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A Study of the Modal Truncation Error in the Component Mode Analysis of a Dual-Rotor System

In the component mode synthesis method, the equation of motion in the generalized coordinates is built upon the undamped eigenvalue data of the component structures. Error is inevitable when truncated modes are used. In this paper, two model truncation schemes were evaluated with regard to the critical speed, stability, and unbalance response of a two-spool gas turbine engine. The numbers of modes required to yield acceptable accuracy in these cases were determined. Guidelines for modal truncation were derived from these results.

Introduction

One of the major considerations in a truncated model analysis is determining the modes to be retained for accuracy and yet maintaining computational efficiency. Some amount of error is always introduced in the use of a number of modes less than the number of degrees of freedom presented in the system. The resulting modal representation will be somewhat "stiffer" than it actually is because the higher modes are omitted. However, despite this slight disadvantage, the modal method will continue to play an important role in the analysis of large dynamic systems because of the substantial saving in computation time associated with the resultant reduction in the problem size.

The modal method has been applied to the analysis of linear and nonlinear rotor systems. Childs performed transient rotor dynamic analyses using undamped normal modes in [1-4]. Dennis, et al. presented a transient solution of an aircraft engine in [5]. Gunter, et al. illustrated and evaluated the accuracy of the planar mode approach in single shaft flexible rotor with bowed shaft and skewed disk effects [6]. Transient motions of a flexible rotor in fluid film bearings were simulated by Choy [7]. Lund [8] used the damped modal coordinates to calculate the unbalance response and the transient motion of a multi-mass rotor caused by a shock loading. In [9] the effects of bearing mass and prescribed base motion was computed by Pilkey et al. using damped modes.

The present paper focuses on the component mode synthesis method using undamped modes. This procedure is used extensively in the aerospace industry for the calculation of the undamped natural frequencies of large air-frame structures (Hurty [10] and Craig et al. [11]) and was applied to multi-component flexible rotor systems by Childs [4] and Li [13,17]. In this method, a large structure is first partitioned into a number of substructures. The modal information for each individual substructure is then derived either analytically or from vibration tests [12]. "Reconstruction" of the original structure is performed in the modal coordinates using only a few modes from each substructure.

In this paper, an aircraft gas turbine engine consisting of two coaxially mounted rotors is used as a vehicle for the comparison of the component mode synthesis method to an accurate method using transfer matrices [13-15]. Efforts are directed to the evaluation of two proposed mode selection criteria [16] and the determination of the number of lower modes that ought to be retained. The accuracy of the component mode method in this application is examined with regard to linear dynamics in the critical speeds, stability, and forced unbalance response.

Description of the Gas Turbine Rotor System

Figure 1 represents a schematic drawing of the two-spool gas turbine engine used in this analysis. The engine consists of two coaxial rotors. The inner core rotor called the power turbine is supported by two main bearings. There are two intermediate differential bearings, Front Differential Bearing (FDB) and Aft Differential Bearing (ADB), connecting the power turbine to a gas generator rotor. The gas generator rotor consists of a two-stage turbine driving an axial compressor. It is supported by four rolling element bearings.

The computer model in Fig. 2 has a total of 38 nodes (lumped masses) - 22 nodes in the power turbine rotor and 16 nodes in the gas generator rotor. Each node has two translational degrees of freedom and two rotational degrees of freedom. Thus, this engine has a total of 152 degrees of freedom.

Gas Turbine Undamped Component Modes

A component mode analysis requires the undamped eigenvalues and the orthonormal mode shapes of each sub-

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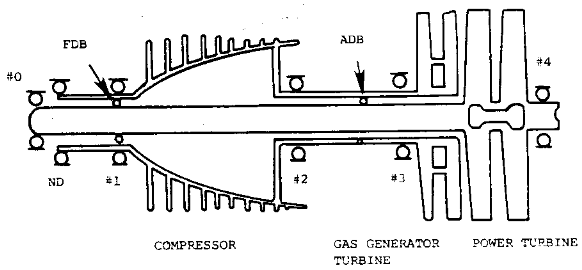

Fig. 1 Two-spool aircraft turbine engine with eight bearings

system to be known. In this case, the engine is considered to be consisting of two subsystems—the gas generator rotor and the power turbine rotor. The modes of the rotors were obtained independently of each other by omitting the intermediate bearings. The effects of disk gyroscopic are excluded by applying zero rotor speed in this calculation.

The method of transfer matrices was used to obtain all undamped normal modes of the gas generator rotor and the power turbine rotor below (1,833 Hz) 110,000 r/min. Below this speed (about 6 times the top operating speed of the rotors), there are a total of five modes in the gas generator rotor and nine modes in the power turbine rotor. The first 5 in each rotor are presented in Figs. 3 and 4.

