Critical Speed Analysis of Offset Jeffcott Rotor Using English and Metric Units

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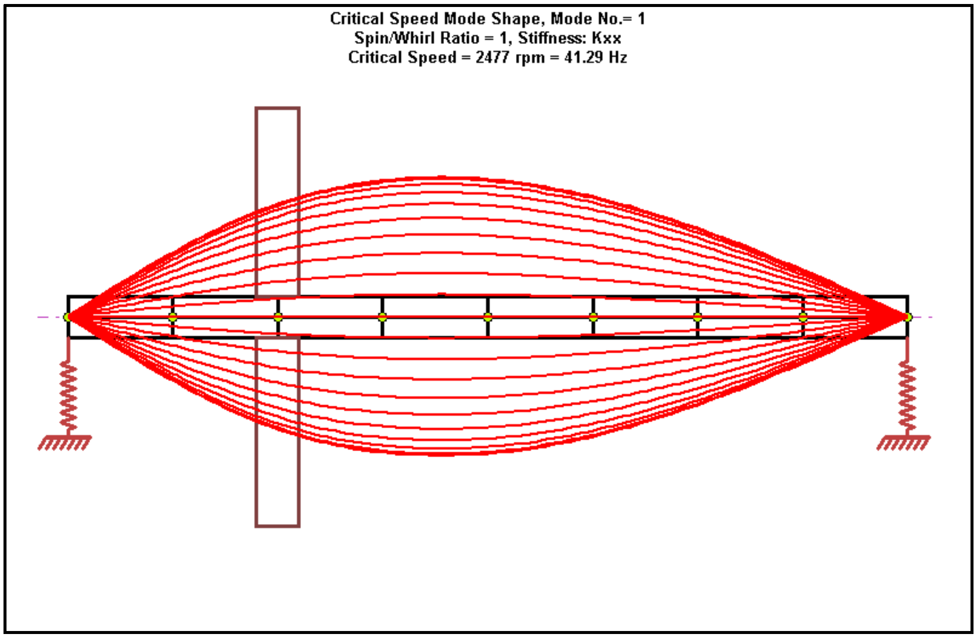

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1 Modeling The Offset Rotor

A uniform rotor of length L=1000 mm and D=50 mm was assumed. A disk of diameter D_{disk}=500 mm and thickness t=50 mm is attached at the quarter span at 250 mm from the left end bearing. The disk and shaft are assumed to be of steel with E=30E6 Lb/in and disk density of 0.283 Lb/in^3

Models were generated with Dyrobes using English and metric system of units. As a check on the accuracy of the calculations, the critical speeds were also computed using the critical speed program CRITSPD-PC which is a transfer matrix program developed by Gunter and Gaston for use by industry.

Selection of Units

There are several basic problems when computing in English and metric units. The problems arise in that there are four basic parameters involved in dynamical analysis regardless of the system of units employed. These fundamental parameters are the units of length L, force F, mass M and time t. These four parameters are not mutually independent as only three are independent. The fourth parameter is defined by Newton's 2nd law of motion ; F=MA. Thus, for example, the English set of units is based on the independent variables as F,L,t and mass M is the defined quantity. The measurement of force is Lb and the mass is derived by M=W/g. The basic measurement of distance is in inches. Therefore the units of gravity is 32.2 ft/sec^2 x 12 in/ft =386.4.

A common mistake in finite element dynamic analysis using standard FEA programs such as NASTRAN or ANSYS is representing the mass with improper units and neglecting to divide by the proper value of gravity when working in English units. When using Unit Set 1 of Dyrobes, the units of mass would be similar to the situation with a standard finite element program. With Unit Set 2, the weights are automatically divided by the proper units of gravity. The specification of mass is in terms of weight measurements in lb. The specification of the dynamic inertia values for gyroscopic effects is in terms of Lb-in^2 units. Thus the properties for steel, for example, using Unit Set 2 would be E=30E6 Lb/In^2 and density =0.283 Lb/In^3. One can then compute the polar and transverse moments of inertia from either the TOOLS section, or by the specification of a rigid disk with the dimensions and density specified.

Specification of Metric Units

There are two sets of metric units that may be specified. These are unit sets 3 and 4. Set 3 is for SI measurements in m. Set 4 for SI units measurements is in mm. It is highly recommended that unit set 4 be used since shaft sections are best measured in mm. The SI units used in Unit Set3 are consistent (Kg,N,m,s). The SI units used in Unit Set 4 are not consistent since the density is specified in Kg/m^3, Ip & Id in Kg-m^2, modulus E in N/mm^2 and bearing stiffness in N/mm. At the lower right corner of each input table is shown the proper set of units. It is of value that one practice using both English (Set 2) and metric units (set 4) on the same problem.

Cross Section of Offset Jeffcott Rotor

Fig 1.1 represents the cross section of the offset Jeffcott rotor with the disk located at the quarter span. Shown is the 9 station model with the disk at station 3. The 9 station model is required in order to insure accuracy of the third critical speed.
Table 1.1 represents the mass and inertia properties of the attached rigid disk and the total weight and inertia of the entire rotor system. Before the critical speeds are computed, one should check that the rotor properties are correct.

