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A Finite Length Bearing Correction Factor for Short Bearing Theory

A rapid method for calculating the general nonlinear response of finite-length plain journal and squeeze film damper bearings is presented. The method incorporates a finite-length correction factor which modifies the nonlinear forces obtained from short bearing theory. The steady-state rotational, precessive squeeze, and radial squeeze forces obtained with the correction factor compare extremely well with the forces obtained from an analytic solution of Reynolds equation using a variational approach up to L/D of 1.25 and hence covers the most commonly encountered L/D ratios. The method is no more time consuming than the short bearing analysis and is especially suited to nonlinear transient analysis of flexible rotors.

1 Introduction

The analysis of present-day rotor-bearing systems often requires the calculation of nonlinear hydrodynamic bearing and seal forces to insure the adequacy of these components in realizing the objective of acceptable machine performance. Very often the nonlinear analysis must be performed in the time domain using time-transient techniques. For example, the determination of the attitude angle of floating ring-type oil seals to obtain the correct force directions in turbo-compressors may require such an analysis [1].

Present interest in squeeze film damper bearings often requires nonlinear bearing analysis. This is primarily due to the fundamental nonlinear nature of squeeze film dampers which, unlike journal bearings, require dynamic journal motion to produce hydrodynamic forces. Thus any linearization which may be done is based on an assumption of the dynamical motion. In most instances, circular synchronous precession of the damper journal about the bearing center is assumed, but the actual motion may be quite different [2] thus requiring time-transient analysis to verify the design parameters.

In most instances, when transient nonlinear hydrodynamic journal bearing analysis is performed, the short bearing form of the Reynolds equation is used because of the time savings over the solution of the complete two-dimensional Reynolds equation. This practice is often justified because the L/D ratio of the bearing or seal being analyzed is less than 0.25. For L/D ratios greater than this, the error introduced by use of the short bearing assumption is often tolerated rather than incur the computational expense of solving the two dimensional Reynolds equation with the usual numerical techniques, i.e., finite-difference and finite-element methods [3, 4]. Analytic solutions to the two-dimensional Reynolds equation using a variational approach with a Fourier-series expansion of the pressure have been made [5–7]. Although computationally faster than the finite-difference and finite-element methods, they are still computationally expensive for nonlinear transient analyses of geometrically complex bearings. Impedance vectors [8], related to mobility vectors [9], have recently been employed to analyze finite-length plain journal and squeeze film bearings. Based on execution time data in reference [8], the impedance method is about six times faster than numerical integration of the short bearing form of the Reynolds equation used in references [10, 11]. Approximate solutions to the two-dimensional Reynolds equation have also been obtained by applying an end-leakage correction factor to the infinitely long bearing solution of Reynolds equation [12].

This present paper presents an efficient method for calculating the hydrodynamic forces for finite-length plain journal bearings, seals, and squeeze film dampers. The method consists of applying a finite-length correction factor to the pressure obtained from the short bearing form of the Reynolds equation [13]. The form of the correction factor is found from the two-dimensional Reynolds equation by assuming a separable solution of the two-dimensional pressure field. For small L/D ratios, the corrected solution reduces to the short bearing solution and justifies the assumption of a separable solution. The basic form of the pressure correction factor in reference [13] has been modified [14] and the method is herein extended to L/D ratios up to 1.25. The hydrodynamic forces found using the finite-length corrected short bearing solution are compared to those calculated using the variational analytic solution [5, 6] and the comparison is found to be quite good. If the finite-length correction factor is used in conjunction with the analytic short bearing solutions, the computer execution times for journal transient motion are approximately 2 to 3 times less than those using the impedance method. The present method is ideally suited for transient analyses of rotor-bearing systems over a very useful range of bearing L/D ratios.