The mode shapes $[\phi]$ are to be arranged in the following fashion.[2]

$$[\phi_1] = \begin{bmatrix} [\phi_G] & [0] \\ [0] & [\phi_G] \end{bmatrix} ;[\phi_2] = \begin{bmatrix} [\phi_p] & [0] \\ [0] & [\phi_P] \end{bmatrix} \quad (1)$$

The accuracy of the undamped mode shapes are evaluated with a test of their orthogonality weighted by the mass matrix. In such a test of product of the matrices $[\phi_i]^T$ $[M]$ $[\phi_i]$, i = 1,2, is computed and the off-diagonal elements of the resultant matrix (which should be equal to zero) are compared to the diagonal elements. For the normal modes calculated, the ratio of the off-diagonal to the diagonal elements is in the order of 10^{-10}. With transfer matrices, each mode requires about 1.8 s of computer time to evaluate.

Gas Turbine Engine Mode Selection

Selection of the component modes from each subsystem to be used in the synthesis of the engine is made according to one of the following criteria.

1 The first criterion is based on a comparison of the subsystem undamped natural frequencies to an upper frequency limit. Only the component modes that are below this limit are retained, those that are above are discarded. In a rotating machine, this upper limit is usually set at several times the maximum rotor speed.

2 The second criterion proposed by Tolani in [16] is based on the strain energy of the subsystems. In this case, an upper limit on the strain energy is set and only the component modes that have strain energy below this value are considered. For each subsystem component mode having the mode shape $\{\phi_i\}$, the strain energy (PE) is calculated from the system stiffness matrix by

$$PE = \frac{1}{2} \{\phi_i\}^T [K] \{\phi_i\} \quad (2)$$

the mode shape $\{\phi_i\}$ is normalized such that the sum of the absolute values of the elements in $\{\phi_i\}$ is equal to one.

[2]Subscripts 1 and 2 refer to subsystem 1 and subsystem 2. $[\phi_G]$ and $[\phi_p]$ represent the planar component modes in the gas generator and the power turbine, respectively.

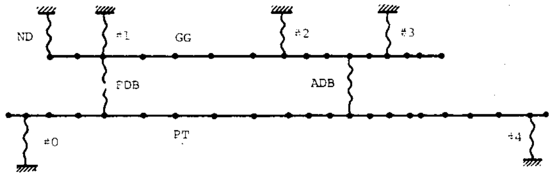

Fig. 2 Two-spool aircraft turbine engine lumped parameter computer model (*PT*-power turbine rotor, *GG*-gas generator rotor)

Table 1 represents the selections of the component modes for the truncated modal problem based on these two criteria. For example, with a problem size of 12 (i.e., taking six component modes in each lateral direction), the modes selected on the basis of frequency include all that are below 500 Hz. This frequency is about 1.5 times the maximum rotor speed. While based on strain energy, a limiting energy of 508 J is imposed. With the strain energy criterion, the tendency is towards selecting more power turbine modes and fewer gas generator modes because the power turbine is more flexible. Note that after the problem size reaches 24, the schemes become identical to each other.

To evaluate the mode selection schemes, the undamped critical speeds of the gas turbine engine are compared for the various reduced problems. Of particular interest in this investigation is the comparison in the suitability of the criteria for this application with regard to solution accuracy.

Here, both rotors are assumed to rotate in the same direction with the gas generator operating at 15,000 r/min and the power turbine operating at 20,000 r/min. The inter-shaft bearings are added to the modal equation in the generalized coordinates [13]. The percentage errors in the system eigenvalues or the engine critical speeds due to modal truncation are established by a comparison to an accurate solution of the full size problem.

Figures 5-6 show the errors of the first four forward engine modes and the first four backward engine modes for the two modal selection schemes. In general, the component mode method predicts the engine modes to be higher in frequency than they actually are. As more component modes are used, the magnitude of the error decreases monotonically. With the exception of the third forward, the first backward, and the fourth backward engine modes, the mode selection scheme based on frequency appears to yield better results than the selection scheme based on strain energy. This is particularly true with the second forward and the second backward engine modes.

When all 28 component modes are used, the largest error of only 1 percent is observed (in the third forward engine mode). The smallest error is 0.01 percent with the second forward mode of the engine.

If only the engine critical speeds below the top speed of the rotors (20,000 r/min) are considered, the upper frequency limit for the modal selection necessary to meet a certain error tolerance may be established. This information is tabulated in Table 2.