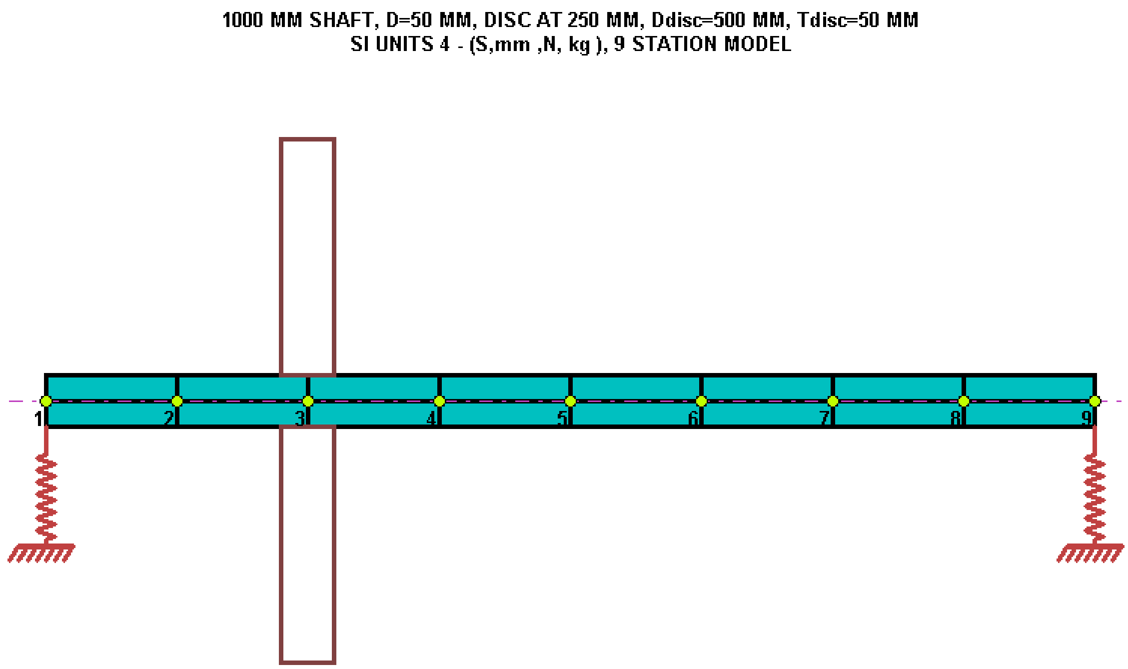


Figure 1.1 Cross Section of Offset Jeffcott Rotor

Material Properties in Metric Units

Table 1.1 Rotor Inertia Properties In metric Units

```
 **************************** Description Headers ****************************
  1000 MM SHAFT, D=50 MM, DISC AT 250 MM, Ddisc=500 MM, Tdisc=50 MM
  ENG UNTIS 4 - (S,mm ,N, kg ), LUMPED DISK INERTIA VALUES
****************************Material Properties ****************************
          Property       Mass          Elastic          Shear
             no         Density        Modulus         Modulus
                       (kg/m^3)       (N/mm^2)        (N/mm^2)

             1          7830.6       0.20682E+06    0.13000E+06
**************************** **************************** *********************
```

In Table 1.1 , the metric Units Set 4 was selected. This unit set specifies the basic unit of length measurement in mm units. In Table 1.1, it is shown that the mass density is given in terms of kg/m^3 rather then in terms of kg/mm^3 since this is the most common method of units for density in the metric system. The units for the elastic modulus is given in terms of N/mm^2.

Table 1.1 Rotor Inertia Properties (Continued)

***************************** Rigid/Flexible Disks *****************************

Station No	Mass (kg)	Diametral Inertia (kg-m^2)	Polar Inertia (kg-m^2)	Skew X (degree)	Skew Y (degree)
3	76.102	1.2168	2.4020	.0000	.0000

****************** Rotor Equivalent Rigid Body Properties ******************

Rotor no	Left End Location (mm)	Length (mm)	C.M. Location (mm)	Mass (kg)	Diametral Inertia (kg-m^2)	Polar Inertia (kg-m^2)	Speed Ratio
1	.0	1000.00	292.02	91.477	3.300	2.40681	1.00

The section table represents the model using English Units 2 set.

Table 1.2 Offset Jeffcott Rotor And Inertia Properties In English Units

```
                 ***** Unit System = 2 *****
        Engineering English Units (s, in, Lbf, Lbm)
                       Model Summary

***************************** System Parameters *****************************

                         1  Shafts
                         8  Elements
                         8  SubElements
                         1  Materials
                         0  Unbalances
                         1  Rigid Disks (4 dof)
                         0  Flexible Disks (6 dof)
                         2  Linear Bearings
                         0  NonLinear Bearings
                         0  Flexible Supports
                         0  Axial Loads
                         0  Static Loads
                         0  Time Forcing Functions
                         0  Natural Boundary Conditions
                         0  Geometric Boundary Conditions
                         0  Constraints
                         9  Stations
                        36  Degrees of Freedom
*****************************************************************************
```

Table 1.2 shows that the rotor model is composed of 8 finite elements and 9 stations. The number of stations is one more than the number of elements. Each station has 4 degrees of freedom in the x and y directions for displacement and rotation. For the analysis of planar and synchronous critical speed modes, the size of the mass and stiffness matrices would be 36.

If the bearings are constrained for pinned end modes, then the number of degrees of freedom are reduced by 4 (x&y at each bearing) for a total of 32 dof. The condition of constrained bearings may be simulated by specifying the bearing stiffness as a very high value in order to simulate the pinned end condition. There is a basic problem with all finite element programs in that one has now introduced very high stiffness or K values along the diagonal of the system stiffness matrix. This can lead to problems in convergence with large models. Hence the stiffness should not be specified as too high a value. The proper value may be seen from a review of the strain energy distribution.

