUDE , Dim');
      title('Forced Response of Single Mass System For Various Damping
      text(.8,.75, 'z=.5');
      text(1.15,5,'z=.1');
      text(1.05,9.6,'z=.05')

pause
subplot(2,1,2)
plot(freq,TRD);
hold on
set(gca,'DefaultTextFont','Times','DefaultFontSize',14)
grid
xlabel(' Frequency Ratio ,Dim');
ylabel('Forces Transmitted , Dim');
title('Forces Transmitted for Single Mass System For Various Damping
```

In Table 3.2, the calculation for the response was computed by using two nested **do** loops for k = 1:5 for the five values of damping, and l = 1:200 for the 200 values of the frequency. With ***MATLAB***, one or more of the **do** or **for** loops may be condensed, but for those familiar with ***FORTRAN***, one can write the program to be similar to ***FORTRAN***. This procedure is recommended until one becomes more familiar with the compact ***MATLAB*** matrix statements.

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3.3 Tuned Vibration Absorber

Figure 3.3 represents a schematic figure of a tuned vibration absorber shown in Thomson, *Theory of Vibration With Application*, page 167. Although the system has only two degrees-of-freedom, the technique used in the solution is applicable to multiple degrees-of-freedom systems with damping.

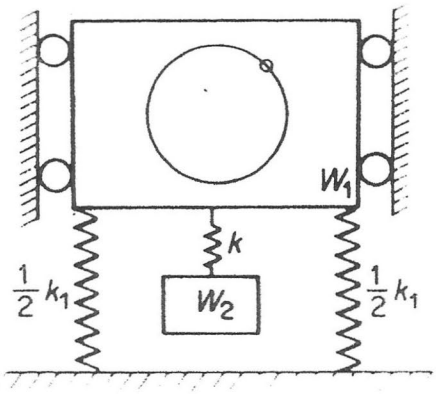


Figure 3.3 Tuned Vibrations Absorber
(*Thomson*)

In the problem shown by Thomson for the vibration absorber, no damping is included. This is easily accomplished by first writing the kinetic, potential, and dissipative energies of the system.

$$T = \tfrac{1}{2} M_1 \dot{X}_1^2 + \tfrac{1}{2} M_2 \dot{X}_2^2$$

$$V = \tfrac{1}{2} K_1 X_1^2 + \tfrac{1}{2} K_2 (X_2 - X_1)^2 \qquad (3.3.1)$$

$$D = \tfrac{1}{2} C_1 \dot{X}_1^2 + \tfrac{1}{2} C_2 (\dot{X}_2 - \dot{X}_1)^2$$

The system equations of motion may be generated by LaGrange's Equation of Motion, shown below in its most generalized form, including the Rayleigh dissipation function and generalized forcing function.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{X}}\right) - \frac{\partial T}{\partial X} + \frac{\partial D}{\partial \dot{X}} + \frac{\partial V}{\partial X} = F_x \qquad (3.3.2)$$

Application of LaGrange's Equation to (3.3.1) leads to the generalization of two coupled equations in X_1 and X_2, as follows:

$$M_1 \ddot{X}_1 + (C_1 + C_2)\dot{X}_1 - C_2 \dot{X}_2$$
$$+ (K_1 + K_2) X_1 - K_2 X_2 = U_1 \omega^2 e^{i\omega t} \qquad (3.3.3)$$

The general matrix form of the coupled linear equations of motion is

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \omega^2 \{U\} e^{i\omega t} \qquad (3.3.4)$$

Let $X(t) = X e^{i\omega t}$ for harmonic motion. The complex equations for forced response are

$$\left[[K] - \omega^2 [M] + i\omega [C] \right] \{X\} = \omega^2 \{U\} \qquad (3.3.5)$$

or

$$[A]\{X\} = \{R\} \qquad (3.3.6)$$

$$\text{Where } [A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\text{and } X^t = [X_1 \ , \ X_2]$$

For the two degree-of-freedom problem shown with included damping, the complex *a* coefficients are given by

$$a_{11} = K_1 + K_2 - \omega^2 M_1 + i\omega (C_1 + C_2) \ ; \ a_{12} = -K_2 - i\omega C$$

$$a_{21} = a_{12} \ ; \ a_{22} = K_2 - \omega^2 M_2 + i\omega C_2$$

Table 3.3 represents the script file for the dynamic vibration absorber. The system is assumed to have a natural frequency at 1,850 RPM. The tuner is designed to be in resonance at 1,800 RPM, which is the operating speed of the rotor. The weight of the absorber is 50 lb and the weight of the rotor is 200 lb. The response is completed using two **for** loops for clarity to make the file similar to ***FORTRAN***. The computation of the absolute value of the complex motion is achieved by using the function **abs**.

