

Review: Rotor balancing

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This article reviews the literature concerning the balancing of rotors including the origins of various balancing techniques including ones that use influence coefficient, modal, unified, no phase, and no amplitude methods to balance. This survey covers the computational algorithms as well as the physical concepts involved in balancing rotating equipment.

Keywords: Balancing, rotor, vibration, influence coefficient, exact point balance, least squares, linear programming, modal, unified

1. Introduction

Balancing a rotating machine is a vital step to ensure that the machine will operate reliably. This balancing generally consists of the addition or removal of small amounts of weight at various axial locations and angular positions along the rotor that contribute rotating forces to the system. This process requires both time and money; the longer the process takes the more costly the operation for critical path machinery, because the machinery is generally out of service for part or all of the balancing effort.

Various aspects relating to efficient balance strategies concern the size of the balance correction weights, modal type weights, the optimal balance planes, angular positions, and the number of balance planes required. Additionally, the balancer may need to balance for vibration on just one machine or bearing in a multi-bearing machinery train, such as a generator, but not have the use of the balance planes in that machine because of the difficulty involved in their use. For instance in the case of a hydrogen cooled generator, the generator hydrogen must be purged before installing a balance weight, and before running the unit under load

the system has to be tested for leaks and refilled with hydrogen – a time consuming and expensive exercise; a balance shot can take from one day to over two days.

2. Literature

A. Föppl (1895) formulated and solved the equations governing the response of a single mass undamped rotor system [35]. His analysis showed that at speeds significantly higher than the critical speed the rotor would turn about its mass center; his undamped analysis predicted infinite response at the critical speed and a transient response at the critical speed frequency. H.H. Jeffcott [63] analyzed the fundamental nature of the response of a single mass flexible rotor to imbalance in 1919. Proper instrumentation to experimentally verify these results would not exist for years. Rieger [123] references electronic and stroboscopic measurements first developed in the 1930's.

Even before Jeffcott explained the fundamental response of rotor systems, balancing machines were in use, Rieger [123]; Martinson developed a balancing machine as early as 1870. On this balance machine the 'heavy' spot was marked by hand. At low speeds (sub-critical) the 'heavy' spot would coincide with the 'high' spot of the whirl according to Jeffcott's analysis.

Early balancers such as S.H. Weaver [159] in 1928 realized that the balance weights and imbalances act as forces to the system. Weaver was aware that the forces at the bearing locations for a rigid rotor changed with the magnitude and phase of the imbalance weights. Later balancers would develop the notion of influence coefficients, though at first they would not use this name.

2.1. Influence coefficient methods

T.C. Rathbone [120], an experimental engineer in the Large Turbine Division of Westinghouse at the South Philadelphia Works, used a *strobe-vibroscope* to study the vibration of a large rotor (50 tons). The *strobe-vibroscope* used a microscope, reflective foil mounted to the bearing housing, and a strobe light. The

strobe activated a neon flash which caused a pink spot to appear on the observed Lissajous figure indicating the angular orientation. A second strobe, synchronized to the first, illuminated an exposed portion of the shaft which had been painted to give an angular reference. In a series of experiments Rathbone applied various size balance weights that showed the amplitude responded linearly; he also placed a trial weight at different angular locations with the vibration amplitude remaining the same but the vibration phase shifting by the same amount as the changes in the trial weight angle. These experiments also found a difference between the housing and shaft vibration.

Rathbone [120] in 1929 described a balancing method that used *unit motions*, similar to influence coefficients for orbits, to reduce the vibration amplitudes at each end of a machine. The technique used linear superposition to simultaneously reduce the elliptical vibration pattern at both ends of the rotor. The *unit motions* were the elliptical motion that resulted from a known (unit) imbalance; the ellipses were referenced at a constant shaft rotation throughout the procedure. The author stated that J.P. Den Hartog, who worked for Westinghouse at this time, had an analytic solution to this problem.

Rathbone showed that the ellipse method produced two solutions, and he reasoned that data from only two vibration directions were needed, either the vertical or horizontal vibration would suffice. Using a shaft reference system, Rathbone expressed the vibration in one direction as an amplitude and phase, a vector.

He then used known calibration weights to derive the rotating “vectors representing the influence of the unit motions alone,” which we now know as the influence coefficients (vectors); these vectors could be denoted by an amplitude and phase. When using the vertical motion, phase would be computed by stroboscopic determination of the angular rotation of a mark on the shaft from the vertical plane at the moment the vibration (motion) reached its peak. This phase convention would correspond to a *lag angle* in today’s terminology. The amplitude of the vector would be the magnitude of the vibration (presumed to be mostly at $1 \times$ rotation).