Material Properties in English Units

*************************** Description Headers ****************************

1000 MM SHAFT, D=50 MM, DISC AT 250 MM, Ddisc=500 MM, Tdisc=50 MM

9 STATION MODEL

****

*************************** Material Properties ****************************

Property no	Mass Density (Lbm/in^3)	Elastic Modulus (Lbf/in^2)	Shear Modulus (Lbf/in^2)
1	0.28300	0.30000E+08	0.12000E+08

****

****************************** Shaft Elements ******************************

Ele no	Sub Ele no	Left End Loc	Length (in)	------ Mass ------ Inner Diameter (in)	Outer Diameter (in)	--- Stiffness ---- Inner Diameter (in)	Outer Diameter (in)	Material no
1								
	1	.000	4.9210	.0000	1.9700	.0000	1.9700	1
2								
	1	4.921	4.9210	.0000	1.9700	.0000	1.9700	1
3								
	1	9.842	4.9210	.0000	1.9700	.0000	1.9700	1
4								
	1	14.763	4.9210	.0000	1.9700	.0000	1.9700	1
5								
	1	19.684	4.9210	.0000	1.9700	.0000	1.9700	1
6								
	1	24.605	4.9210	.0000	1.9700	.0000	1.9700	1
7								
	1	29.526	4.9210	.0000	1.9700	.0000	1.9700	1
8								
	1	34.447	4.9210	.0000	1.9700	.0000	1.9700	1

****

*************************** Rigid/Flexible Disks ***************************

Stn No	Mass (Lbm)	Diametral Inertia (Lbm-in^2)	Polar Inertia (Lbm-in^2)	Skew X (degree)	Skew Y (degree)
3	167.89	4159.0	8209.4	.0000	.0000

****

The English Units Set 2 is used to generate the above properties. In this unit set, the specific weight is used such as the specification of steel as 0.283 Lb/In^2. All masses qre entered as weights. The mass moments of inertia are specified in physical English units of Lb-In^2 units.

```
****************** Rotor Equivalent Rigid Body Properties ******************
 Rotor Left End             C.M.              Diametral     Polar    Speed
  no   Location  Length  Location    Mass      Inertia     Inertia   Ratio
         (in)     (in)     (in)     (Lbm)    (Lbm-in^2)  (Lbm-in^2)

   1     .000    39.368   11.498    201.85   0.1129E+05    8225.84   1.0000

****************************************************************************

************************** Bearing Coefficients *************************
 StnI,  J   Angle   rpm     ---------------- Coefficients ----------------

   1    0     .00                    (Linear Bearing)
                       Kt: Lbf/in, Ct: Lbf-s/in;  Kr: Lbf-in, Cr: Lbf-in-s

 Kxx  Kxy  Kyx  Kyy    .100000E+09  .000000      .000000      .100000E+09
 Cxx  Cxy  Cyx  Cyy    .000000      .000000      .000000      .000000
 Krr  Krs  Ksr  Kss    .000000      .000000      .000000      .000000
 Crr  Crs  Csr  Css    .000000      .000000      .000000      .000000

   9    0     .00                    (Linear Bearing)
                       Kt: Lbf/in, Ct: Lbf-s/in;  Kr: Lbf-in, Cr: Lbf-in-s

 Kxx  Kxy  Kyx  Kyy    .100000E+09  .000000      .000000      .100000E+09
 Cxx  Cxy  Cyx  Cyy    .000000      .000000      .000000      .000000
 Krr  Krs  Ksr  Kss    .000000      .000000      .000000      .000000
 Crr  Crs  Csr  Css    .000000      .000000      .000000      .000000

****************************************************************************

********************** Gravity Constant (g) (in/s^2) **********************

             X direction =  .000000         Y direction = -386.088
```

Note that the bearings are specified as Kb=100.0E6 Lb/In. This is more then sufficient to represent a rigid bearing. It should be noted that it is almost physically impossible for a bearing to exceed a stiffness value of Kb>10.0E6 Lb/In. The reason for this is foundation flexibility. Bearing stiffness values specified greater than the 0.1E9 Lb/In value shown in the above table can result in numerical round off errors.

It is important to review the total calculated weight of the model to insure accuracy. This is necessary before one begins more elaborate calculations on forced response and stability.

2 Critical Speeds of Offset Jeffcott Rotor

The critical speeds were computed for various models assuming that the bearings are stiff. The case of finite stiffness bearings will be considered later. The object of the current analysis is to demonstrate the use of English and metric units and to review the accuracy of the various methods of calculation. Regardless of the system of units, it is very easy to have errors in the specification of such properties as the mass density, external masses and disk inertia properties.

Also later, to be examined, will be the generation of a critical speed map, and review of the potential and kinetic energy distributions in the various modes. The evaluation of the energy distribution provides information on the effectiveness of the bearing design, rotor design, and balancing requirements.

2.1 Rotor 1st Critical Speed

Fig. 2.1 represents the first critical speed mode of the offset Jeffcott rotor. A very high bearing stiffness was assumed in order that the bearings are node points. An example of this would be high stiffness rolling element bearings. The synchronous 1st critical speed is shown to be at 2477 RPM. show At this speed, high amplitudes of motion would be encountered since the stiff bearings will