Table 3.3 Vibration Absorber Script File

```
% V2dABS.M  Vibration Absorber For Rotor Operating at 1800 rpm
% Prob 5.37 pg 151 Thomson
% Rotor Weight W1 =200 lb , W2=50 lb absorber weight
% Absorber tuned for 1800 RPM
% k1 value tuned for critical speed at 1850 RPM
% c2= absorber damper
clf;  clear
W1=200;       W2=50;        % Main weight & absorber weight
m2=W2/386;    m1=W1/386;
c2=[ 2  5  10   20  50 ]; %  Values of damping-Lb-Sec/In
Ntuned=1800  ;              %  absorber frequency tuned to running speed
om2=(Ntuned/9.55)^2  ;     %
k2= m2*om2             ;    %  Absorber spring rate tuned to 1800 RPM
omcr=1850/9.55         ;
k1=m1*omcr^2          ;  % k1= major spring rate system tuned to 1850 RPM
um=2.0/386              ;          % lb-in/g unbalance
freq=linspace(1000,3000,200);   % Starting speed=1000 ,final=3000,200 cases
for n=1:5;                      % Loop thru 5 damping  values
   for l=1:200;                 %  Loop thru 200 speed cases
      omega=freq(l)/9.55 ;      % Speed in Rad/sec
      omega2=(freq(l)/9.55 )^2    ;    %
      fu=omega2*um          ;    % Rotating unbalance force , Lb
      a(1,1)= k1 + k2 -m1*omega2 + i*omega*c2(n) ; % Computer complex coef
      a(2,1)=-k2 -i*omega*c2(n)    ;
      a(1,2)= a(2,1)      ;
      a(2,2)= k2-m2*omega2+ i*omega*c2(n)  ;  % influence coefficient matr:
      b=inv(a);                          % Invert complex matrix
      z1(n,l)=b(1,1)*fu          ;
      z1mils(n,l)=abs(z1(n,l))*1000 ;
      z2(n,l)=b(2,1)*fu  ;
      z2r(n,l)=z1(n,l)-z2(n,l)      ;   % Relative absorber motion
      z2rmils(n,l)=abs(z2r(n,l))*1000; % Amplitude in mils
      z2mils(n,l)=abs(z2(n,l))*1000;
      ftr=z1(n,l).*k1    ;
      ftrans(n,l)=abs( ftr)           ;     % force traaansmmmitted
  % trd(n,l)=k(n)/kd ; %  ftran(n,l)./fu  ; % dynamic traansmissibility
    end;
end;
subplot(2,1,1)
plot(freq,z1mils,'linewidth',1), grid
xlabel('Rotor Speed, RPM' )
ylabel( 'Amplitude , Mils')
title('RESPONSE With VARIOUS TUNED ABSORBER DAMPING VALUES')
legend('C2=2','C2=5','C2=10','C2=20','C2=50')
axis([1000  3000  0  800])

subplot(2,2,1)
plot(freq,ftrans,'linewidth',1), grid
xlabel('Rotor Speed, RPM' )
ylabel( 'Forces Transmitted , Lbs')
title(' FORCES TRANSMITTED  With Various ABSORBER DAMPER RATES')
axis( [ 1000  3000  0  3000 ] )
```

The solution to the complex vector displacements {X} is given by

$$\{X\} = [A]^{-1} \{R\} \tag{3.3.7}$$

In ***MATLAB***, this is performed either by direct inversion, such as the statement

$$X = Inv\ (A) * R$$

or

$$X = A / R$$

In the first method of solution, an inversion of the complex matrix A is performed similar to the theoretical solution as shown in Eq. (3.3.7). Such a solution method can be time consuming for a system with hundreds of degrees-of-freedom. A quicker solution is obtained by the second method. In this method (shown with the backslash), an **LU** decomposition is performed, rather than a direct inversion of the A matrix. This procedure is very efficient and may be used on systems with thousands of degrees-of-freedom. This is the procedure of choice used in the majority of large finite element system solvers.