2 Analysis

2.1 Correction Factor. Subject to the normal assumptions of laminar, incompressible isoviscous flow, Reynolds equation for an

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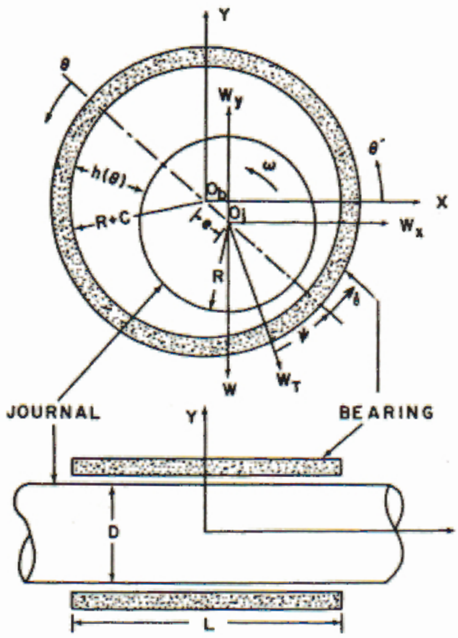

Fig. 1 Schematic of plain journal bearing

axially aligned finite-length journal bearing shown in Fig. 1 is

$$6\mu B - \frac{1}{R^2}\frac{\partial}{\partial \theta}\left[h^3 \frac{\partial p}{\partial \theta}\right] - h^3 \frac{\partial^2 p}{\partial z^2} = 0 \tag{1}$$

where

$$B = (\omega - 2\dot{\phi})\frac{\partial h}{\partial \theta} + 2\frac{\partial h}{\partial t}$$

Denoting the pressure for instantaneous values of h and $\partial h/\partial t$ by [13]

$$p(\theta, z) = f(z)p_c(\theta) \tag{2}$$

where $p_c(\theta)$ is the circumferential pressure around the axial centerline, equation (1) can be written

$$b + \left(\frac{G}{R}\right)^2 f - \frac{d^2 f}{dz^2} = 0 \tag{3}$$

where

$$b = \frac{6\mu B}{p_c h^3}$$

$$G^2 = \frac{-1}{p_c h^3}\frac{\partial}{\partial \theta}\left[h^3 \frac{\partial p_c}{\partial \theta}\right]$$

The boundary conditions for equation (3) are

$$f = 0 \text{ at } z = -L/2$$

$$f = 0 \text{ at } z = +L/2$$

and the solution to equation (3) becomes

$$f = -b\left(\frac{R}{G}\right)^2\left[1 - \frac{\cosh(Gz/R)}{\cosh(GL/D)}\right] \tag{4}$$

The pressure in the bearing is therefore

$$p(\theta, z) = \frac{-6\mu B}{h^3}\left(\frac{R}{G}\right)^2\left[1 - \frac{\cosh(Gz/R)}{\cosh(GL/D)}\right] \tag{5}$$

For small L/D (and hence z/D), the term

$$\left(\frac{R}{G}\right)^2\left[1 - \frac{\cosh(Gz/R)}{\cosh(GL/D)}\right]$$

approaches $L^2/4 - z^2$, and equation (5) reduces to

$$p(\theta, z) = \frac{-3\mu B}{h^3}\left[\frac{L^2}{4} - z^2\right] \tag{6}$$

which is the pressure obtained from the short bearing form of Reynolds equation [15].

The term

$$\left(\frac{R}{G}\right)^2\left[1 - \frac{\cosh(Gz/R)}{\cosh(GL/D)}\right]$$

in equation (5) has the function of modifying the centerline pressure obtained from the short bearing solution to Reynolds equation for finite L/D, and the remainder of the equation is proportional to the short bearing centerline pressure, i.e.,

$$\frac{\mu B}{h^3} = \frac{-4}{3L^2}p_{oc} \tag{7}$$

Equation (5) can therefore be written

$$p(\theta, z) = 2p_{oc}\left(\frac{D}{GL}\right)^2\left[1 - \frac{\cosh(G\bar{z}L/D)}{\cosh(GL/D)}\right] \tag{8}$$

where the substitution, $\bar{z} = 2z/L$, has been made. Equation (8) suggests a form of a correction factor which may be used to modify the pressure obtained from the short bearing solution of Reynolds equation for finite length bearings.

2.2 Steady-State Analysis. A solution to equation (8) can be obtained once the functional form of G is known. Observing that as $L/D \to \infty$, $df/dz \to 0$, $f \to 1$ and from equation (3), G^2 becomes

$$G^2 = g^2 = \frac{-6\mu BR^2}{p_\infty h^3}, \ L/D \gg 1 \tag{9}$$

where p_∞ is the pressure obtained from the infinitely long approximation of Reynolds equation. Under steady-state operation,

$$B = \omega\frac{\partial h}{\partial \theta}$$

$$p_\infty = 6\mu\left(\frac{R}{c}\right)^2\frac{\omega\epsilon(2 + \epsilon\cos\theta)\sin\theta}{(2 + \epsilon^2)(1 + \epsilon\cos\theta)^2}$$

and equation (9) becomes

$$g^2 = \frac{(2 + \epsilon^2)}{(1 + \epsilon\cos\theta)(2 + \epsilon\cos\theta)} \tag{10}$$