Gas Turbine Lateral Critical Speeds

There are two possible modes of operation in a two-spool rotor system: one is that both rotors corotate simultaneously in the same direction, the other is that the rotors counterrotate with respect to each other. In this analysis, only the case of corotation is considered. The gas generator and the power turbine are assumed to be operating at 15,000 r/min and 20,000 r/min, respectively. With the transfer matrix method, the first five forward and the first five backward engine critical speeds are calculated (the backward modes are obtained by specifying negative rotor speeds in the computer

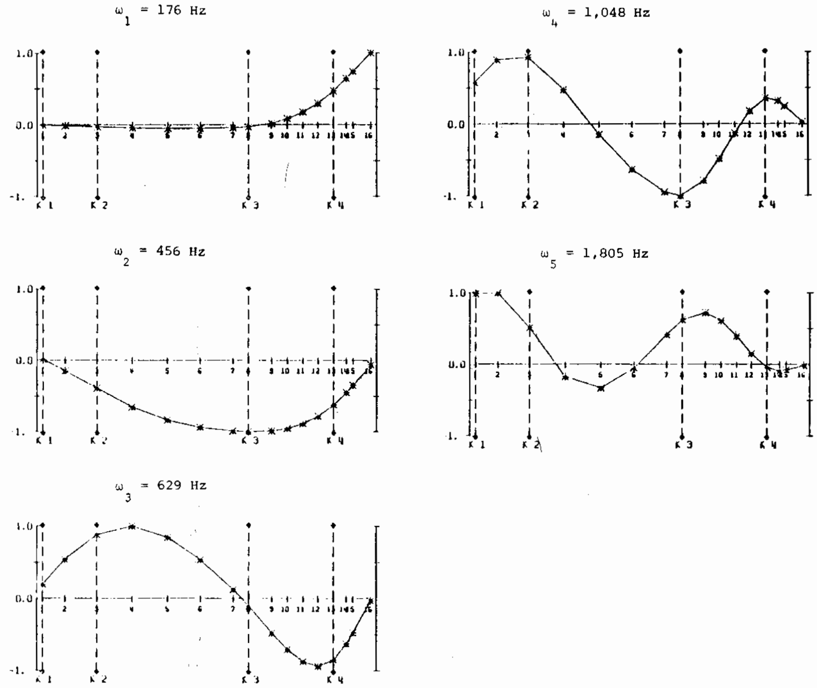

Fig. 3 Gas generator rotor undamped component mode 1 to mode 5. $K1 = 1.75 \times 10^8$ N/m; $K2 = 1.75 \times 10^8$ N/m; $K3 = 0.88 \times 10^8$ N/m; $K4 = 0.70 \times 10^8$ N/m.

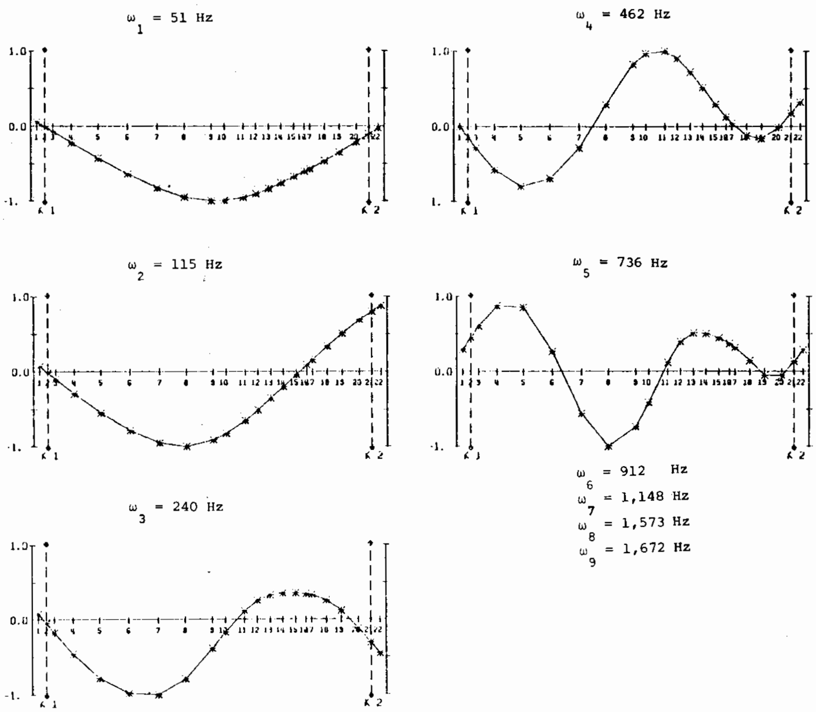

Fig. 4 Power turbine rotor undamped component mode 1 to mode 5. $K1 = 0.88 \times 10^8$ N/m; $K2 = 1.05 \times 10^7$ N/m

program). This result, calculated to an error of less than 1.0×10^{-5} percent, is used to assess the accuracy of a truncated component mode analysis for the following reduced problems using: (*a*) two gas generator modes and four power turbine modes, (*b*) three gas generator modes and six power turbine modes, (*c*) five gas generator modes and nine power turbine modes. The component modes have been selected according to the frequency criterion described previously. The critical speeds are calculated with a stability program assuming zero damping. The result of this investigation is tabulated in Table 3. Good accuracy of the modal method is obtained. Errors less than 1 percent are achieved with the largest problem size. Reasonably accurate results are also obtained for the problem size of 18 with the use of three gas generator modes and six