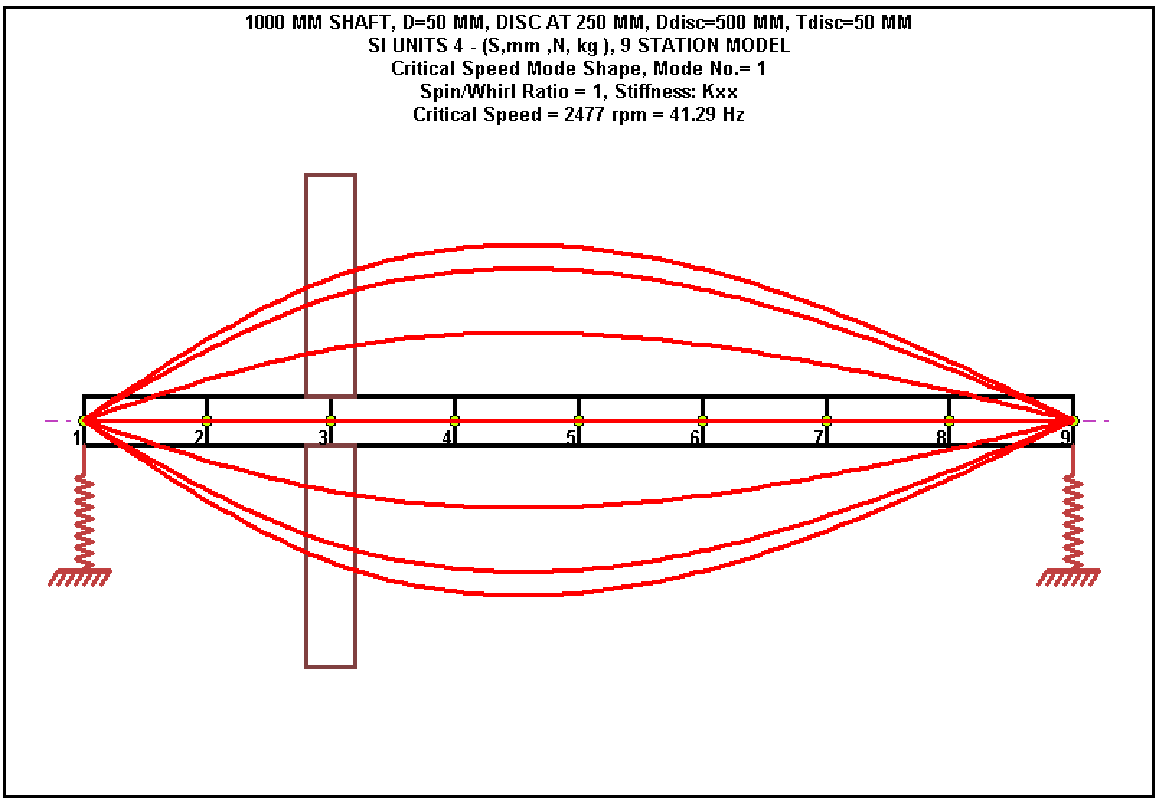


Figure 2.1 1st Critical Speed With At 2,477 RPM

provide little effective damping. In an actual rotor system, the main damping would be provided by windage losses on the disk. The effect of windage losses may be simulated by the specification of a third bearing acting at station 3.

It should be of interest to note that a smooth curve is obtained with only 9 node points. This is because the curve is generated by a cubic spline curve fit. The cubic spline curve exactly matches beam theory and the curvature provides information on the shaft relative stresses.

2.2 Rotor 2nd Critical Speed

Fig 2.2 represents the 2nd critical speed of the offset Jeffcott rotor at N_{cr2}=16,526 RPM. As can be seen from the mode shape, the bearings have zero amplitude. This represents a rigid bearing critical speed. After the rotor passes through the first critical speed, the disk mass center becomes a node point. This implied that radial unbalance at the disk center will have little influence on exciting this mode. Balancing applied at the disc will have little influence in controlling this mode. This mode could be excited by skew in the disk.