Rathbone then used an iterative graphical technique to reduce the vibration at both ends of the rotor. However, he knew that his solution involved the solution of linear equations, and although he did not present the results, he stated that the mathematical solution which was “quite involved” had been solved by an undergraduate at the University of Copenhagen named Nils O. Myklestad.

Not only does a linear solution for the two plane balance problem as stated in [120] represent a two plane exact point balance, but Rathbone also showed how to apply the method to multi-bearing rotors. In particular he examined a four bearing system, such as a turbine-generator set. He stated that an initial run and four trial runs were required for a balance, and he gave an example of this iterative solution for the four planes. One of the article’s discussers, M. Stone, acknowledged that Rathbone had the logistics for multi-plane balancing; however, he (Stone) doubted the practicality of obtaining a correct balance solution.

E.L. Thearle [148] of the General Electric Company presented a two plane semi-graphical balancing procedure based on a linear rotor system. Thearle’s included an analytical solution compared to Rathbone’s iterative solution [120]. This technique comprised what we would now call a two point exact point balance; the balance computation included one speed and two vibration sensors. For many years, until computing devices progressed, a two plane balance computation would be the practical limit for most field balancing. J.G. Baker [8] and a later discussion of this paper by T.C. Rathbone [121] generalized Thearle’s work. Baker suggested using groups of trial weights which affect the vibration at only one bearing (at the one speed) at a time. Baker explored the use of this on machinery involving both two and three bearings; with such a technique one could balance in more than two planes using essentially a single plane balance computation.

In 1932 K.R. Hopkirk [51]¹ stated the defining principle for what we call influence coefficients. In Hopkirk’s own words

Thus, if the [mass] eccentricity is represented by the vector a , referred to a direction fixed in the moving mass and serving as the positive real axis, the displacement of each support may be represented by

$$u = A_1 a$$

where A_1 is a vector or a complex number The support displacement is thus represented as a complex number referred to a direction fixed in the rotor. If the system in motion includes several masses, each eccentrically mounted mass will give

¹Translated from French by L. Kitis, a faculty member in the Mechanical Engineering Department at the University of Virginia.

rise to a similar support displacement, such that the total displacement will be represented by

$$u = A_1a + B_1b + \dots = \sum_i A_i a$$

each term corresponding to a mass in motion.

The above formulates the balancing problem for multiple weight additions, although no runout or noise is present in these equations. Hopkirk also states the problem for two measurement locations, u_1 and u_2 .

Hopkirk goes on to state,

The equation given above for the displacement of a point, whose terms are functions of unbalanced forces, is the basis of all methods of balancing, and its solution depends on whether it is possible to determine the vector u . The vector u is completely determined by its magnitude (modulus) and its phase with respect to a point fixed in the moving part. The magnitude can be measured with sufficient accuracy by means of one of a number of instruments used today [1930's] for this purpose. The measurement of the phase, if it is to be accurate, requires a stroboscopic method; the various instruments used have not become standard like those intended only for measuring the magnitude. It is not necessary to know the vibration phase to obtain a solution, although the work involved would be considerably easier.

In the above method Hopkirk would have computed a vibration to cancel the existing vibration. He follows by solving the problem using just the magnitudes of the vibration.

K.R. Hopkirk [52] formulated the two plane balance using influence coefficients in the same manner as more modern treatments; he called the influence coefficients transmission constants and used vector notation to denote these coefficients. He presented an analytical solution in terms of vectors (complex valued quantities).

T.P. Goodman [42] substantially improved balancing technology in 1964 when he introduced the least squares balancing procedure. This method used data from multiple speeds and measurement locations, more measurements than balance planes, to minimize a weighted sum of the squares of the residual vibration. The technique sought a weighting scheme that would minimize the maximum residual vibration through an iteration of the weighting used for the least squares optimization.

In various combinations the authors A.G. Parkinson, M.S. Darlow, A.J. Smalley, and R.H. Badgley [21,23,24,113] have explored what they call unified balancing. This approach uses influence coefficients to compute modal trial weight sets that have little or no effect on modes of vibration that have already been balanced. Balancing the next mode is equivalent computationally to a single plane balance with the balance weights applied in more than one plane simultaneously.