Nomenclature

- $b = 6\mu B/p_c h^3$, L^{-2}
- $B = (\omega - 2\dot{\phi})\partial h/\partial\theta + 2\partial h/\partial t$, LT^{-1}
- c = bearing radial clearance, L
- D = bearing diameter, L
- f = axial pressure function, dim
- F_r = radial bearing force, F
- F_t = tangential bearing force, F
- g, G = defined by equation (3) and equation (10), dim
- g_r, g_t = defined by equations (11), (12), (23), (24), dim
- h = bearing film thickness, L
- L = bearing length, L
- p = bearing pressure, FL^{-2}
- p_{oc} = short bearing center-line pressure, FL^{-2}
- p_r = radial pressure component, FL^{-2}
- p_t = tangential pressure component, FL^{-2}
- R = bearing radius, L
- S = Sommerfeld number, dim
- $S_s = 4S(L/D)^2$, dim
- W = journal weight, F
- z = axial coordinate, L
- $\bar{z} = 2z/L$, dim
- $\gamma_{1,2}$ = angles defined by equations (23), (24), dim
- ϵ = eccentricity ratio, dim
- $\dot{\epsilon} = d\epsilon/dt$, T^{-1}
- θ = circumferential coordinate, dim
- μ = dynamic viscosity, FTL^{-2}
- ψ = journal attitude angle, dim
- ω = journal angular velocity, T^{-1}
- $\dot{\phi}$ = journal precession rate, T^{-1}

Noting that for small L/D equation (8) reduces to equation (6) independent of the functional form of g, and that equation (10) is valid for large L/D, it is assumed here that equation (10) is reasonably valid for intermediate values of L/D. Equation (10) indicates that g is a function of both ϵ and θ. In the present form, direct substitution of g into equation (8) would present severe problems in obtaining an analytic expression for the hydrodynamic forces by integrating the pressure on the journal surface. To allow for a variation in the steady state journal attitude angle, the following radial and tangential components of g are defined

$$\bar{g}_r = g \cos\theta$$

$$\bar{g}_t = g \sin\theta$$

To eliminate explicit dependence of g_r and g_t on θ, "average" values could be defined. However, closed form integration of $\bar{g}_r$ and $\bar{g}_t$ would be difficult (if not impossible) to perform. Instead, weighted average values, g_r^2 and g_t^2, are defined which permit relatively easy closed form integration. They are

$$g_r^2 = \frac{1}{\pi}\int_0^\pi w(g\cos\theta)^2 d\theta \tag{11}$$

$$g_t^2 = \frac{1}{\pi}\int_0^\pi w(g\sin\theta)^2 d\theta \tag{12}$$

A weighting function, $w = \pi$, has been found to give best results.

The averaging is performed over the range $0 \le \theta \le \pi$ since during steady-state operation the pressure field is antisymmetric about the journal-bearing line of centers using the half-Sommerfeld cavitation condition. In terms of the components of g^2 defined by equations (11) and (12), the radial and tangential components of the pressure can be written

$$p_r = \frac{2p_{oc}\cos\theta}{g_r^2(L/D)^2}\left[1 - \frac{\cosh(g_r\bar{z}L/D)}{\cosh(g_rL/D)}\right] \tag{13}$$

$$p_t = \frac{2p_{oc}\sin\theta}{g_t^2(L/D)^2}\left[1 - \frac{\cosh(g_t\bar{z}L/D)}{\cosh(g_tL/D)}\right] \tag{14}$$

The evaluation of g_r^2 and g_t^2 results in

$$g_r^2 = \frac{\pi(2+\epsilon^2)}{\epsilon^2}\left[1 + \frac{1}{(1-\epsilon^2)^{1/2}} - \frac{4}{(4-\epsilon^2)^{1/2}}\right] \tag{15}$$

$$g_t^2 = \frac{\pi(2+\epsilon^2)}{2}\left[\frac{1}{(1-\epsilon^2)^{1/2}} - \frac{1}{(4-\epsilon^2)^{1/2}}\right] \tag{16}$$