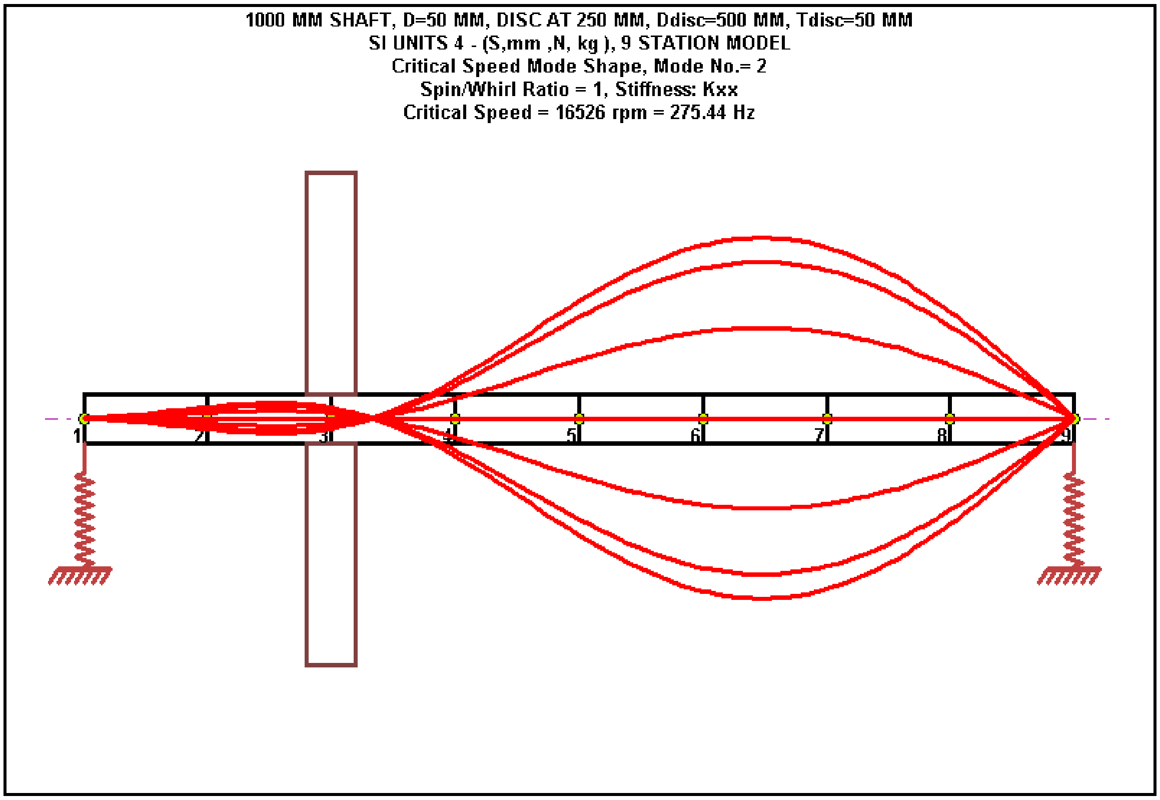


Figure 2.2 2nd Critical Speed At 16,526 RPM

2nd Kinetic Mode Energy Distribution

Fig. 2.3 represents the kinetic energy distribution for the 2nd mode. The kinetic energy of translation of the shaft is 83% and only 4% in the disk. The other components of kinetic energy represent the kinetic energy of rotary inertia and gyroscopic energy.

The total energy of translation is 82.86% for the shaft and only 4.02% for the disk for a total translation energy of 86.88%. The net gyroscopic energy in the shaft is 0.27 %R and-0.54%G for a net gyroscopic energy of -.27%. A negative gyroscopic energy implies that this effect will raise the critical speed. In this case there is little to no difference between Bernoulli-Euler beam assumptions and the more complex Timoshenko beam.

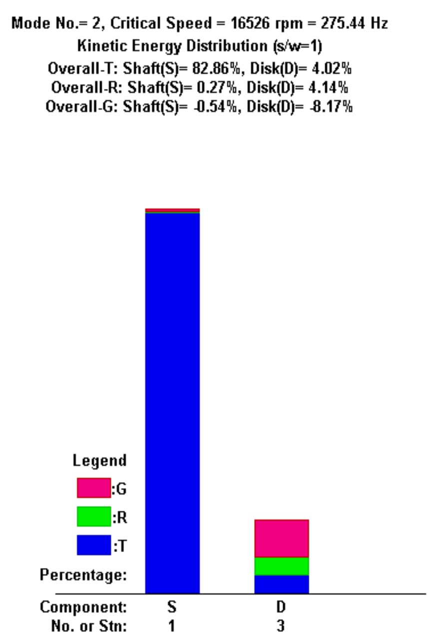


Fig 2.3 2nd Mode Energy Distribution

The disk net gyroscopic energy for the second mode is 4.14% R and -8.17% G for a net total of -4.03%. The negative value implies that the disc 2nd critical speed will be slightly elevated by the effective gyroscopic moment. It should be noted that during a backward whirling motion, the gyroscopic energy G=-8.17 will be reversed in sign. In this case the total net gyroscopic energy will be a positive 12.31%. The higher the positive value of net gyroscopic energy implies the greater the backward whirl mode will be ***reduced*** from the synchronous critical speed value.

2.3 Rotor 3rd Critical Speed

Fig 2.3 represents the offset rotor 3rd critical speed at 53,982 RPM. Note that the third mode shows 3 node points of zero amplitude. The disk center mass also remains a nodal point.

Kinetic Energy Distribution For 3rd Mode

Fig 2.4 represents the kinetic energy of the 3rd critical speed mode shape as seen in Fig. 2.4. The

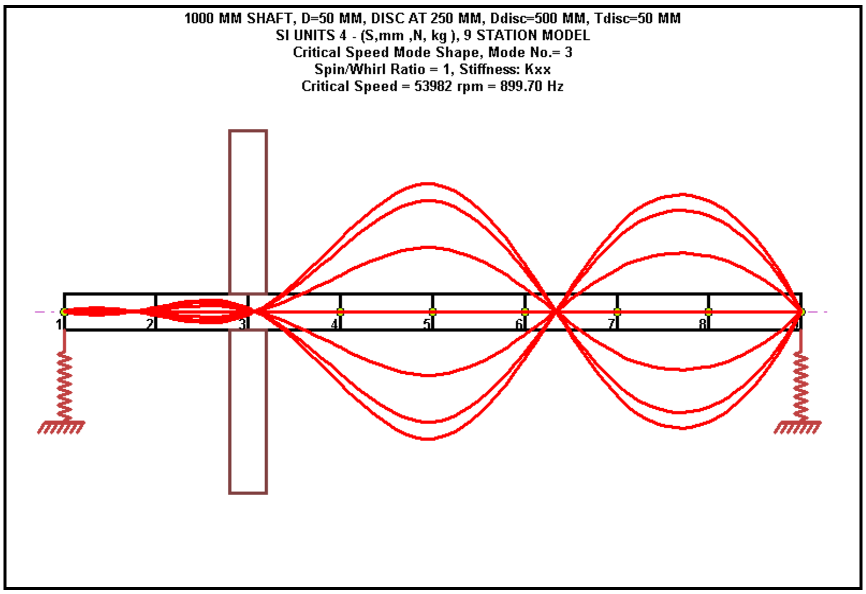


Fig 2.3 3rd Critical Speed At 53,982 RPM

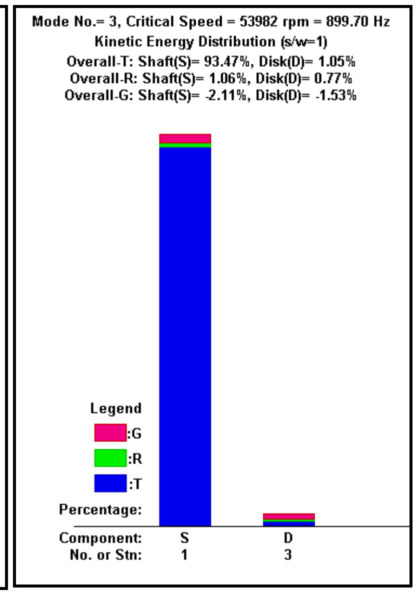


Fig 2.4 3rd Mode Kinetic

shaft has 93.5 % of the total system energy and the translation energy of the disk is only 1%. The net gyroscopic energy of the disk is only -0.76%. Hence we would expect to see little separation between the forward and backward modes for this system.