Figure 3.4 represents the motion of the machine or rotor mass over a speed range with the vibration absorber tuned to 1,800 RPM.. The response was run with five cases of damping, ranging from 2 to 50 lb-sec/in damping in the absorber. Without any damping in the absorber, the motion at 1,800 RPM would be zero. However, the mass of the absorber M_2 creates resonance frequencies below and above the tuned absorber frequency of 1,800 RPM. The new frequencies are approximately 1,430 RPM and 2,330 RPM. For the case of no absorber damping, the amplitude of motion at these two resonance frequencies would be unbounded. These responses become finite with absorber damping applied.

As the absorber damping is increased, the amplitude of motion at the tuned frequency of 1,800 RPM increases. Therefore, the introduction of absorber damping is a compromise in order to limit the motion at 1,430 and 2,330 RPM. There are two speeds in which the amplitude of motion is independent of damping. These two speeds are at 1,500 RPM and 2,300 RPM. These two points were referred to as the P and Q points for an absorber as first described by Den Hartog. An optimum damping for the system could be chosen such that the slope is zero through the first (or P) point. This would require a damping of about 12 lb-sec/in in the absorber. As the absorber damping exceeds 50 lb-sec/in, the damper becomes locked up. The new natural frequency of the locked-up system drops to 1,650 RPM due to the added mass of the absorber. Damping values in excess of 50 lb-sec/in would result in large amplitudes of motion and bearing forces transmitted.

Figure 3.5 represents the forces transmitted to the foundation. At running speed, the unbalance force is 184 lb. Note that with only 2 lb-sec/in absorber damping, the transmitted force at the 1st resonance is over 2500 lb. However if the absorber could be controlled magnetically, then the absorber could be locked until the P point is reached. It is then unlocked until the Q point is reached. At speeds above the Q point the dynamic vibration absorber is again locked up before it moves into the second resonance speed. The transient motion of the rotor and absorber at P & Q points due to unlocking and locking may be studied by the method outlined in the next section.