The radial and tangential hydrodynamic force components are found by integrating the pressure components over the positive pressure region, i.e.,

$$F_r = \frac{LR}{2}\int_{-1}^{1}\int_0^\pi p_r d\theta d\bar{z} \tag{17}$$

$$F_t = \frac{LR}{2}\int_{-1}^{1}\int_0^\pi p_t d\theta d\bar{z} \tag{18}$$

which results in

$$F_r = \frac{-3\mu\omega LD^3\epsilon^2}{2c^2g_r^2(1-\epsilon^2)^2}\left[1 - \left(\frac{D}{g_rL}\right)\tanh(g_rL/D)\right] \tag{19}$$

$$F_t = \frac{3\mu\omega LD^3\pi\epsilon}{8c^2g_t^2(1-\epsilon^2)^{3/2}}\left[1 - \left(\frac{D}{g_tL}\right)\tanh(g_tL/D)\right] \tag{20}$$

The steady-state operating eccentricity ratio and attitude angle become

$$S = \frac{1}{24\pi}\left\{\left[\frac{\epsilon^2}{2g_r^2(1-\epsilon^2)^2}\right]^2\left[1 - \left(\frac{D}{g_rL}\right)\tanh(g_rL/D)\right]^2 + \left[\frac{\pi\epsilon}{8g_t^2(1-\epsilon^2)^{3/2}}\right]^2\left[1 - \left(\frac{D}{g_tL}\right)\tanh(g_tL/D)\right]^2\right\}^{-1/2} \tag{21}$$

$$\tan\psi = \left[\left(\frac{g_r}{g_t}\right)^2\frac{\pi(1-\epsilon^2)^{1/2}}{4\epsilon}\right]\left[\frac{1 - \left(\frac{D}{g_tL}\right)\tanh(g_tL/D)}{1 - \left(\frac{D}{g_rL}\right)\tanh(g_rL/D)}\right] \tag{22}$$

where the Sommerfeld number, S, is defined as

$$S = \frac{\mu\omega LD}{2\pi W}\left(\frac{R}{c}\right)^2$$

2.3 Radial Journal Motion. The preceding section describes the application of the finite-length correction factor to steady-state motion. It is also applicable to steady or instantaneous values precession of the journal (nonzero $\dot{\phi}$) without modification. Nonzero radial journal motion can be handled in a completely analogous manner, although the functions g_r^2 and g_t^2 are different from those used for zero radial motion. By using the infinitely long bearing pressure for radial motion in equation (9) and integrating over the positive pressure region, the functions g_r^2 and g_t^2 become

$$g_r^2 = \frac{2}{\epsilon^2}\left[\pi + \frac{2(\pi-\gamma_1)}{(1-\epsilon^2)^{1/2}} - \frac{8(\pi-\gamma_2)}{4-\epsilon^2)^{1/2}}\right] \tag{23}$$

$$g_t^2 = 2\left[\frac{(\pi-\gamma_1)}{(1-\epsilon^2)^{1/2}} - \frac{(\pi-\gamma_2)}{(4-\epsilon^2)^{1/2}}\right] \tag{24}$$

where

$$\gamma_1 = \tan^{-1}\left[\frac{(1-\epsilon^2)^{1/2}}{\epsilon}\right]$$

$$\gamma_2 = \tan^{-1}\left[\frac{(4-\epsilon^2)^{1/2}}{\epsilon}\right]$$

2.4 General Dynamic Motion. To calculate the general dynamic motion of a journal within its bearing, the forces due to instantaneous values of journal position and velocity are obtained by superposing the pressures due to rotation-precession and radial squeeze motion and integrating over the resulting net positive pressure region. The instantaneous forces are then used in an appropriate dynamical integration routine to predict a subsequent position and velocity. The instantaneous forces are

$$F_r = \frac{D^3}{L}\left[\frac{1}{g_{r1}^2}\left\{1 - \left(\frac{D}{g_{r1}L}\right)\tanh(g_{r1}L/D)\right\}\int_{\theta_1}^{\theta_2} p_{oc1}\cos\theta d\theta + \frac{1}{g_{r2}^2}\left\{1 - \left(\frac{D}{g_{r2}L}\right)\tanh(g_{r2}L/D)\right\}\int_{\theta_1}^{\theta_2} p_{oc2}\cos\theta d\theta\right] \tag{25}$$