Balancing Considerations For The 3rd Mode

From a balancing standpoint, the third mode can not be balanced either by the placement of a balance weight on the disk at station 3. Since the translational energy of the disk is so low for this mode, then correction weights on the disk or impeller will be ineffectual. A couple set of weights would also have a minor effect on correction of this mode since the gyroscopic energy is also insignificant.

For proper balancing of this mode, balancing collars would need to be placed at locations 5 and 8. The placement of these collars would cause a reduction in frequency of the 3rd mode, and also to some extent in the first two modes as well. A single correct weight could be placed at the balancing collar at station 5, but this would cause a strong excitation at the first mode as well. Two balancing weights placed out of phase at stations 5 and 8 could be used in balancing this mode. The use of balancing collars has been employed on long LP gas turbine engine rotors and also on drive shafts of helicopter engines.

2.4 Critical Speed Summary

Table 2.4.1 represents a summary table of the various computations of the Jeffcott offset rotor critical speeds. The rotor critical speeds were computed using English and metric units with the finite element based program **DYROBES** and with English units using the transfer matrix program **CRTSPD-PC**.

Table 2.4.1 Comparison of Critical Speeds Computed With English and Metric Units of DYROBES and CRTSPD Computer Programs

Critical Speeds of 1000 mm Lx 50 D Shaft With 500mm x 50 Offset Disk at Quarter Span						
Model	**DYROBES Units 2 English**	**DYROBES Units 2 English**	**DYROBES Units 4 SI - mm**	**CRITSPD Transfer Matrix**	**CRITSPD Transfer Matrix**	**CRITSPD Transfer Matrix**
Stations	5 St	9 St	9 St	5 St	9 St	9 St
N_{c1} RPM	2,480	2,478	2,477	2,470	2,486	2,468
N_{c2} RPM	16,545	16,486	16,526	15,567	16,601	16,145
N_{c3} RPM	54,024	53,603	53,982	***43,952***(1)	54,295	50,121
Kb Lb/In	1.0E7 Lb/In	1.0E7 Lb/In	**.175E6 N/mm**	1.0E7 Lb/In	1.0E7 Lb/In	1.0E7 Lb/In
ρ	0.283 Lb/In^3	0.283 Lb/In^3	7830 Kg/m^3	0.283 Lb/In^3	0.283 Lb/In^3	0.283 Lb/In^3
E	30E6 Lb/In^2	30E6 Lb/In^2	**206920N/mm^2**	30E6 Lb/In^2	30E6 Lb/In^2	30E6 Lb/In^2
Shear Def Shaft Inertia	YES	YES	YES	YES	NO	YES

Note (1) - Error in 3rd mode with transfer matrix method due to insufficient number of stations -7 req.

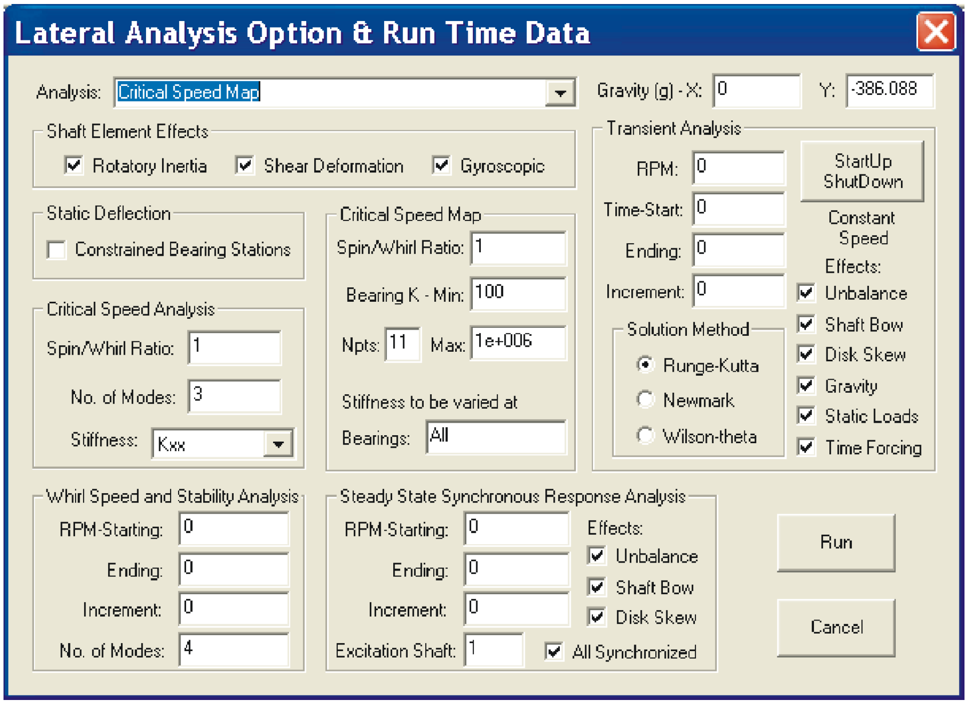
The first case is with **DYROBES** with a 5 station model in English Units set 2. In the second model, the number of stations was increased from 5 to 9 stations. There is observed only a small change in the third critical speed. This is not the case with run 4 which is the results obtained with **CRTSPD-PC**, the transfer matrix based critical speed program. The reason for this difference is that the mass modeling in **CRTSPD-PC** is lumped mass. At least 7 stations are required for accuracy with a lumped mass model in order to accurately compute the third critical speed.