2.2. Generating influence coefficients

Most authors and balancers who use an influence coefficient method, starting with Rathbone [120] and Thearle [148], have used a reference run with no balance weights and a calibration or trial run with a balance weight or weight set attached to the rotor to generate the influence coefficients. For actual experimental rotor balancing, experimentally generated influence coefficients are preferred.

J. Lund and J. Tonnesen [93] used two trial weight runs for each balance plane in order to identify and reduce measurement errors. This technique would be most useful in the laboratory, since it requires extra trial weight runs that would delay any balancing.

Jeffrey V. LeGrow [86] presented a technique to generate the influence coefficients for an actual rotor using a computer model. The advantage in time and cost of such an approach could be substantial; however, this method could not balance the test rotor adequately. LeGrow reported that further tests were being conducted which showed some promise.

A.H. Church and R. Plunkett (1961) [18] used a mobility (modal) method to generate influence coefficients without trial weights. Church and Plunkett excited the non-rotating shaft with a shaker, and they tested this theory on a very flexible shaft, whose first three resonances occurred at 550 cpm, 2000 cpm, and 4180 cpm, mounted in stiff ball bearings. While quite efficient in theory this method did not produce good results, and furthermore, one could have significant difficulties in applying this method to an actual machine, as is pointed out in the discussion by Josef K. Sevcik. J. Tonnesen and J. Lund [151] used impact excitation to determine the influence coefficients. This method has some limitations as well as sources of error and would not be practical for field balancing.

Lars-Ove Larsson [82] generated influence coefficients using a statistical technique. The imbalance runs corresponded to statistical trials, and the influence coefficients assumed the role of the regression co-

efficients. This approach allows one to ‘average’ the data when constructing the influence coefficient matrix; also it would enable one to update the influence coefficients upon the additional of balance weights to a machine.

2.3. Modal balancing

Modal balancing generally assumes that the rotor system has planar modes of vibration. Balancing one mode should not effect any other mode; although higher modes that are not being considered may be adversely effected. Often the procedure can be quite efficient. There are variations on this theme.

K.R. Hopkirk [51] wrote of the first two modes being effected by

two forces can be expressed as a static unbalanced force P and a couple T The critical speed is produced by grouping linear combinations such as

$$K_1P + K_2T$$

Hopkirk [51] illustrated what happens to a two mode rotor with imbalance at one end. He showed the excitation of the first critical, and its traversing of a 180° phase change due to the natural frequency. Next he showed correctly that the end with the imbalance has the lowest (he showed zero) vibration between the first and second modes; an idea used in polar plot balancing. Also, Hopkirk shows super critical operation above the second mode. This seems rather advanced for the early 1930’s considering the limitations of the available instrumentation.

In 1953 L.P. Grobel [43] used so-called static, weights in-phase at either end of the shaft, and couple (or dynamic) balance weights, weights placed at each end but at 180° phase angle with respect to each other. These balance weight combinations were used to successfully balance the rotor one mode at a time by progressing from the lower modes.

J.R. Lindsey [88] used static and couple balance weights combined with a sensitivity factor and a *high spot number*. The modal component of the vibration is determined by graphical means using vibration data from each end of a machine in one plane. The first mode (one loop) vibration is taken to be the vector average of the two vibrations, and the second mode component is taken to be $1/2$ the vector difference of the two vibration components. (Three modes can also be accommodated.) Sensitivity factors relate to the mag-

nitude of either the first or second mode components of vibration calculated as previously stated.

The *high spot number* relates to the phase angle. Sensitivities and high spot numbers have been developed over time for a variety of machines. The strength of this technique derives from using this historical data, and the method is most often used as a one shot balance method. General Electric Company’s field engineers use this technique, as do many utilities to balance their turbine generators. From 1960 until the presentation of the paper [88], Lindsey said that this technique had been used to balance more than one hundred rotors.

The desire with this method is to arrive at an adequate balance – usually not the best achievable balance – in an efficient manner; the technique provided good results with turbine generator units consisting of multiple rotors with long spans. A weakness of the method involves its lack of concern with the cross effect of a static weight on the couple vibration and the effect of a couple weight on the static component of the vibration. Lindsey stated, “When extensive coupling exists among unbalances in several rotors, the methods errs significantly.” He also remarked on difficulties the approach had in distinguishing between one and three-loop modes, the first and second critical speeds.