Table 1 Gas turbine engine mode selection schemes

REDUCED PROBLEM DIMENSION	MODE SELECTION BASED ON FREQUENCY		MODE SELECTION BASED ON STRAIN ENERGY	
	GAS GENERATOR MODES RETAINED	POWER TURBINE MODES RETAINED	GAS GENERATOR MODES RETAINED	POWER TURBINE MODES RETAINED
8	1	3	0	4
10	2	3	0	5
12	2	4	1	5
14	3	4	2	5
16	3	5	2	6
18	3	6	2	7
20	4	6	2	8
22	4	7	3	8
24	4	8	4	8
26	4	9	4	9
28	5	9	5	9

Table 2 Modal truncation cut-off frequency versus percentage error in the undamped modes (* modes retained in each lateral plane)

UPPER FREQUENCY LIMIT BELOW WHICH MODES ARE RETAINED (MULTIPLES OF TOP ROTOR SPEED)	GAS GENERATOR MODES * RETAINED	POWER TURBINE MODES * RETAINED	ERROR IN THE UNDAMPED MODES BELOW TOP ROTOR SPEED
1.5	2	4	< 20%
2.5	3	5	< 6%
4	4	6	< 3%
5	4	8	< 1.5%
6	5	9	< 1%

power turbine modes. In this case, the third forward mode has the largest error of 3.29 percent.

The computer time requirement for the calculation of the undamped critical speeds using transfer matrices is about 3.8 s for each mode computed. A direct comparison of the computational efficiency of the two methods based on the result in this section was not possible because the eigenvalues calculated with the method of component modes were generated from a stability computer program using complex arithmetic. Furthermore, an eigenvalue extraction scheme based on similarity transformations was used such that all existing eigenvalues including both forward and backward modes are computed simultaneously. For example, with a problem size of 18, 18 conjugate pairs of roots with zero real parts are produced in 77 s. In contrast, the transfer matrix computer program calculates only the modes required by iterating on the frequency determinant in real arithmetic. If a similar procedure is employed in the modal calculation, it is anticipated that the computer time requirement of the component mode method will be comparable to, if not less than that of the transfer matrix method.

The mode shapes of the first, second, and the third forward modes obtained with component modes and transfer matrices are presented in Figs. 7-9. For the first and second modes, very little error due to modal truncation is observed. When a reduced modal problem consisting of only six component modes per lateral plane is used, some inaccuracy is indicated in the third forward mode shapes. The percentage error in this mode is 20 percent according to data in Table 3.

Gas Turbine Lateral Stability

In the stability analysis, two fluid film components are added to the system. The No. 3 gas generator bearing is replaced by a squeeze film damper assumed to have a stiffness of 2.15×10^7 N/m (123,000 lb/in.) and a damping of 1.7×10^4 N-s/m (100 lb-sec/in.). In comparison to the original stiffness of 7.0×10^7 N/m (400,000 lb/in.) used in the component normal mode evaluation, this modification

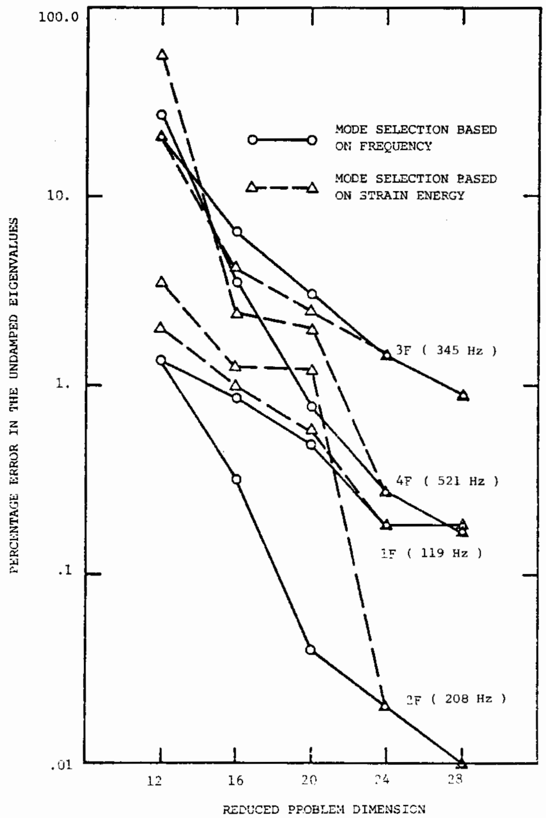

Fig. 5 Percentage error in the gas turbine undamped forward modes due to modal truncation