In case 3, the SI metric units 4 using mm was employed. The slight differences in the various cases is due to the slight errors in translating the material properties from English to metric values. Another metric case was computed using Unit set 3 with consistent metric units with m instead of mm. This case is not shown, but the results are identical to case 3. Normally, it is undesirable to specify detailed shaft properties in terms of m. When using metric units, set 4 should be used with shaft measurements in mm. One must be careful in the proper specification of density of ρ (Kg/m^3), Youngs modulus E(N/mm^2) and bearing stiffness Kxx,yy(N/mm) when using SI unit set 4.

2.5 Critical Speed Map

A critical speed map may be generated to show the influence of the variation of bearing stiffness on the various critical speeds.

Table 2.5-1 Specification of Input Parameters For Critical Speed Map



In Table 2.5-1, under analysis options, Critical Speed Map is selected. For synchronous critical speeds, the spin/whirl ratio is selected as 1. The next step is to select the range of bearing stiffness values. For SI mm units, the bearing stiffness would be in units of N/mm. A critical speed map may be generated by the variation of one bearing at a time or all of them. In this case, ALL is selected.

Fig. 2.5-1 represents the critical speed map for the Jeffcott rotor for the first three modes. The critical speeds are plotted as a function of bearing stiffness in units of N/mm. A great deal of information may be obtained from observation of the slopes of the various modes on the critical speed map. For example, above a stiffness of K=1.0E5N/mm, all modes show little increase in the critical speed with further increase in bearing stiffness. The bearings act as rigid pinned supports. Operation of a rotor through a critical speed with such bearing conditions is dangerous as the bearings will provide no damping to attenuate the motion while passing through the critical speed. As the bearing stiffness reduces, we see that below K=1.0E3, the slope on the log-log plot is constant. This implies that the critical speed is controlled by bearing stiffness. Under these circumstances, the first mode may be balanced by a single plane and two plane balancing may be used on the second mode.

Also shown on the critical speed map are some labels of speed range and labeling of modes. This is accomplished by printing the file to a file (it will be a .bmp file) and then adding the labels with any standard graphics program. One may also plot the variation of bearing stiffness vs speed on the map. At a crossover of the bearing stiffness curve with one of the rotor modes would be the occurrence of a critical speed. Table 2.5-2 represents options for plotting and display of the critical speed map.

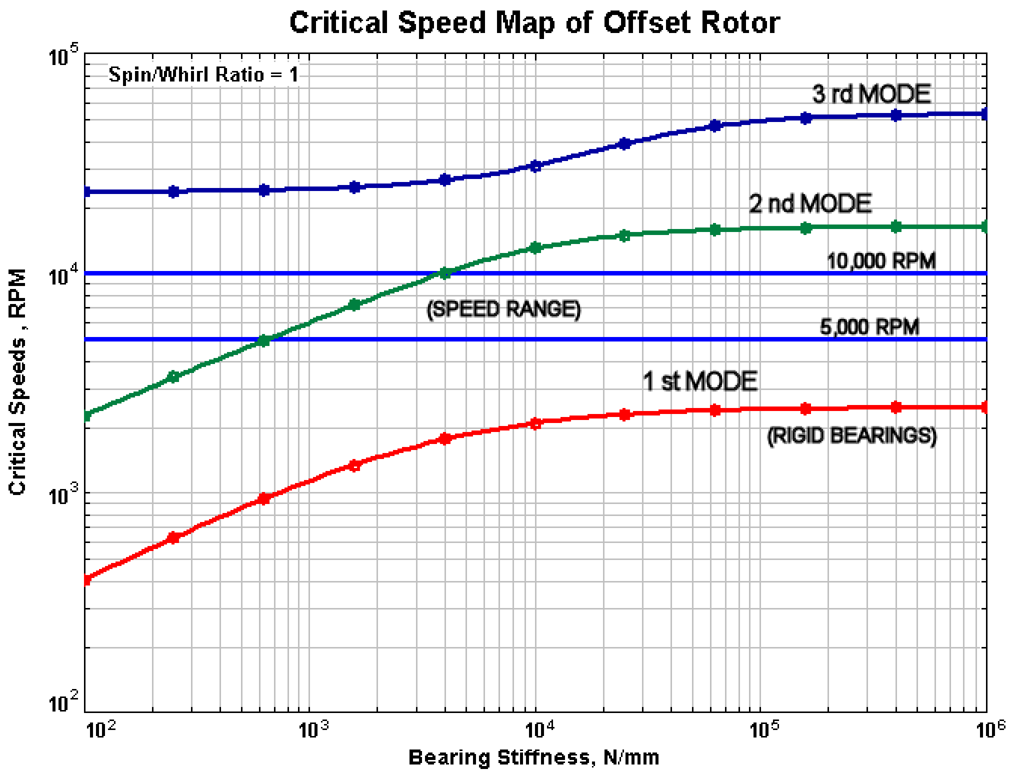


Figure 2.5-1 Critical Speed Map For Offset Jeffcott Rotor For Various Values Of Bearing Stiffness , N/mm