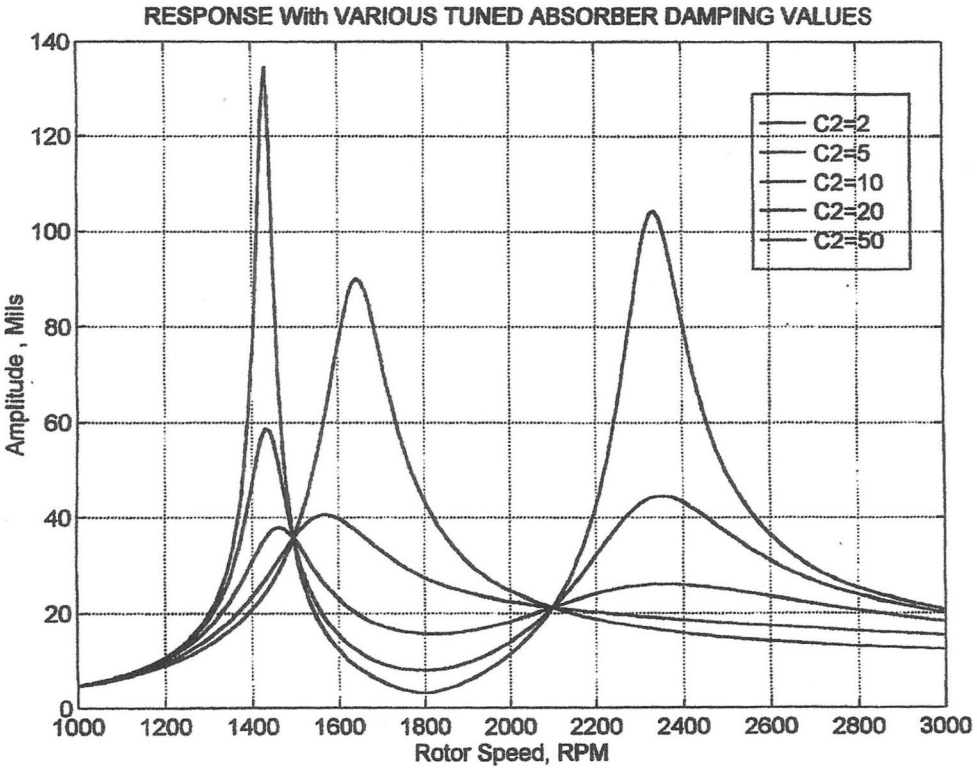


Figure 3.4 Amplitude of Rotor vs. Speed With Tuned Vibration Absorber for Various Values of Absorber Damping

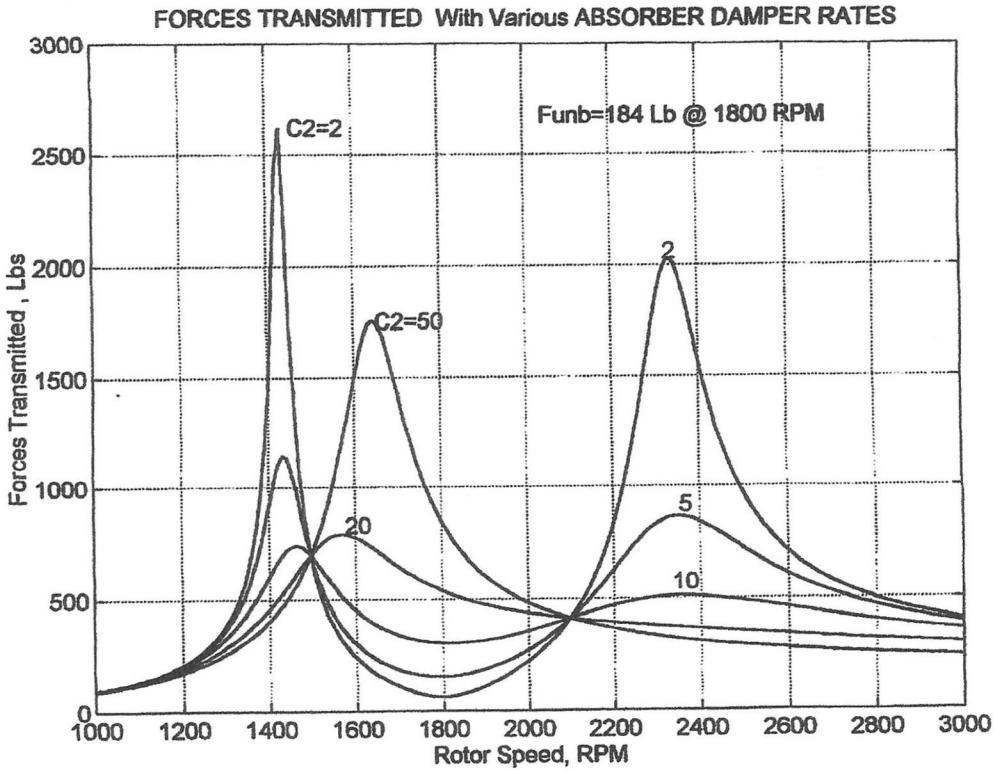


Figure 3.5 Forces Transmitted vs. Speed For Various Values of Absorber Damping

3.4 Transient Response of Vibration Absorber

The initial transient motion of the absorber may be computed by writing the equations in state space formulation with four coupled first order equations in Y as follows

$$\dot{Y} = f\,(Y,t) \tag{3.4.1}$$

The derivatives of Y are given by

$$\dot{Y} = \{\, y\, d\,\} = [\ \ddot{X}_1\ ,\ \dot{X}_1\ ,\ \ddot{X}_2\ ,\ \dot{X}_2\ ] \tag{3.4.2}$$

and the Y vector is given by

$$Y = [\ \dot{X}_1\ ,\ X_1\ ,\ \dot{X}_2\ ,\ X_2\ ] \tag{3.4.3}$$

MATLAB has a number of numerical integration procedures. In this example, the numerical integration function ***ODE23*** will be used. This is a low order numerical integration routine based on a 2nd order Runge-Kutta integration method. For more information on this and other numerical methods, at the ***MATLAB*** >> prompt type

>>help ODE23

In order to apply the function ***ODE23*** or any of the other numerical integration procedures, a separate function file must be generated to have the derivatives of Y. The function file, called absorber.m is shown in Table 3.4. The first line of the function file must start with the word "function" and the file must be saved corresponding the function call. In this case, the file is saved as absorber.m. The function call differentiates the function file from the main body, which is a script file. The majority of the function calls of ***MATLAB*** are ASCII files that may be modified.