R.E.D. Bishop [11] in 1959 formulated equations for the displacement amplitudes of a circular rotor with distributed mass and elasticity. His solution for a rotor’s vibration looks like a power series of Jeffcott rotors. Also in 1959, R.E.D. Bishop and G.M.L. Gladwell [12] introduce what is generally thought of as modal balancing. In this paper they show the inadequateness of low speed balancing (rigid rotor balancing) for high speed flexible rotors, analyze the effects of shaft bow, and study the effects of the rotor’s weight. They applied the analysis in reference [11] to a uniform shaft through two modes.

Gladwell and Bishop (1959) [40] used the results of Bishop’s [11] paper to analyze an axisymmetric shaft of non-uniform diameter along its length. Bishop and Gladwell discuss forced and free vibration as well as methods to determine the natural frequencies and characteristic modal functions. R.E.D. Bishop and A.G. Parkinson [14] investigated problems associated with closely spaced critical speeds or when a substantial modal component from one mode affects another. Bishop and Parkinson show two methods of adapting C.C. Kennedy and D.D.P. Panco’s [73] method for resonance testing to modal balancing.

A.G. Parkinson, K.L. Jackson, and R.E.D. Bishop (1963) in two papers [114,115] further discussed the

theory of modal balancing and tested this theory on several thin shafts. A good discussion of modal balancing a bowed rotor and a bowed rotor with mass imbalance is presented in the first paper. The modal method was effective on the test rotors which had an initial bend. A.L.G. Lindley and R.E.D. Bishop [87] discuss the application of modal balancing to large steam turbines in the range of 120–500 MW where the bearing stiffness is large compared to the shaft stiffness. Their experience is complimented by the work of several other balancers. A.G. Parkinson and R.E.D. Bishop in 1965 [112] consider the residual vibration due to higher modes after modal balancing.

Parkinson summarized much of the work on modal balancing and forced response from a modal view in a 1967 reference [109].

M.S. Hundal and R.J. Harker [54] presented a method similar to those described above. They showed that one could balance several modes simultaneously as well as just one mode at a time.

2.4. Balancing using amplitude only

In the early days of vibration measurements, it was difficult to obtain the phase of the $1\times$ vibration accurately. Although the vibration should be filtered to $1\times$ rotation often the overall vibration amplitude was used; sometimes it was measured using a mechanical indicator, vibrometer (see Rathbone [120] for an example). Techniques were developed to balance using only the amplitude of the vibration; this practice continues even today. G.B. Karelitz [68], in the Research Department of Westinghouse Electric & Manufacturing Company, used a three trial weight to balance turbine generators. This graphical technique used an *unbalance finder* to locate the mass imbalance; the *unbalance finder* consisted of four transparent strips held together with a pivot at one end. The method could be used with trial weights of unequal magnitudes.

F. Ribary [122] presented a graphical construction that balanced using only the amplitude taken from an initial run and three trial weight runs. I.J. Somerville [141] considerably simplified the graphical construction of Ribary [122]. Somerville's construction is also known as the four circle method of balancing without phase. The four circle method, as it is generally used now, can be found in C. Jackson [62].

Balancing with only amplitude has had several extensions. K.R. Hopkirk [52] presented an analytical solution for using only amplitude to perform a two plane exact-point balance; this technique took seven runs.

L.E. Barrett, D.F. Li, and E.J. Gunter [9] adapted the technique to balance a rotor through two modes using modal balance weights; E.J. Gunter, H. Springer, and R.R. Humphris [46] used modal balancing without phase to balance a rotor through three modes. Two plane balancing using only the amplitude was also done independently by L.J. Everett [30] who appeared to be unaware of the earlier publications.

The number of runs required for a balance using amplitude only makes this method inherently less efficient than an equivalent influence coefficient method. Furthermore, after completing such a balance one has no information that would help to trim balance or in the future perform one-shot balancing. A trim balance requires another four runs.

2.5. Balancing using phase only

Phase data may be obtained by directly marking the shaft as was done on some of the early balancing machines from the 1800's as described by N.F. Rieger [123] or F. Ribary [122]. C. Jackson [61, 62] described methods of obtaining phase using a pencil to mark the shaft and using orbit (Lissajous) analysis; Jackson then incorporated the physics of the rotor, whether it is above, below, or near a critical speed, to balance. This technique could require some iterations to find a solution depending upon the knowledge and experience of the balance practitioner.