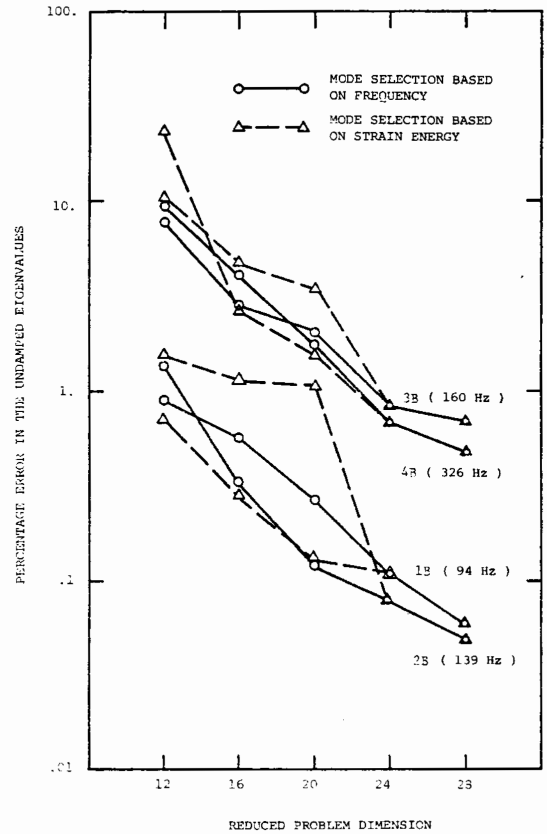

Fig. 6 Percentage error in the gas turbine undamped backward modes due to modal truncation

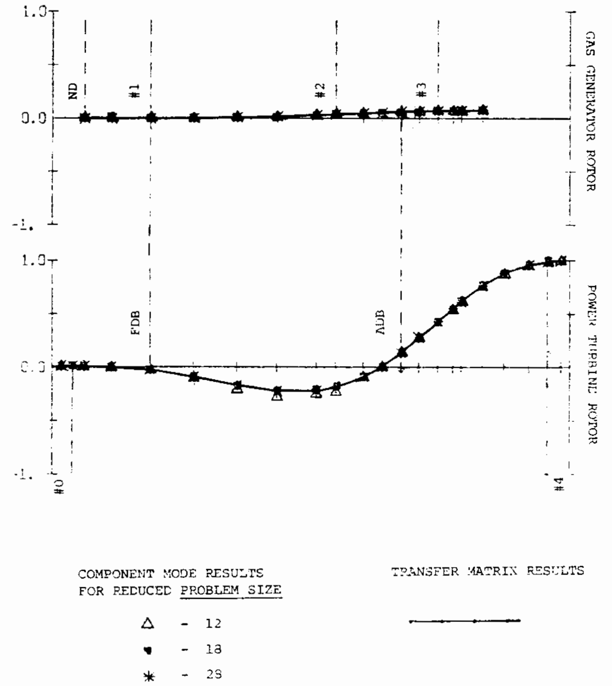

Fig. 7 Gas turbine engine critical speed mode shape error due to component mode truncation—first system forward mode (119 Hz)

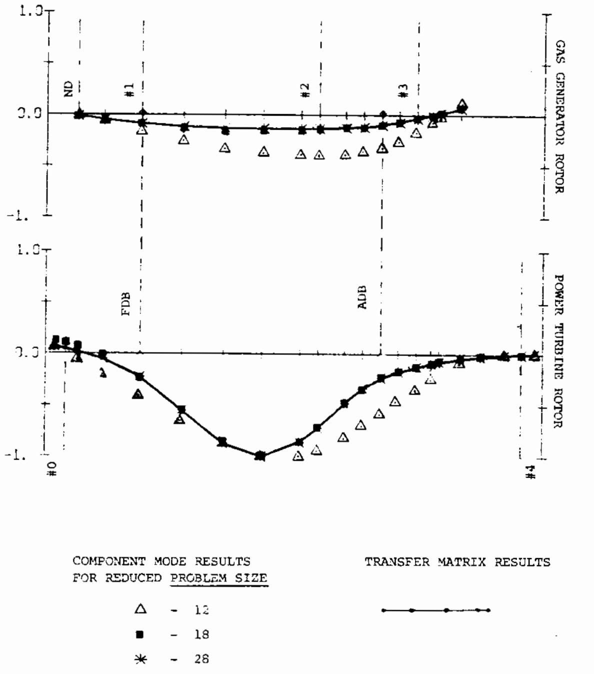

Fig. 9 Gas turbine engine critical speed mode shape error due to component mode truncation—third system forward mode (345 Hz)

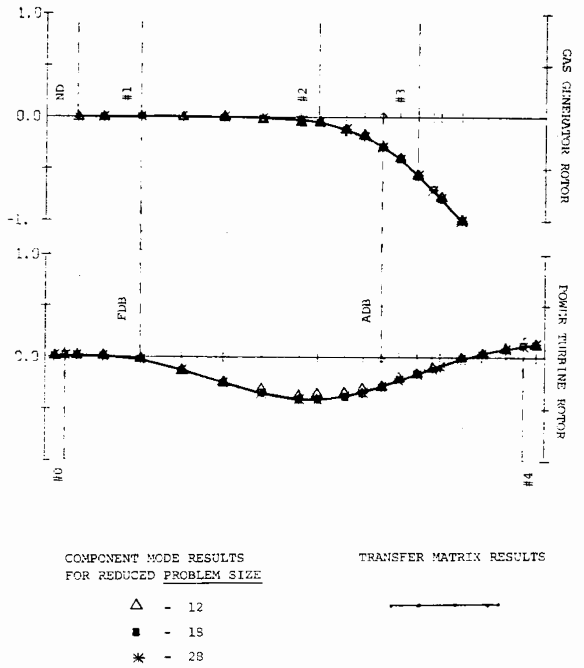

Fig. 8 Gas turbine engine critical speed mode shape error due to component mode truncation—second system forward mode (208 Hz)