Table 3.4 represents the functional file, which is stored as absorber.m. Note the important statements

[rows, cols] = size (y)
yd = zeros (rows, cols)

These statements set dimension size and initialize the derivative matrix yd. The values of y(1) through y(4) are not single-valued functions, but are variable vectors with the dimension of t. As the absorber function is called, the variables are increased in dimension. The number of rows in the matrices yd and y are determined by the number of time steps. One has only to set the initial time (0) and the final time (1 sec) and the increment and number of time steps are automatically computed. This is in considerable contrast to a ***FORTRAN*** program in which the size of the matrices must be initially set. This variable or dynamic dimensioning of the matrices is one of the powerful features of ***MATLAB***. The author has encountered on many occasions the statement of ***DIMENSION OUT OF BOUNDS*** in a FORTRAN transient analysis program.

Table 3.4 Function File for Absorber

```
function yd=absorber(t,y)
% 2 mass system vibration absorber,M1 main mass, K1 main stiffness
% M2 & K2 tuned to running speed at 1,800 RPM (omega=194 rad/sec)
% Ncr = system natural frequency at 1,850 RPM
% y(2)=X1 motion ,   y(4)= X2 absorber absolute  motion
% y(1)=X1 velocity,  y(3)= X2 absorber absolute  velocity
%global M1  M2  K1  K2  C1 C2  omega  Fu
M1=200/386 ;  M2=50/386;
C1=0.05;                        %Damping in rotor
C2=2  ;                         %Relative damping in absorber
K1 = 19444      ;               % System spring lb/in-resonance at 1850 RPM
omega=1800/9.55 ;               % Operating speed, rad/sec
K2=M2*omega^2 ;                 % Absorber spring tuned to 1800 rpm
Fu=200;                         % Applied Forcing function, Lb
[rows,cols]=size(y);            % * Set Matrix size, t rows, 4 columns
yd=zeros(rows,cols);            % * Set size matrix

% Specify the 1 st order functional form
yd(1)=-((C1+C2)*y(1)-C2*y(3) +(K1+K2)*y(2)-K2*y(4))/M1+Fu*sin(omega*t)/M1;
yd(2)=  y(1);      % X1 velocity
yd(3)=-( C2*( y(3)-y(1) ) + K2*( y(4) -y(2) ) )/M2; % X2 acceleration
yd(4)=y(3);        % X2 velocity
% end absorber function
```

Table 3.5 represents the main program or script file that calls the numerical integration procedure ODE23. Note that ODE23 calls the function absorber which contains the system first order derivatives. The response is plotted for one sec of motion, as shown in Figure 3.6. The first plot, using the ***subplot*** command, is for the motion of the major mass. The array y (:;2) represents the second column, which is the motion of X_1 for all the time values. In Figure 3.6, the top figure represents the major mass.

It is seen that the maximum amplitude of motion after two cycles is approximately ± 20 mils. After one sec of motion, the amplitude reduces to ± 4 mils. In this example, an absorber damper value of $C_2 = 2$ lb-sec/in was assumed. Although the steady state response of the rotor mass is less than 4 mils, it is seen that the sudden application of the dynamic load may cause excessive amplitudes of motion. Therefore the final damping selected for the absorber may be a compromise based on range of operation and transient considerations.

The second figure represents the absolute motion of the absorber. Note that after two cycles of motion, the absorber peak amplitude is over ± 70 mils of motion. Thus, the absorber transient motion is almost four times as great as the main mass. After one sec of motion, the system is approaching steady-state conditions in which the absorber is moving with ± 40 mils of motion. In Fig 3.6 (b) there appears to be a beating motion caused by the suddenly applied unbalance. This nature of the beating motion may be evaluated by performing an FFT analysis on the absorber.