$$F_t = \frac{D^3}{L}\left[\frac{1}{g_{t1}^2}\left\{1 - \left(\frac{D}{g_{t1}L}\right)\tanh(g_{t1}L/D)\right\}\int_{\theta_1}^{\theta_2} p_{oc1}\sin\theta d\theta + \frac{1}{g_{t2}^2}\left\{1 - \left(\frac{D}{g_{t2}L}\right)\tanh(g_{t2}L/D)\right\}\int_{\theta_1}^{\theta_2} p_{oc2}\sin\theta d\theta\right] \tag{26}$$

where p_{oc1} and p_{oc2} are the center-line pressures obtained from short bearing theory for rotation-precession and radial squeeze motion respectively and where g_{r1}, g_{t1}, g_{r2}, and g_{t2} are defined by equations (15), (16), (23), and (24), respectively. The angles θ_1 and θ_2 define the region of positive pressure and are dependent on the journal motion and cavitation assumptions.

3 Results

To evaluate the accuracy of the finite-length correction factor, a comparison of the steady-state and radial squeeze forces obtained using the correction factor was made with the forces obtained from an analytic solution of finite-length journal bearings (5, 6). Figs. 2 through 4 show the results of the steady-state force comparison for L/D ratios of 0.5, 1.0, and 1.25. The steady-state operating eccentricity is plotted as a function of the short bearing Sommerfeld number, S_s. It is observed that the results with the finite-length correction factor provide a very close approximation to the analytic results even at L/D = 1.25. The correlation is best in the middle range of eccentricity ratios ($0.2 \le \epsilon \le 0.8$) and hence covers the range most useful in transient analyses. It is also noted that even for L/D = 0.5 the uncorrected short bearing results are significantly in error, particularly at high eccentricity ratios. A comparison of attitude angle is shown in Fig. 5.

Figs. 6 through 8 compare the radial forces obtained for pure radial squeeze motion using corrected short bearing theory with those ob-