Table 3 Error in the gas turbine critical speeds using truncated component modes. (*Problem size is the dimension of the second order modal equation.)

UNDAMPED MODE	TYPE	CRITICAL SPEED, r/min (Hz)	% ERROR FOR THE REDUCED*PROBLEM SIZE		
			12	18	28
1st Mode	Forward	7,115.7 (119)	1.38	0.52	0.18
2nd Mode	Forward	12,471.0 (208)	1.31	0.32	0.01
3rd Mode	Forward	20,677.7 (345)	20.17	3.29	0.89
4th Mode	Forward	31,283.1 (521)	26.88	1.38	0.17
5th Mode	Forward	35,403.9 (590)	23.44	2.43	1.22
1st Mode	Backward	5,656.4 (95)	0.90	0.28	0.06
2nd Mode	Backward	8,311.7 (139)	1.38	0.30	0.05
3rd Mode	Backward	9,596.3 (160)	7.86	2.04	0.70
4th Mode	Backward	19,528.6 (326)	9.67	2.42	0.48
5th Mode	Backward	27,563.9 (460)	28.22	1.43	0.39
Computer Time Required to Calculate the Component Modes			11 s	16 s	25 s
Computer Time Required to Calculate all Eigenvalues Using a Transformation Method			35 s	77 s	179 s

requires a negative stiffness of 4.85 × 10^7 N/m (277,000 lb/in.) to be applied at the No. 3 bearing location in the modal synthesis process. In addition, an inter-shaft fluid-film bearing is incorporated near the aft differential bearing. The bearing is 38 mm long, 66 mm in diameter and has a radial clearance of 0.25 mm. Turborotor speeds are taken to be 15,000 r/min for the gas generator and 17,000 r/min for the power turbine rotating in the same direction. The dynamic coefficients are computed according to the short bearing theory. A bearing eccentricity ratio of 0.3 is assumed. The function of the fluid-film bearings is to provide some of the damping required to relieve the high loading experienced by the aft differential bearing when the rotor is operated near a power turbine mode. In this calculation, all the rolling element bearings are taken to have damping of 350 N-s/m (2 lb-sec/in.) each.

The complex eigenvalues of the damped system are computed with an error of less than 0.01 percent by the transfer matrix method. This result forms the basis for evaluating the accuracy of the component mode approach. For the purpose of comparison, all engine modes below 51.7 Hz (31,000 r/min), including four forward modes and five backward modes, are considered. With the use of transfer matrices, each complex root requries about 3.9 s to compute.

Table 4 shows the truncation error for the three different

Table 4 Error in the gas turbine engine stability using truncated component modes. (*Problem size is the dimension of the second order modal equation.)

DAMPED MODE	DAMPING EXPONANT, 1/s	DAMPED FREQUENCY, r/min (Hz)	% ERROR IN THE DAMPING EXPONENT FOR THE REDUCED PROBLEM SIZE *			% ERROR IN THE DAMPED FREQUENCY FOR THE REDUCED PROBLEM SIZE*		
			12	18	28	12	18	28
1st Forward	-19.85	6,978.3 (116)	-43.1	-1.64	-1.55	2.00	0.39	0.19
2nd Forward	-155.2	8,926.0 (149)	-13.3	-3.17	-0.94	8.33	1.74	0.23
3rd Forward	-14.48	20,758.1 (346)	216.	13.1	-0.61	18.01	3.16	0.85
4th Forward	-52.71	30,257.3 (504)	416.	5.17	1.29	31.19	1.42	0.29
1st Backward	-9.71	5,924.8 (99)	-5.60	-2.16	-0.47	0.45	0.12	0.02
2nd Backward	-87.03	6,167.7 (103)	-14.5	-2.94	-1.32	1.46	0.44	0.35
3rd Backward	-1.91	10,093.8 (168)	79.6	53.8	13.94	8.51	2.01	0.70
4th Backward	-136.2	18,674.5 (311)	-15.6	-1.91	0.00	8.38	2.01	0.25
5th Backward	-194.0	25,970.1 (433)	-91.2	-7.21	-3.08	25.60	2.18	0.57
Computer Time Required to Calculate the Undamped Component Modes						11 s	16 s	25 s
Computer Time Required to Calculate All Eignevalues Using a Transformation Method						32 s	66 s	177 s

problem sizes using component modes. The component mode selection scheme is the same as those for the critical speed analysis in the last section. A common feature of the modal result is that the resonant frequencies of the damped engine system have a positive error. This means that these frequencies are always overestimated due to modal truncation. In contrast, tendency of the error in the damping exponent does not seem to show any definite pattern.