Table 3.5 Script File Tranabs.m for Transient Analysis

```
% tranabs.m transient motion of two mass absorber with suddenly
% applied unbalance
% 11/19/96   E.J. Gunter
% y(2)= x1 main mass motion, y(4)= x2 absorber motion
% global M1  M2  K1  K2  C1  C2  omega  Fu
clf
clear
M1=200/386 ;  % main mass
M2=50/386  ;
C1=0.05    ;
C2=2.0     ;
N= 1800 ; omega=N/9.55 ;  % speed rad/sec
K2=M2*omega^2  ;          % tuned absorber to running speed
K1=19444       ;          % resonance at 1850 RPM original
Fu=200         ;

t0=0.0  ;  tf= 1.0  ;    % initial & final time
% set initial conditions as column vector
y0 = [0.0 ; 0.0 ; 0.0 ; 0.0] ;   % v1 , x1  ,v2 ,x2
[t,y]=ode23('absorber',t0,tf,y0) ;  % Call Numerical integrater

subplot(2,1,1)
plot(t,y(:,2)*1000,'linewidth',1.5),grid
xlabel('Time, Sec')
ylabel('Amplitude, Mils')
title('Motion With Suddenly Applied Unbalance With Tuned Absorber,C2=2')

subplot(2,1,2)
plot(t,y(:,4)*1000,'linewidth',1.5),grid
xlabel('Time, Sec')
ylabel('Amplitude, Mils')
title('Absorber Motion With Suddenly Applied Unbalance ,C2=2')
```

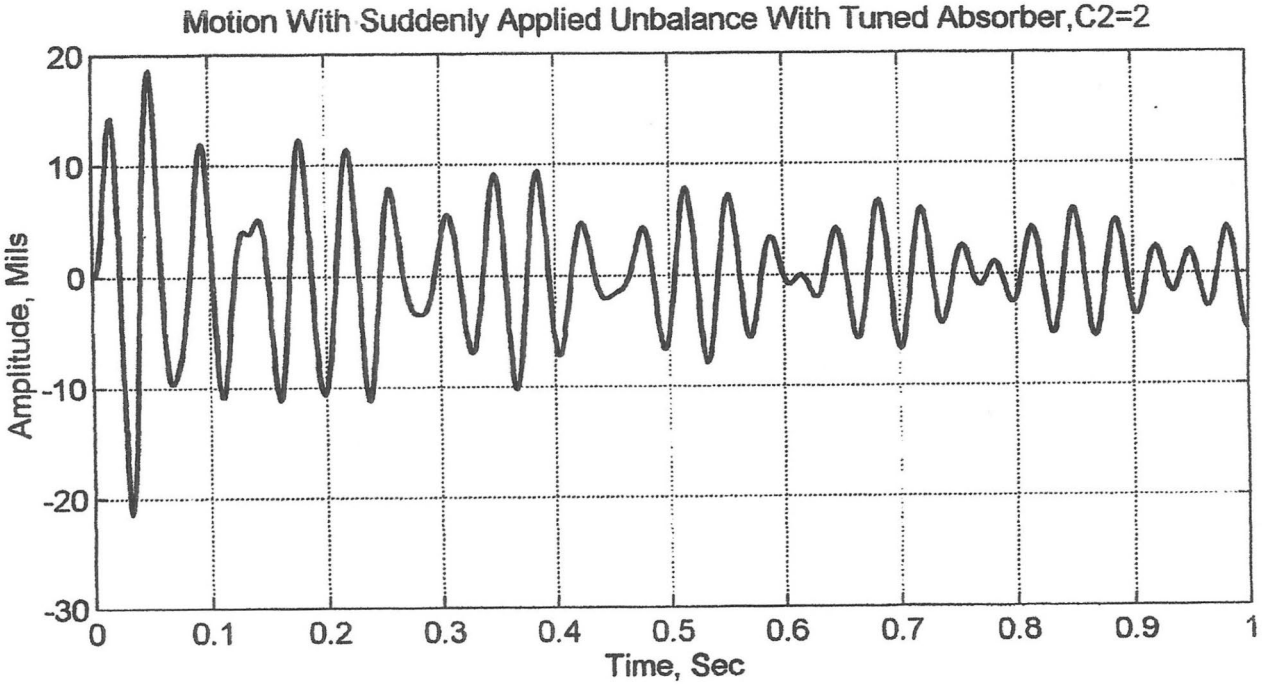
The classical method of solution of a transient response is by means of a convolution integral of the form:

$$y(t) = \int_0^t f(\zeta)\, h(t-\zeta)\, d\zeta$$

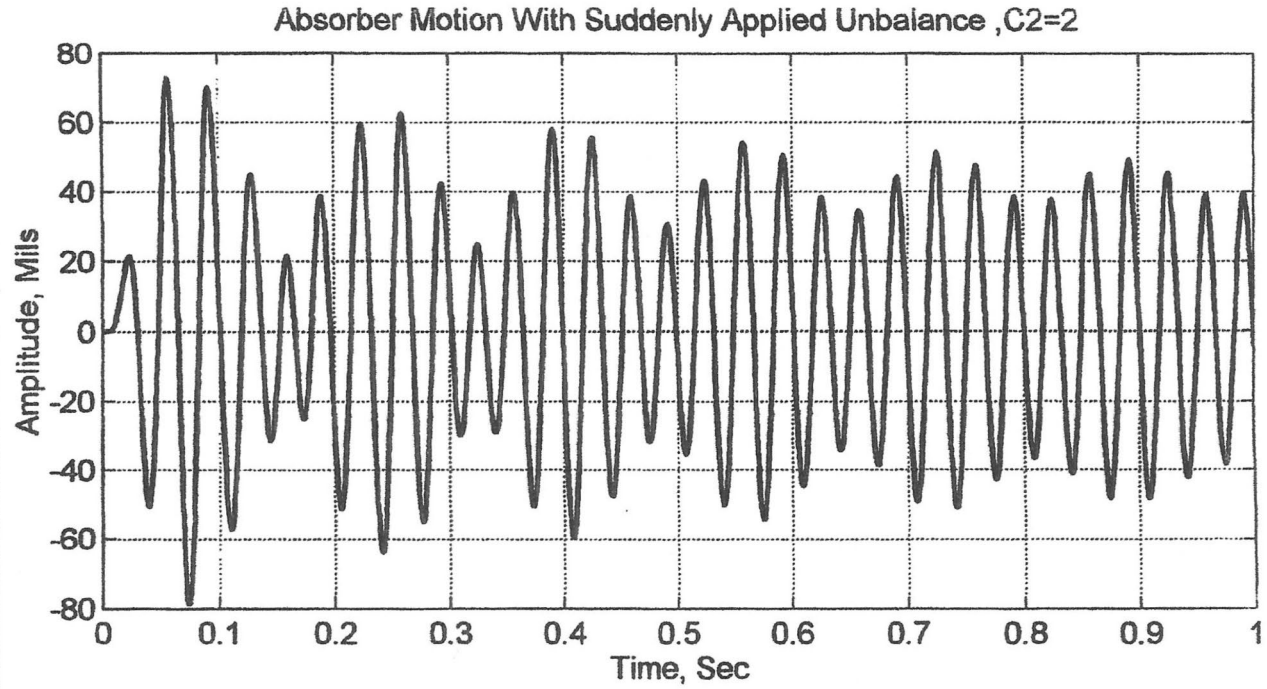
For the case of a damped single degree of freedom system, the convolution function of the integral equation, h(t) is given by:

$$h(t) = \frac{e^{-\zeta\omega_n t}}{M\omega_n\sqrt{1-\zeta^2}} \sin(\sqrt{1-\zeta^2}\,\omega_n t)$$

The convolution integral is difficult to evaluate for even the simplest of forcing functions. The theory also is applicable only to linear systems. For more complex systems, a damped eigenvalue analysis of the roots is first required. Therefore the classical method of transient analysis by convolution integral is obsolete. It should also be pointed out that a computer program is always required to plot out the results. ***MATLAB*** has a variety of equation solvers to handle nonlinear and stiff equations.



(a) Initial Transient Motion of Rotor For 1 Second



(b) Initial Transient Motion of Absorber For 1 Second

Figure 3.6 Transient Motion of Rotor and Absorber With Suddenly-Applied Unbalance

3.5 fft Analysis of Transient Motion

In order to better understand the transient motion and the beating effect observed in the absorber motion, an *fft* (fast Fourier Transform) analysis using ***MATLAB*** was performed . Table 3.6 represents the additional commands required to perform the fft. Since this function is included in only the student version, the Signal Processing Toolbox is required. Figure 3.7 shows the fft analysis of the absorber. The frequencies seen in the signal are 30 Hz, running speed, and the two resonance frequencies at approximately 24 Hz and 39 Hz. Without the use of the fft it would be difficult to determine the presence of the second resonance frequency at 39 Hz .