K.R. Hopkirk [52] derived a technique for two plane balancing using only phase information. Hopkirk's method comprises a two plane exact-point balance, and the procedure required five trial runs including the initial one. I.J. Somerville [141] presented a graphical means to solve for unbalance on a disc (single plane) using only the phase information.

W.C. Foiles and D.E. Bently [34] found both analytical and graphical solutions for single-plane and multi-plane balancing using only phase information; their solution used a type of influence coefficient applicable to balancing using this partial information. Whereas single plane balancing without phase requires three trial weights; this technique uses just two trial weight runs. Methods were developed for both single plane and multi-plane balancing. The Foiles and Bently paper [34] allowed for trial weights of different magnitudes. This paper presented both analytical and graphical solutions for a single plane balance (or the influence coefficients for a multi-plane balance), and the authors applied the technique to a cooling tower fan. Somer-

vaille's [141] graphical technique for single plane balancing was a better graphical method.

Similar to the techniques that use only amplitude, these methods require additional balance runs compared to influence coefficient methods that use full information, both amplitude and phase. Also, after a balance, one has no useful residual data for trim balancing or one-shot balancing in future efforts. These deficiencies result in serious inefficiencies for the techniques that use only partial information in balancing, either only amplitude or only phase.

2.6. Linear programming techniques

R.M. Little's dissertation [90] and R.M. Little and W.D. Pilkey [90] described a linear programming method of balancing that enables one to place constraints on the magnitude of the balance weights; however, this technique requires at least as many balance weights as measurement observations which is in general not possible. Little's first balance computation (analytical model) resulted in an unbounded solution when he used eight balance planes; he bounded the magnitudes of the balance weights to produce a solution. M.S. Darlow [23] discussed the problem of redundant balance planes when more than the required number of balance planes are used. Often large balance weights will be computed because the influence matrix is ill-conditioned. This occurs, because the columns of the influence coefficient matrix are or are nearly linearly dependent.

W.D. Pilkey and J.T. Bailey [118] corrected the deficiencies of the previous linear programming approach by using a different formulation for the problem. Pilkey and Bailey separated their techniques into time independent and time dependent algorithms. The techniques investigated were the following:

1. *Linear Sum*. Minimize the sum of all the computed residual measurements (absolute value of the residuals).
2. *Min-max*. Minimize the maximum residual measurement.
3. *Least Squares*. Minimize the sum of the squares of all residual measurements including constraints on the magnitudes of the corrective balance weights. This leads to a quadratic program.

The time independent techniques only view the response with the shaft in its 0° position for balance weights placed on the x and the y axes, because the linear programming techniques use real valued influence

coefficients. The time dependent techniques are similar to the above with the inclusion of constraints relating to other orientations of the shaft or equivalently the balance weights with the shaft in its initial position.

E. Woomer and W. Pilkey [163] explored a quadratic formulation to the balancing problem. They use a shift for the inequalities on the balance weights. This shift guarantees the positivity of the new variables so that quadratic programming techniques can be used directly.

2.7. Number of planes required to balance

K.R. Hopkirk begins his 1940 article [52] with the following statement,

... in order to balance a cylindrical type rotor, two planes spaced apart axially are necessary and sufficient for the addition of correcting weights. This statement is incorrect only when applied to machines running above the second critical speed.

This agrees more-or-less with the later ideas from modal balancing as established by Bishop and Gladwell [12] as well as others.

J.P. Den Hartog [25] stated the following:

Theorem. *A rotor consisting of a straight, weightless shaft with N concentrated masses along its length, supported in B bearings along its length, and afflicted with an arbitrary unbalance distribution along the shaft (not restricted to the location of the concentrated masses) can be perfectly balanced at all speeds by placing appropriate small correction weights in $N + B$ different planes along the length of the shaft. In case the location of a mass happens to coincide with that of a bearing, only one of the two is to be counted.*

Additionally, Den Hartog extended his result to more realistic rotors as: "nearly perfect balance at all speeds can be obtained by balancing in $N + B$ planes where N now means the number of rotor critical speeds in the speed range from zero to four times the maximum service speed of the machine." He also stated that such a balance would be independent of the bearing support parameters.

W. Kellenberger [71] believed that a flexible rotor with two bearings requires $N + 2$ balance planes where N is the number of modes. The rotor was balanced initially as a rigid rotor, and then modal balance weight sets were computed that were orthogonal to the rigid body modes. This orthogonality condition constrains the nature of the balance weights and thus requires two

additional balance planes. One should note that with this technique only N linearly independent balance weight sets are used after the initial rigid rotor balance.