The amount of error in the damped frequency of the reduced modal problems is of the same order of magnitude as that in the undamped critical speeds computed previously. For a problem size of 28, in which 14 component modes in each lateral plane are used, the largest error in the frequencies is only 0.85 percent. But when the problem size is reduced to 18, the amount of error increases. In this case, the largest error of about 3 percent is observed in the third forward mode. A further decrease in the accuracy is indicated for the problem size of 12. If only the damped modes that are below 334 Hz (20,000 r/min) are considered, an error of up to 18 percent can be seen.

The real part of the eigenvalues is shown to have considerably larger error than the damped frequency. Even when all of the 28 undamped planar modes are employed, an error as high as 14 percent in the exponent of the third backward mode is obtained. For the same problem size, the largest inaccuracy in the forward modes is 1.6 percent. With a reduction of the problem size to 18, the largest overall error is 54 percent and the largest forward mode error is 13 percent. Unacceptable error is produced in the damped exponents for the problem size of 12. This general inaccuracy of the real part of the complex eigenvalues is also reported by Gunter, et al. in [6].

It should be noted that all four backward modes appear to be less stable than the forward modes. Since the majority of the damping comes from the No. 3 gas generator squeeze film damper bearing, it is reasoned that the result reflects a lower efficiency of the squeeze-film damper bearing in the backward modes.

Gas Turbine Lateral Unbalance Response

The forced response of the gas turbine engine due to mass unbalance in the power turbine rotor is computed with the same bearing configurations as those described in the last section. For this analysis, two unbalances of 35 g.cm each are assumed to be present at the mid-span of the power turbine rotor and at the second stage turbine 180 deg out of phase from each other. The gas generator is taken to be balanced and operating at 15,000 r/min.

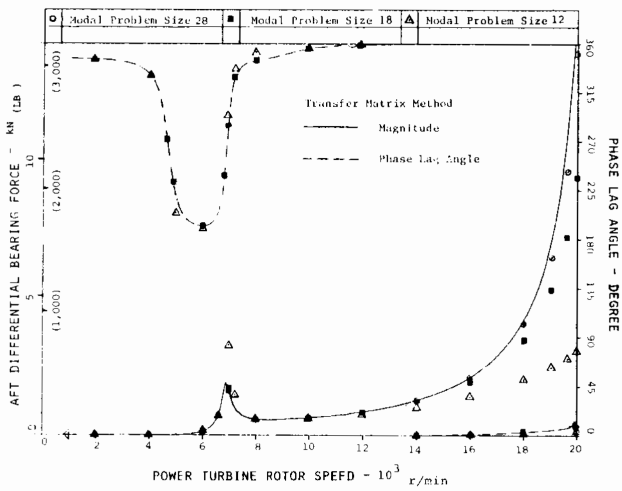

Fig. 10 Component mode truncation error in the aft differential bearing force due to coupled power turbine unbalance

The dynamic coefficients of the bearings, including those of the squeeze film damper and the intershaft journal bearing, are considered to be constant with speed. Again, the unbalance response problem is solved first with the transfer matrix method, then with component modes using a truncated number of modes.

Because mass unbalance in a rotor generates circular forces that rotate in the same direction as the rotor spin vector, only the forward modes are excited. According to Table 4, the first three damped forward modes of the engine occur at 116 Hz (6,978 r/min), 149 Hz (8,926 r/min), and 346 Hz (20,758 r/min). Therefore, within the speed range of 1,000 r/min to 20,000 r/min the rotors have to operate through the first two modes and below, but close to the third mode of the system.

In general, there are two predominant factors that determine whether the response amplitudes and the bearing forces will be higher or lower than the actual value due to modal truncation. The first factor depends on the error introduced in the damping available for each mode due to the omission of some of the higher component modes. As it is concluded in the last section, the tendency of the error in this respect does not have a definite pattern. If the error tends towards producing less damping, a response of lesser amplitude will be computed. The reverse will be true if the error tends toward producing more damping in a particular mode. The second factor is determined by the shift of the damped resonant frequency towards a higher value due to the loss of system flexibility. Consequently, the amplitude of vibration computed by a modal method will be lower on the low-speed side but higher on the high-speed side of the actual response peak.

The steady-state forces transmitted to the outer raceway of the differential bearing are presented in Fig. 10. Due to the large damping coefficient assumed at the No. 3 gas generator bearing, the second mode, which shows predominant gas generator motion in the mode shape, has been completely damped out. The magnitude of the bearing forces shows a peak of about 1.8 kN at the first critical speed and starts to build up rather rapidly as the power turbine rotor is operated toward the third system mode. A bearing force higher than 14 kN is indicated at rotor speeds above 19,000 r/min.