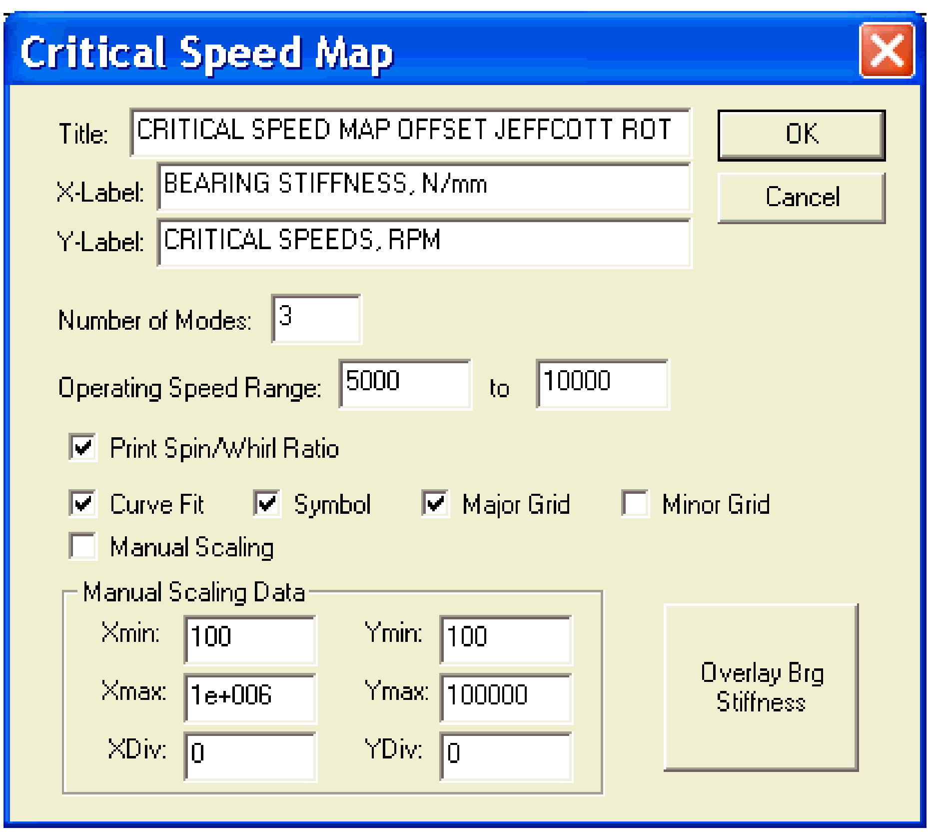


Table 2.5-2 Specifications Of Options For Critical Speed Map

2.6 Summary And Conclusions

The offset Jeffcott rotor was analyzed using both a finite element and matrix transfer program for the evaluation of the rotor critical speeds. The model was evaluated in **DYROBES** using both English and SI units. The computations were made in order to demonstrate the process of modeling a rotor in English and metric units. For those who are most comfortable in English units, the transfer to a metric formulation can be most formidable. To assist those who are working in English units, it is recommended that Units Set 2 be used. With this unit set, the units of weights and inertias are divided by gravity. The weights and moments of inertia are specified in terms of Lb and Lb-in^2 units. The material density is specified as specific weight such as 0.283 Lb/In^3 for steel.

The modeling of a rotor in metic units is best performed using Unit Set 4 which uses mm. The operation and modeling in metric SI units can be confusing since the mass unit is the Kg and the units of force is the Newton N, which is a derived quantity. For example, metric weights are measured in grams or kilograms. The unit of metric force is the Newton which is related to English Lb forces by $\mathbf{F}_{Newton} = 4.448 \times \mathbf{F}_{lb}$. This can be most confusing for those who have modeled in the earlier cgs units, for example, with bearing stiffness values expressed in such terms as Kg/cm.

A major problem for modeling in the SI unit system is the specification of the proper units for material density, Youngs modulus, and bearing stiffness and damping. By modeling a rotor model such as the offset Jeffcott rotor in both English and metric units, one can then verify that the correct units of material properties and density have been employed. Additional modeling of the offset Jeffcott model is desirable for the application of proper units of bearing stiffness and damping coefficients and unbalance for the analysis of stability (damped eigenvalues) and forced response A later presentation will review the procedure to determine the optimum bearing stiffness and damping and design of a damper bearing for the offset rotor for maximum log decrement.

A critical speed map was generated for the rotor model. From an observation of the behavior of the slopes of the critical speed plot vs bearing stiffness, one can see the stiffness range at which bearing lockup occurs. This implies that a further increase in bearing stiffness will not result in a rise in the critical speed. Operation of a rotor through speed ranges with locked up bearings(rigid) should be avoided as failure may occur. For example, a tilting pad fluid film bearing may have very high damping, but if the stiffness of the bearing causes a locked up condition, then that bearing must be redesigned. The high bearing damping will be reduced to almost zero effective damping under lockup conditions.

In the analysis presented was also a comparison between the **DYROBES** finite element program and the computations using an earlier critical speed program **CRITSPD-PC**. The transfer matrix program has been widely used by industry for over 20 years. Good agreement is seen between these two programs. It is also seen that for the same number of nodes, the finite element method has higher accuracy. There are also convergence problems with the transfer matrix method that is not encountered with the finite element method. This includes loss of modes and problems in convergence. Convergence problems were even encountered with the transfer matrix method for computing the third mode. When flexible supports are included , these problems become more pronounced. The finite element method is now the preferred method for rotor dynamics analysis.