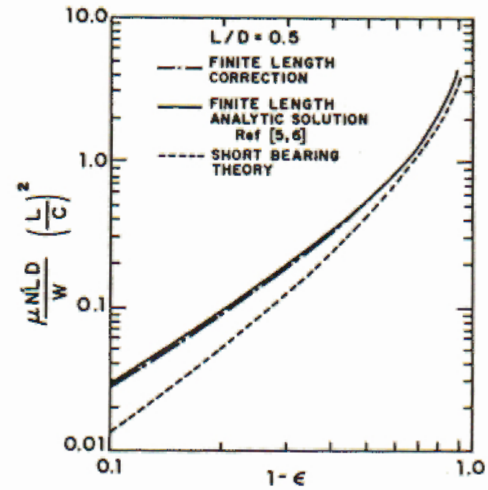

Fig. 2 Comparison of steady state load capacities for L/D = 0.5

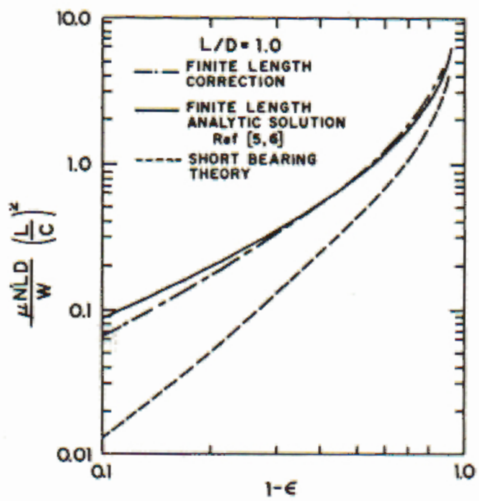

Fig. 3 Comparison of steady state load capacities for L/D = 1.0

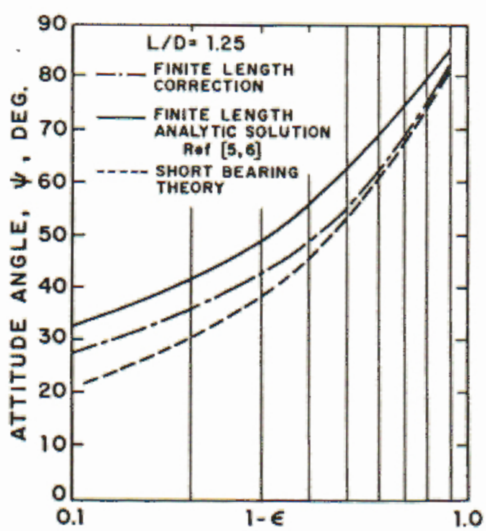

Fig. 4 Comparison of steady state load capacities for L/D = 1.25

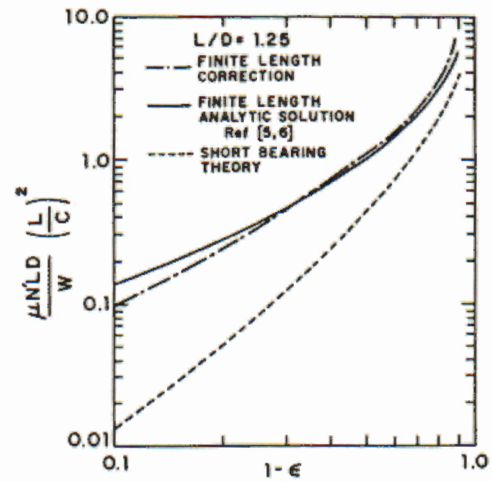

Fig. 5 Comparison of steady state attitude angles for L/D = 1.25

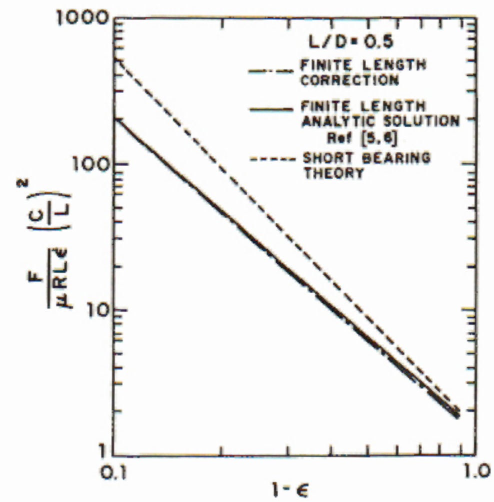

Fig. 6 Comparison of radial squeeze forces for L/D = 0.5

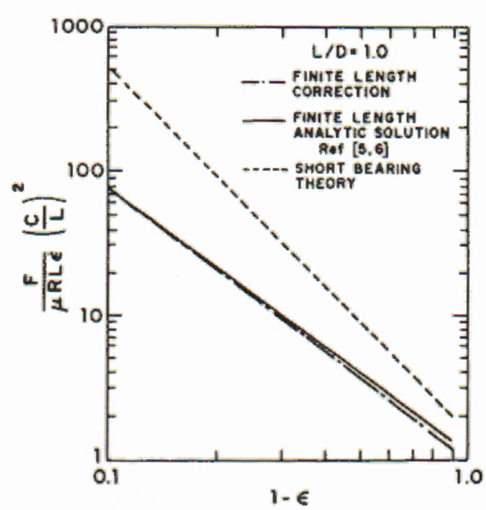

Fig. 7 Comparison of radial squeeze forces for L/D = 1.0

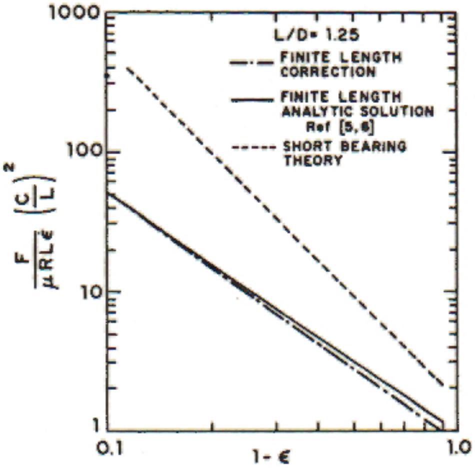

Fig. 8 Comparison of radial squeeze forces for L/D = 1.25

tained analytically for finite length bearings and by uncorrected short bearing theory. Again, the finite-length correction factor compares very favorably with the analytic solution over a wide range of eccentricity and L/D ratios. For $L/D \leq 0.25$, the results for both steady-state and radial squeeze motion using the three methods are nearly identical, as is expected, and are not shown here.

The finite length correction factor described herein does not result in any appreciable increase in the computational time required for short bearing analysis. The effect of increased L/D ratios in plain journal bearings, seals, and squeeze film damper bearings can, therefore, be efficiently treated in rotor-dynamic transient analysis.

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