The symbols in Fig. 10 represent the bearing forces calculated by the component mode method using the reduced problem sizes of 12, 18, and 28. With the problem size of 12, (two gas generator modes and four power turbine modes used) the force magnitude at the first critical speed is almost 70 percent higher than the correct value. This appears to be in correlation with the damped eigenvalue result which indicates an error of 43 percent less damping available to this mode for the same reduced problem size. As the rotor is operated near the third mode at about 20,000 r/min, a large error in the bearing force as much as 400 percent is indicated. The observation of a lower response amplitude and more gentle response slope associated with this mode is consistent with the eigenvalue error in the damping exponent of 216 percent as shown in Table 4. This combines with the shift towards a higher resonant frequency producing the observed result.

Excellent correlation between the transfer matrix method and the modal method for the problem size of 18 and 28 is indicated for bearing force in the first mode. When the rotor operates in the vicinity of the third mode, good accuracy can still be seen with the larger problem size of 28; however, about 40 percent error can be seen at 20,000 r/min with the problem size of 18.

Figure 11 shows the rotor response at the second stage turbine of the gas generator rotor. It is seen that the rotor has rather high amplitudes when it is operated near the first and third critical engine speeds (power turbine modes). Again, the amplitude at the second engine mode (gas generator mode) is almost damped out by the large amount of damping at the No. 3 gas generator bearing. According to the mode shapes in the power turbine modes (Figs. 7 and 9), the gas generator bearing location has very little motion. Therefore, damping applied to the gas generator at this location does not effectively reduce the vibration amplitude in these modes. Very large error is indicated with the problem size of 12, particularly near the first mode and the third mode. The shift of the second mode towards a higher frequency is clearly seen. When the problem size is increased to 18, the error is reduced to a more reasonable level. Good accuracy is again obtained with the problem size of 28 in which five gas generator modes and nine power turbine modes are included. The modal method appears to yield more accurate results in the phase angles than in the vibration amplitudes in this unbalance response analysis.

The computer time required in the calculation of the above results using the modal approach is 0.45 s for a problem size of 12, 0.67 s for a problem size of 18, and 1.47 s for a problem size of 28. These figures represent one steady-state response computation, excluding computer time taken to calculate the components modes. In comparison, the transfer matrix method requires 0.87 s per calculation.

Conclusions

Based on the results in this analysis, the selection of component modes according to an upper frequency limit is recommended. Furthermore, all component modes that have frequencies below 4 to 5 times the top rotor speed must be included in a critical speed analysis to give sufficient accuracy. Component mode synthesis is a fast and economical method for the calculating the undamped modes in a large rotor system.

Stability analysis using component modes must be performed with extreme caution. Accuracy in the damping exponent can only be obtained when all modes that lie below at least six times the highest rotor speed are retained in the calculation

For the computation of the steady-state unbalance response near the lower modes and up to the second mode in the operating speed range, the component mode method is more economical than the transfer matrix method both in computation time and accuracy. However, for higher mode computation, the transfer matrix method is superior.

Due to modal truncation, the resonant frequencies are always overestimated. But because the tendency of the error in the damping (relating to the real part of the eigenvalue) does not show any definite pattern, the unbalance response peaks may either be higher or lower than the actual values. It is shown that the damping exponents of the eigenvalues and the unbalance peak response always exhibit the largest error.

Because the modal method still provides the only efficient way to perform transient analysis of complex rotor systems, precaution must be taken in the interpretation of computer-simulated transient solutions using truncated modes. When the quantitative result is of interest, the analyst must ascertain that enough number of modes is included. An accurate solution is reached only when no significant change in the result can be achieved by the addition of more component modes.

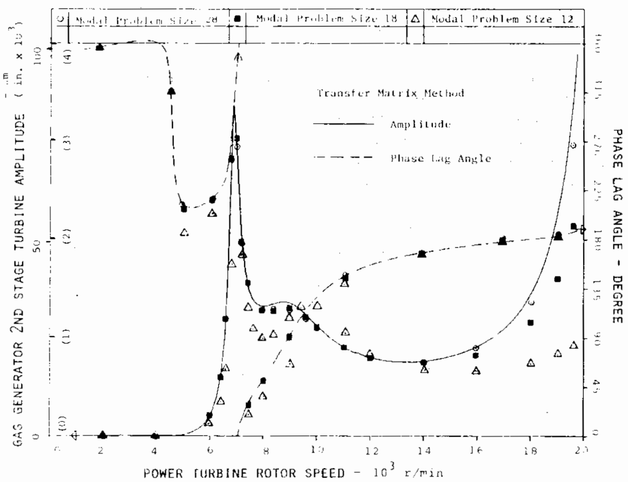

Fig. 11 Component mode trunction error in the gas generator response amplitude due to coupled power turbine unbalance

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