Table 3.6 FFT Analysis Statements

```
% perform fft on main mass using 1024 points, use whos to insure that
% ntotal >1024 to use fast fourier transform, else must use slow dff

n=1024 ;
ttotal=t(n); fs=1024/ttotal ; % sample frequecy
ts=1/fs     ;  % time per sample
k=0:n-1     ;  % counter for plotting
dhz=fs/n    ;  % frequency increment of fft
f1complex=fft(y(:,4),n) ;  % compute complex fft for frequency content
f1=abs(f1complex)       ;  % convert to absolute magnitude
hertz=k*dhz             ;  % frequency steps
subplot(2,1,2)          ;  % plot fft vs hZ
stem( hertz(1:n/2) , f1(1:n/2) )  % plot using vertical lines
xlabel('Frequency ,Hz')
ylabel('Amplitude')
title('FFT Analysis of Absorber Mass')
axis([ 0 60  0  20])
```

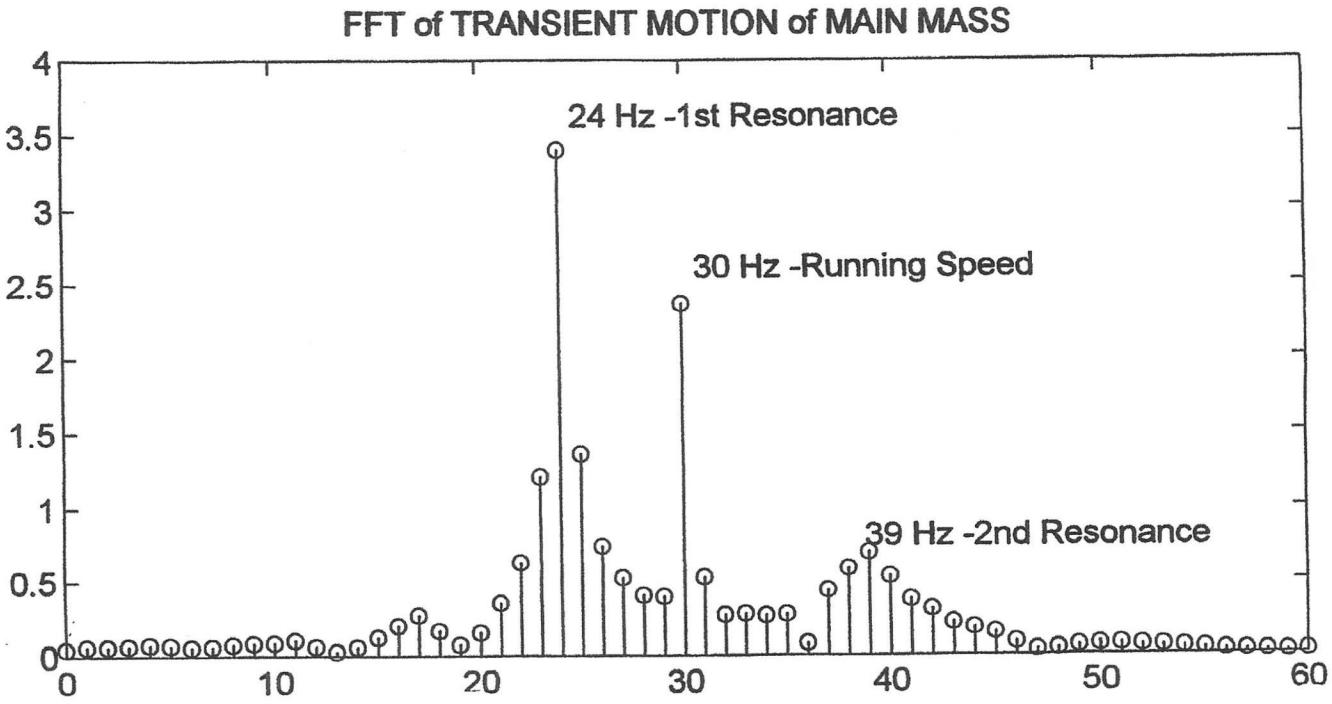


Figure 3.7 FFT Analysis of Transient Motion of Dynamic Vibration Absorber

ACKNOWLEDGMENT

The author wishes to acknowledge Robert L. (Rob) Howes, Senior Engineer- ***Acoustics and Vibrations*** of Cessna Aircraft, Wichita, Kansas for demonstrating to me the various applications of ***MATLAB*** in the aerospace industry and directing me towards the study and application of this language to vibration analysis and data processing.

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