R.E.D. Bishop and A.G. Parkinson in a discussion to Kellenberger's paper [71] pointed out that the higher modes cause the residual vibration and neither their N plane method nor Kellenberger's $N + 2$ plane method can guarantee the effect on these modes "*for better or worse.*" To this aim they present a simple example where the N plane method works better than the $N + 2$ plane method; however, their point is that one can not tell how the higher modes will be affected.

Also, Bishop and Parkinson agreed with Kellenberger that his technique complicates the procedure. One can see that this complication restricts the locations of weight placement which in general results in weights that have less effect (per amount of weight) on the mode being balanced. Also, not mentioned is the fact that for a rotor with flexible bearings, like a real rotor, the deflection at the bearings is proportional to the force there. Hence the force at the bearings would be linearly dependent (given the assumptions used) on the modal displacements; so, balancing the modes also would result in balancing the forces at the bearings. Only when the bearings take the form of a rigid constraint (i.e., rigid bearings) could the modes be balanced and non-zero forces occur at the bearing locations.

H.F. Black and S.M. Nuttal [15] investigated the unbalance response of rotors with non-conservative cross coupling such as results from hydrodynamic bearings. They claim that it takes $2N$ balance planes to balance a rotor with N modes. They do point out that this may result in ill conditioned balancing equations. They show that the modes in such a rotor would not be real (normal modes) but would be complex; so planar modes would not exist. Also, they mention that the eigenvalues and mode shapes depend upon the speed of the rotor – this is not accounted for by the modal methods.

2.8. Bowed rotor

A.G. Parkinson, K.L. Jackson, and R.E.D. Bishop [114,115] experimented with the effects of shaft bow on modal balancing. J.C. Nicholas, E.J. Gunter, and P.E. Allaire [104,105] analyzed the response and balancing of a single mass rotor with shaft bow. In [105] they examined three methods of balancing a bowed rotor. Method I reduced the shaft deflection to zero at the balance speed, method II minimized the elastic shaft deflection (not including shaft bow) at the balance

speed, and method III balanced the shaft to zero total shaft deflection at the critical speed. W.C. Foiles [33] balanced an experimental rotor that had a shaft bow using conventional balance weights and demonstrated that an analytic model of a rotor with shaft bow could be balanced through three modes by using either conventional or modal balance weights. Foiles used a multiple speed procedure instead of the single speed procedure of Nicholas, Gunter, and Allaire.

2.9. Redundant balancing planes

M.S. Darlow [23] provided information on an important problem in balancing that results in ill-conditioning of the balance equations. When columns, corresponding to the balance planes, of the influence coefficient matrix form a linear (or nearly) dependent set of vectors very large correction weights can be computed. The majority of the effects of these weights cancel each other; this results from the ill-conditioning of the influence coefficient matrix. Darlow showed with the aid of examples that the problem can be solved by using fewer balance planes. In reference [23] Darlow gave an algorithm to compute the balance planes that should be used.

2.10. Other methods

G.A. Hassan [48] presented a statistical approach to balancing that used regression analysis to reduce the vibration amplitude (dependent variable) at one or two bearings using the balance weight magnitude and angular location as the independent variables. This approach uses a number of balance runs and has applications to rotors that have a non-linear response. It seems that it would be more efficient to use a linear model and iterate for a rotor system that exhibited response nonlinearities and measurement uncertainties; most rotor systems are more linear than not.

Y. Kang, C.P. Liu, and G.J. Sheen [65] derived a method to balance non-symmetric rotors; the asymmetries include both mass and stiffness properties. The method requires two trial weights per balance plane and uses the forward precession component of the rotor's response. Y. Kang and G.J. Sheen and S.N. Wang [67] formulated a unified balancing algorithm for non-symmetric rotor systems.

3. Current balancing practices

Today, most field balancing uses either an exact point procedure or a least squares error method to compute the correction weights. Trial weight sets may consist of more than one weight, such as modal (or unified) balance weight sets or 'static' and 'couple' balance weights. Balancing using partial information (just the amplitude or phase of the vibration) is also practiced; although this is usually on machines which one can easily attach balance weights. Agreeing upon a goal for the balance helps to establish a stopping point; one need not necessarily balance a rotor to the lowest achievable levels in order to have a satisfactory balance.

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